Expressing Measurement Uncertainty in Software Models

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http://atenea.lcc.uma.es/index.php/Main_Page/Resources/DataUncertainty

Uncertainty [Merriam-Webster]

Definition of UNCERTAIN

- not exactly known or decided : not definite or fixed *(underspecified)*
- not sure : having some doubt about something *(unprecise)*
- not definitely known *(unknown)*

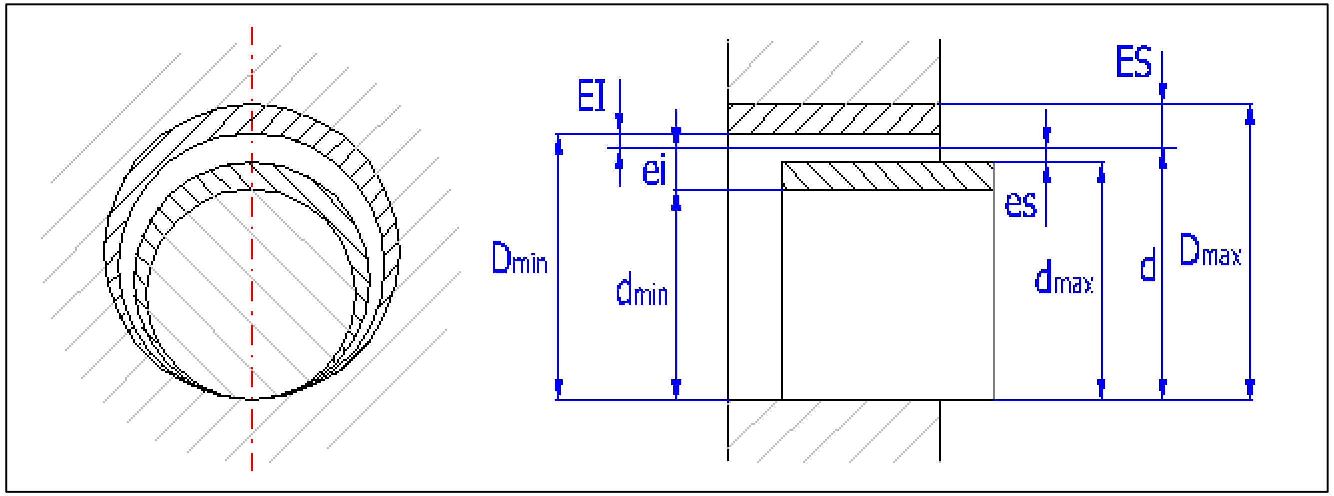
Uncertainty

- **Uncertainty**: Quality or state that involves imperfect and/or unknown information. It applies to predictions of future events, estimations, physical measurements, or unknown properties of a system, due to:
  - Underspecification
  - Lack of knowledge of the system actual behavior or underlying physics
  - Numerical or measurement errors
  - Numerical approximations because values are too costly to measure
  - Associated properties are not directly measurable or accessible.

- **Measurement of Uncertainty**: A set of possible states or outcomes where probabilities are assigned to each possible state or outcome – this also includes the application of a probability density function to continuous variables.

Use cases

GBP/USD 90 Day History
Apr 30 00:02 AM GMT
1.4615
1.4411
1.4208
1.4004
1.38
Feb 01
Mar 01
Mar 31
Apr 29
(c) Exchangerates.org.uk



EI
ei
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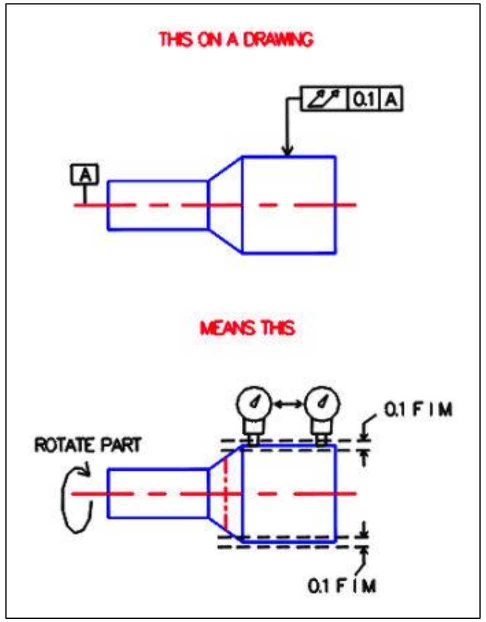
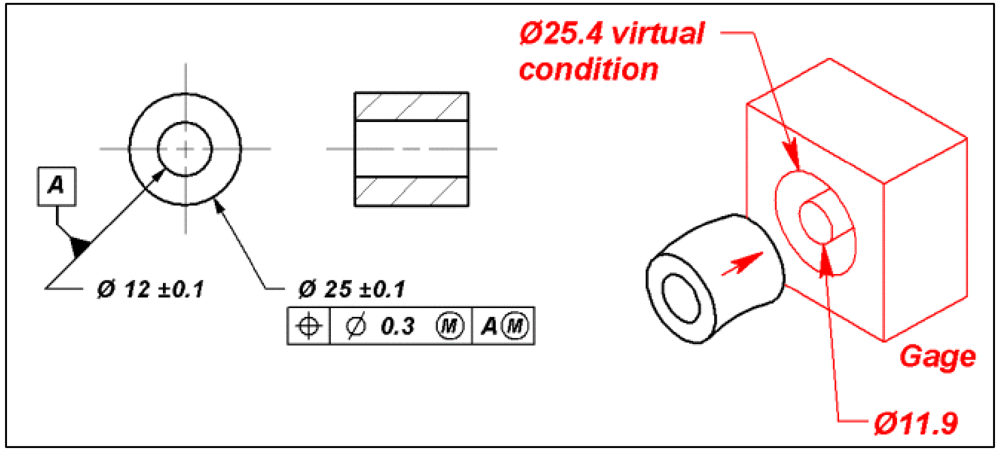
Use cases


THIS ON A DRAWING
0.1
A
A
MEANS THIS
ROTATE PART
0.1 F I M
0.1 F I M


Ø25.4 virtual condition
A
Ø 12 ±0.1
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Ø 0.3 Ⓜ A Ⓜ
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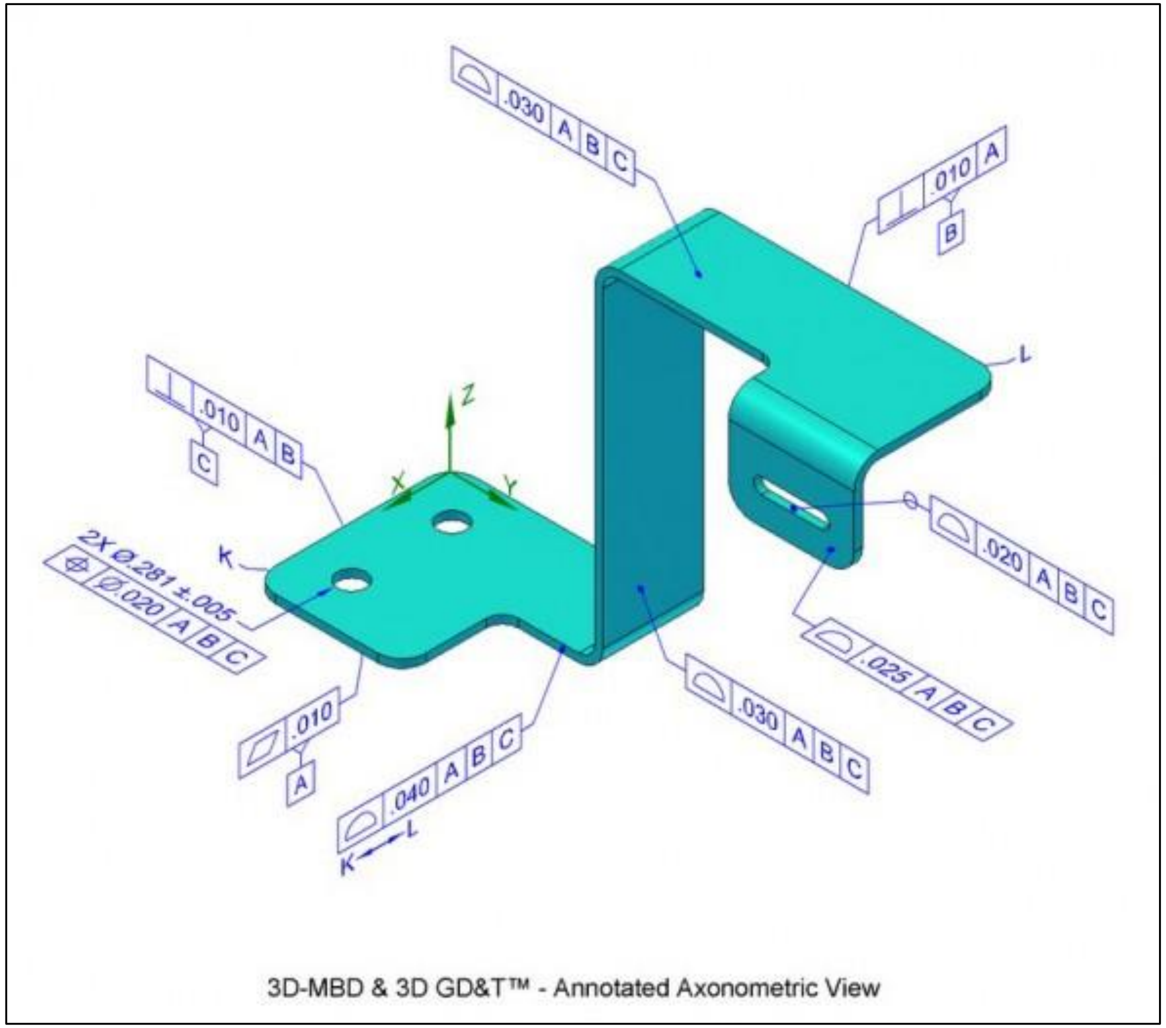
Ø11.9


.030 A B C
.010 A
B
.010 A B
C
2X Ø.281 ±.005
Ø.020 A B C
K
Z
X
Y
L
.020 A B C
.025 A B C
.030 A B C
.010
A
.040 A B C
K L
3D-MBD & 3D GD&T™ - Annotated Axonometric View

Use cases

Measurement uncertainty

- The ISO document "**Guide to the Expression of Uncertainty in Measurement**" (GUM) describes the procedure for calculating measurement uncertainty, as used by most Engineering Disciplines (*but Software*)

- Uncertain values may be grouped into two categories according to the method used to estimate their numerical values:
  - Type A, those evaluated by statistical methods
  - Type B, those evaluated by other means, e.g., by assigning a probability distribution, intervals defined by experts' *gess*timations, etc.

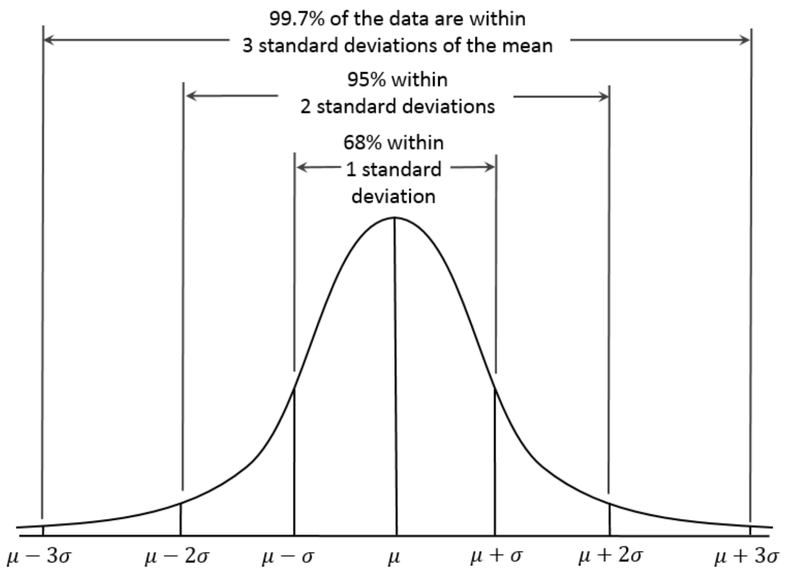
JCGM 100:2008: Evaluation of measurement data -- Guide to the expression of uncertainty in measurement (GUM). Joint Committee for Guides in Metrology. (2008). http://www.bipm.org/utils/common/documents/jcgm/JCGM_100_2008_E.pdf

Representation of Measurement Uncertainty

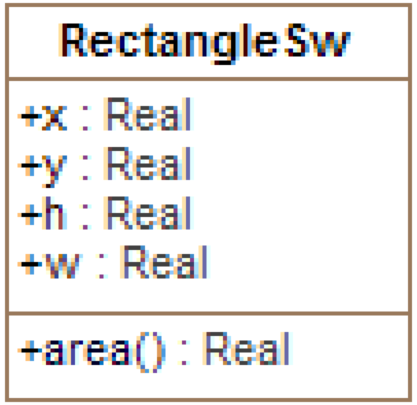
- Instead of giving a single number x to model a measurement result, engineers normally use **$x \pm u(x)$** to represent the *result*, being u(x) the *associated standard uncertainty*
- In Type A evaluation of measurement uncertainty, the estimated value x is taken as the **mean** of the measured values $\{x_1,\ldots,x_n\}$ and u(x) as the **experimental standard deviation** of such values.
- In Type B evaluation, is possible to estimate only upper and lower bounds for the values of X, i.e., $x \in [a,b]$. Then in this case,

  $x = (b-a)/2$

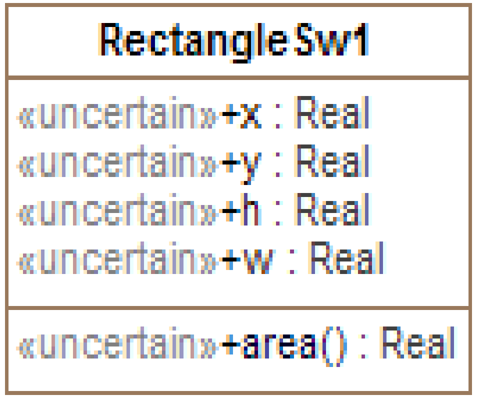
  $u(x) = (b-a)/\mathrm{sqrt}(3)$

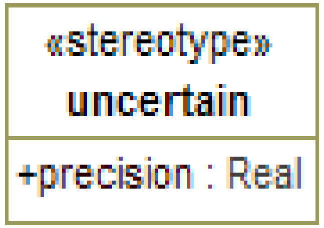


Measurement Uncertainty in Software Models



context RectangleSw::area() : Real = h*w





Our proposal

- New Type “UncertainReal”
  - Part of the UML/OCL type system
  - Supertype of Real
  - Values are pairs: (x,u)

- Define an algebra of operations to work with them and make calculations

UncertainReal
+x : Real = 0.0 +u : Real = 0.0
+add(r : UncertainReal) : UncertainReal +minus(r : UncertainReal) : UncertainReal +mult(r : UncertainReal) : UncertainReal +divide(r : UncertainReal) : UncertainReal +abs() : UncertainReal +neg() : UncertainReal +power(s : Real) : UncertainReal +lessThan(r : UncertainReal) : Boolean +lessEq(r : UncertainReal) : Boolean +equals(r : UncertainReal) : Boolean

```
context Rectangle::fitsIn (base : Rectangle) : Boolean
post: result = base.x.lessEq(self.x) and
      self.x.add(self.w).lessEq(base.x.add(base.w)) and
      base.y.lessEq(self.y) and
      self.y.add(self.h).lessEq(base.y.add(base.h))
```

Operations

$$(x_1, u_1) + (x_2, u_2) := \left(x_1 + x_2, \sqrt{u_1^2 + u_2^2}\right)$$

$$(x_1, u_1) - (x_2, u_2) := \left(x_1 - x_2, \sqrt{u_1^2 + u_2^2}\right)$$

$$(x_1, u_1) * (x_2, u_2) := \left(x_1 * x_2, \sqrt{x_2^2 u_1^2 + x_1^2 u_2^2}\right)$$

$$(x_1, u_1)/(x_2, u_2) := \left(\frac{x_1}{x_2} + \frac{x_1}{x_2^3} u_2^2, \sqrt{\frac{1}{x_2} u_1^2 + \frac{x_1^2}{x_2^4} u_2^2}\right)$$

$$power((x, u), r)) := \left(x^r + \frac{r(r-1)}{2} x^{r-2} u^2, r x^{r-1} u\right)$$

$$\sqrt{(x, u)} := \left(\sqrt{x} - \frac{u^2}{8x^{3/2}}, \frac{u}{2\sqrt{x}}\right)$$

$$\text{abs}((x, u)) := (\text{abs}((x), u)$$

$$-(x, u) := (-x, u)$$

Operations (cnt'd)

$$(x_1, u_1) = (x_2, u_2) := \max(x_1 - u_1, x_2 - u_2) < \min(x_1 + u_1, x_2 + u_2)$$

$$(x_1, u_1) <> (x_2, u_2) := \text{not}\,((x_1, u_1) = (x_2, u_2))$$

$$(x_1, u_1) < (x_2, u_2) := x_1 < x_2 \wedge (x_1 + u_1) < (x_2 - u_2)$$

$$(x_1, u_1) > (x_2, u_2) := x_1 > x_2 \wedge (x_1 - u_1) > (x_2 + u_2)$$

$$(x_1, u_1) \leq (x_2, u_2) := (x_1, u_1) < (x_2, u_2) \vee (x_1, u_1) = (x_2, u_2)$$

$$(x_1, u_1) \geq (x_2, u_2) := (x_1, u_1) > (x_2, u_2) \vee (x_1, u_1) = (x_2, u_2)$$

$$\max((x_1, u_1), (x_2, u_2)) := \max(x_1, x_2)$$

$$\min((x_1, u_1), (x_2, u_2)) := \min(x_1, x_2)$$

Implementations

1) In OCL, assuming Normal or Rectangular distributions

```
context UncertainReal::add(r:UncertainReal) :UncertainReal
post: result.x = self.x + r.x and
      result.u = (self.u*self.u + r.u*r.u).sqrt()

context UncertainReal::minus(r:UncertainReal):UncertainReal
post: result.x = self.x − r.x and
      result.u = (self.u*self.u − r.u*r.u).sqrt()

context UncertainReal::mult(r:UncertainReal) :UncertainReal
post: result.x = (self.x*r.x) and
      result.u = (r.u*r.u*self.x*self.x +
                  self.u*self.u*r.x*r.x).sqrt()

context UncertainReal::div(r:UncertainReal) :UncertainReal
  pre:  not r.equals(0,0)
  post: result.x = (self.x/r.x +
                    (self.x*r.u*r.u)/(r.x*r.x*r.x)) and
        result.u = ((self.u*self.u/r.x) +
                    ((r.u*r.u*self.x*self.x)/
                     (r.x*r.x*r.x*r.x))).sqrt()

context UncertainReal::abs() :UncertainReal
  post: result.x = (self.x).abs() and result.u = self.u

context UncertainReal::neg() :UncertainReal
  post: result.x = −self.x and result.u = self.u

context UncertainReal::power(s : Real) :UncertainReal
  post: result.x = self.x.power(s) + ((s*(s−1))/2)*
                     self.x.power(s−2)*(self.u*self.u) and
        result.u = s*self.u*self.x.power(s−1)
```

Implementations

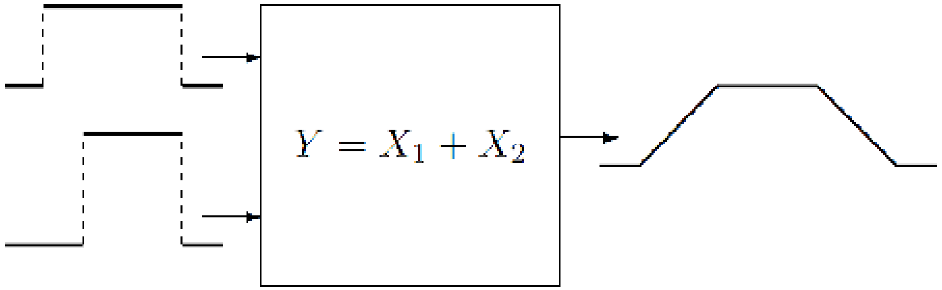
1) In OCL, assuming Normal or Rectangular distributions

2) In Java, when not all variables follow the same distribution (and no analytical solutions exist)

- Using Arrays of 10^6 elements (expected to deliver a 95% coverage interval for the output quantity) and Montecarlo simulations
- Permit different probability distributions in the constructors (Uniform, Triangular, Truncated, Normal and U-shaped)
- Permit propagating uncertainty of variables with mixed distribution probabilities

Two types of uncertainty quantification

- Forward uncertainty propagation
  - Quantification of uncertainties in system output(s) propagated from uncertain inputs.



- Inverse uncertainty quantification, which estimates:
  - The discrepancy between some experimental measurements of a system and some computer simulation results from its mathematical model (***bias correction***)
  - The values of unknown parameters in the model if there are any (***parameter calibration*** or simply ***calibration***).

Other issues

- Increasing tolerance
  - Extended uncertainty (GUM)

```
context UncertainReal::lessThan(r : UncertainReal, k : Real) : Boolean
    post: result = (self.x < r.x) and ((self.x+k*self.u)<(r.x-k*r.u))
```

- Stability of operations
  - Good news: No Unbounded growth of *Relative Error* (ε = |u(x)|/|x|)
  - But… Unbounded growth of *Absolute Error*

Propagation of uncertainty

	X			Y		X+Y		X-Y		X*Y		X/Y	
X	5	0.0000001	Y	2	0.0000001	7	1.41421E-07	3	1.41421E-07	10	5.38516E-07	2.5	1.0025E-06
X+Y	7	1.41421E-07	X-Y	3	1.41421E-07	10	0.0000002	4	0.0000002	21	1.07703E-06	2.33333333	2.971E-06
X*Y	10	5.38516E-07	X*Y	10	5.38516E-07	20	7.61577E-07	0	7.61577E-07	100	7.61577E-06	1	5.3852E-05
X*Y	10	5.38516E-07	X/Y	2.5	1.0025E-06	12.5	1.13798E-06	7.5	1.13798E-06	25	1.0115E-05	4	2.5065E-05
X	5	0.0000001	X	5	0.0000001	10	1.41421E-07	0	1.41421E-07	25	7.07107E-07	1	2.5004E-06
X*X	25	7.07107E-07	X	5	0.0000001	30	7.14143E-07	20	7.14143E-07	125	4.33013E-06	5	1.2504E-05
X*X	125	4.33013E-06	X*X	125	4.33013E-06	250	6.12372E-06	0	6.12372E-06	15625	0.000765466	1	0.06765823
X^4	15625	0.000765466	X^4	15625	0.000765466	31250	0.001082532	0	0.001082532	244,140,625.00	16.91455867	1	186881.236
X	5	0.0001	Y	2	0.0001	7	0.000141421	3	0.000141421	10	0.000538516	2.50000001	0.0010025
X+Y	7	0.000141421	X-Y	3	0.000141421	10	0.0002	4	0.0002	21	0.001077033	2.33333334	0.00297097
X*Y	10	0.000538516	X*Y	10	0.000538516	20	0.000761577	0	0.000761577	100	0.007615773	1	0.05385192
X*Y	10	0.000538516	X/Y	2.5	0.001002497	12.5	0.001137981	7.49999999	0.001137981	25.00000006	0.010114964	4.00000063	0.02506474
X	5	0.0001	X	5	0.0001	10	0.000141421	0	0.000141421	25	0.000707107	1	0.0025004
X*X	25	0.000707107	X	5	0.0001	30	0.000714143	20	0.000714143	125	0.004330127	5	0.012504
X*X	125	0.004330127	X*X	125	0.004330127	250	0.006123724	0	0.006123724	15625	0.765465545	1	67.6582347
X^4	15625	0.765465545	X^4	15625	0.765465545	31250	1.082531755	0	1.082531755	244140625	16914.55867	1	186881236
X	5	0.001	Y	2	0.001	7	0.001414214	3	0.001414214	10	0.005385165	2.50000063	0.01002497
X+Y	7	0.001414214	X-Y	3	0.001414214	10	0.002	4	0.002	21	0.01077033	2.33333385	0.02970971
X*Y	10	0.005385165	X*Y	10	0.005385165	20	0.007615773	0	0.007615773	100	0.076157731	1.00000029	0.53851917
X*Y	10	0.005385165	X/Y	2.5	0.010024969	12.5000006	0.011379807	7.49999938	0.011379807	25.00000625	0.101149642	4.00006332	0.25064742
X	5	0.001	X	5	0.001	10	0.001414214	0	0.001414214	25	0.007071068	1.00000004	0.025004
X*X	25	0.007071068	X	5	0.001	30	0.007141428	20	0.007141428	125	0.04330127	5.0000002	0.12503999
X*X	125	0.04330127	X*X	125	0.04330127	250	0.061237244	0	0.061237244	15625	7.654655446	1.00000012	676.582347
X^4	15625	7.654655446	X^4	15625	7.654655446	31250	10.82531755	0	10.82531755	244140625	169145.5867	1.00000024	1868812365

Ongoing / Future work

- Integration with fUML and Papyrus
- Further validation
- Performance issues
- Connection with other analysis tools