MSDL
Modelling, Simulation and Design Lab

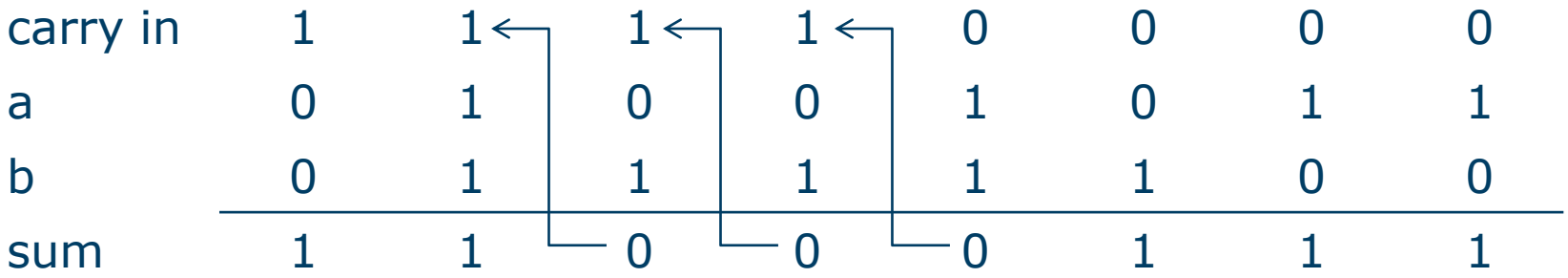
Adders

12 oktober 2009

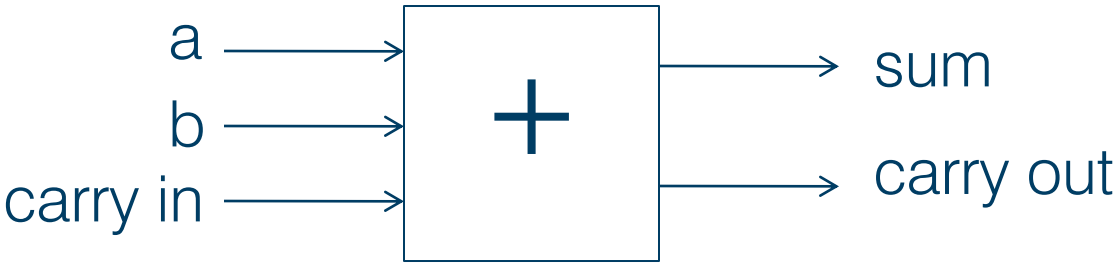
Computersystemen en –architectuur

2009-2010

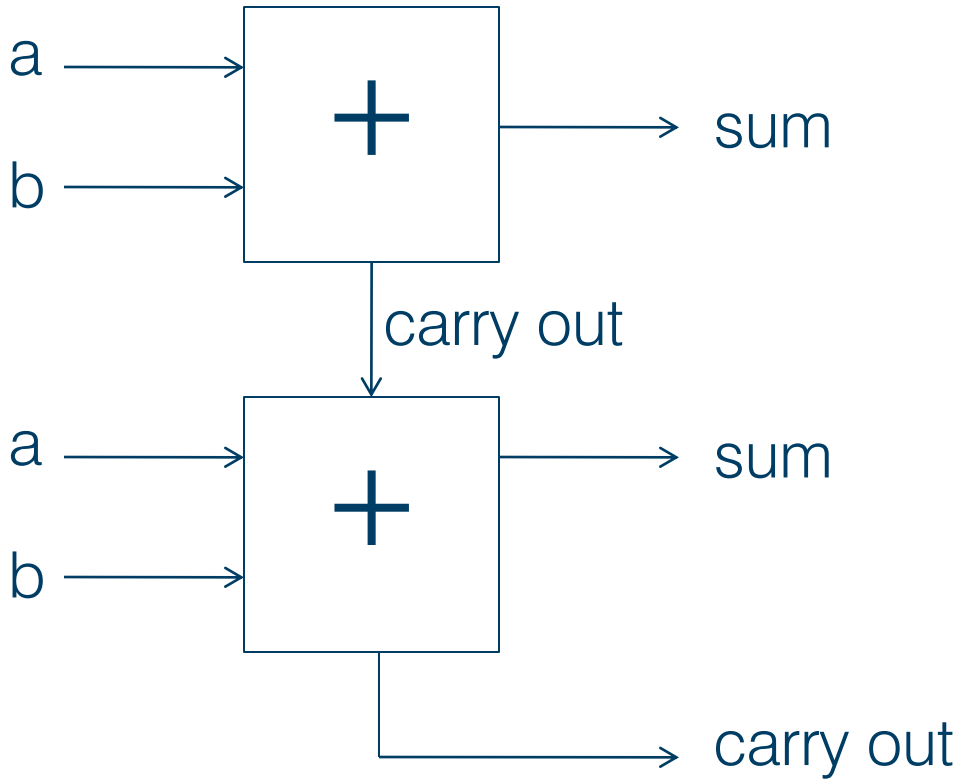
Binary addition



1 Bit Full adder



2 Bit Full adder



2's complement addition

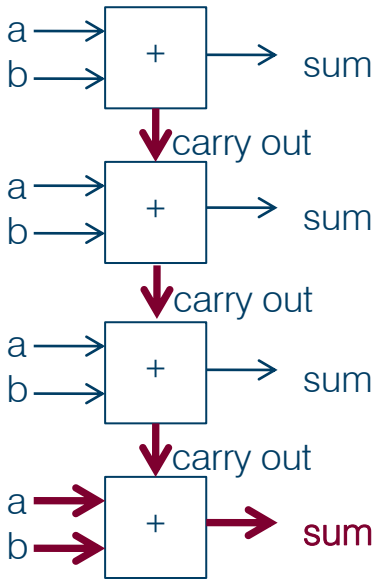
carry in	1	1	1	1	0	0	0	0	
a	0	1	0	0	1	0	1	1	75_{ten}
b	0	1	1	1	1	1	0	0	124_{ten}
sum	1	1	0	0	0	1	1	1	-57_{ten}

Overflow!

Ripple carry adder

- Addition of bits i+1 depends on result of addition of bits i
  - Calculation spans (too) many gates!
  - n gates for calculating carry out in 1 bit adder

    -> i*n gates for i'th bit!



"Ripple Carry"

Carry lookahead adder

- Carry lookahead!
- 1 bit adder

$$CarryOut = a \,.\, b$$

- 1 bit full adder

$$CarryOut = (a \,.\, carryIn) + (b \,.\, carryIn) + (a \,.\, b)$$

- In general

$$\begin{aligned} c_{i+1} &= (b_i \,.\, c_i) + (a_i \,.\, c_i) + (a_i \,.\, b_i) \\ &= (a_i \,.\, b_i) + (a_i + b_i) \,.\, c_i \end{aligned}$$

Carry lookahead adder

- Suppose

$$g_i = a_i \,.\, b_i \qquad \text{(generate)}$$
$$p_i = a_i + b_i \qquad \text{(propagate)}$$

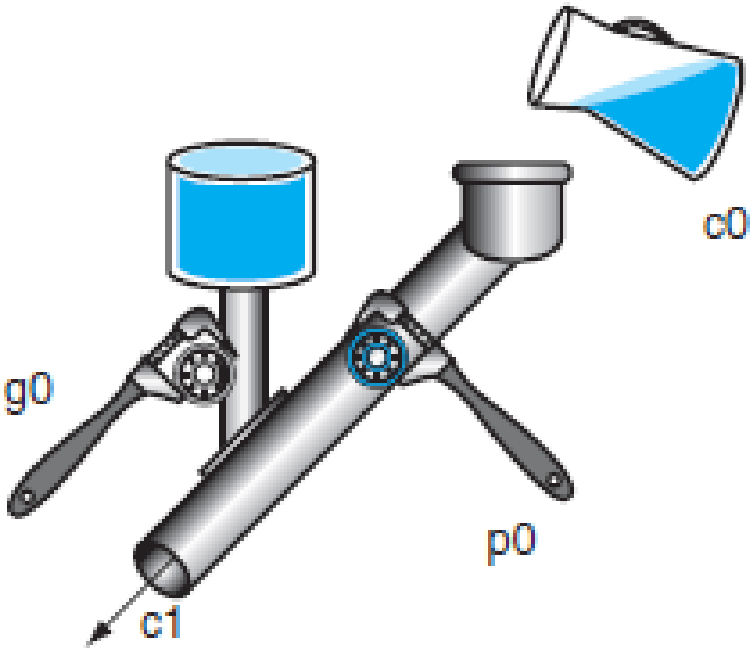
- Then

$$c_{i+1} = g_i + p_i \,.\, c_i$$

$$c_1 = g_0 + (p_0 \,.\, c_0)$$
$$c_2 = g_1 + (p_1 \,.\, c_1)$$
$$= g_1 + (p_1 \,.\, (g_0 + (p_0 \,.\, c_0)))$$
$$= g_1 + (p_1 \,.\, g_0) + (p_1 \,.\, p_0 \,.\, c_0)$$
$$c_3 = g_2 + (p_2 \,.\, g_1) + (p_2 \,.\, p_1 \,.\, g_0) + (p_2 \,.\, p_1 \,.\, p_0 \,.\, c_0)$$
$$c_4 = g_3 + (p_3 \,.\, g_2) + (p_3 \,.\, p_2 \,.\, g_1) + (p_3 \,.\, p_2 \,.\, p_1 \,.\, g_0) + (p_3 \,.\, p_2 \,.\, p_1 \,.\, p_0 \,.\, c_0)$$

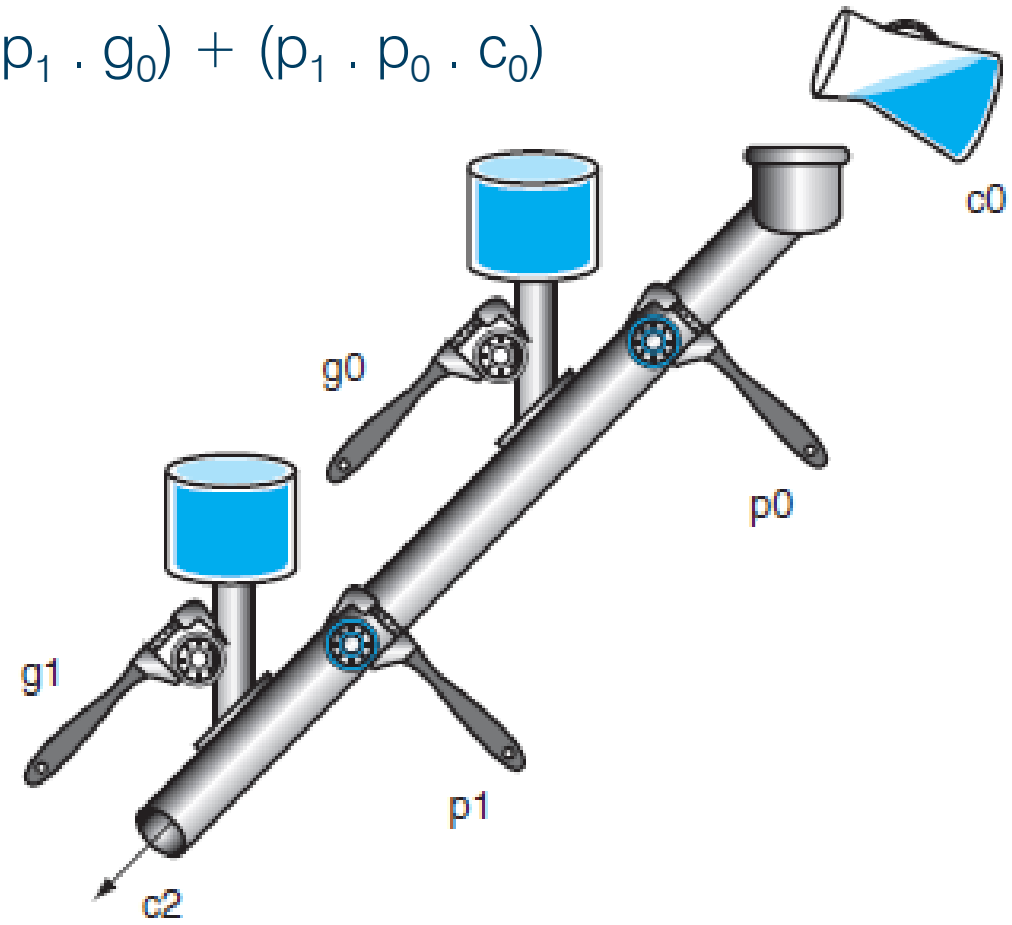
Carry lookahead adder

– "generate" and "propagate"

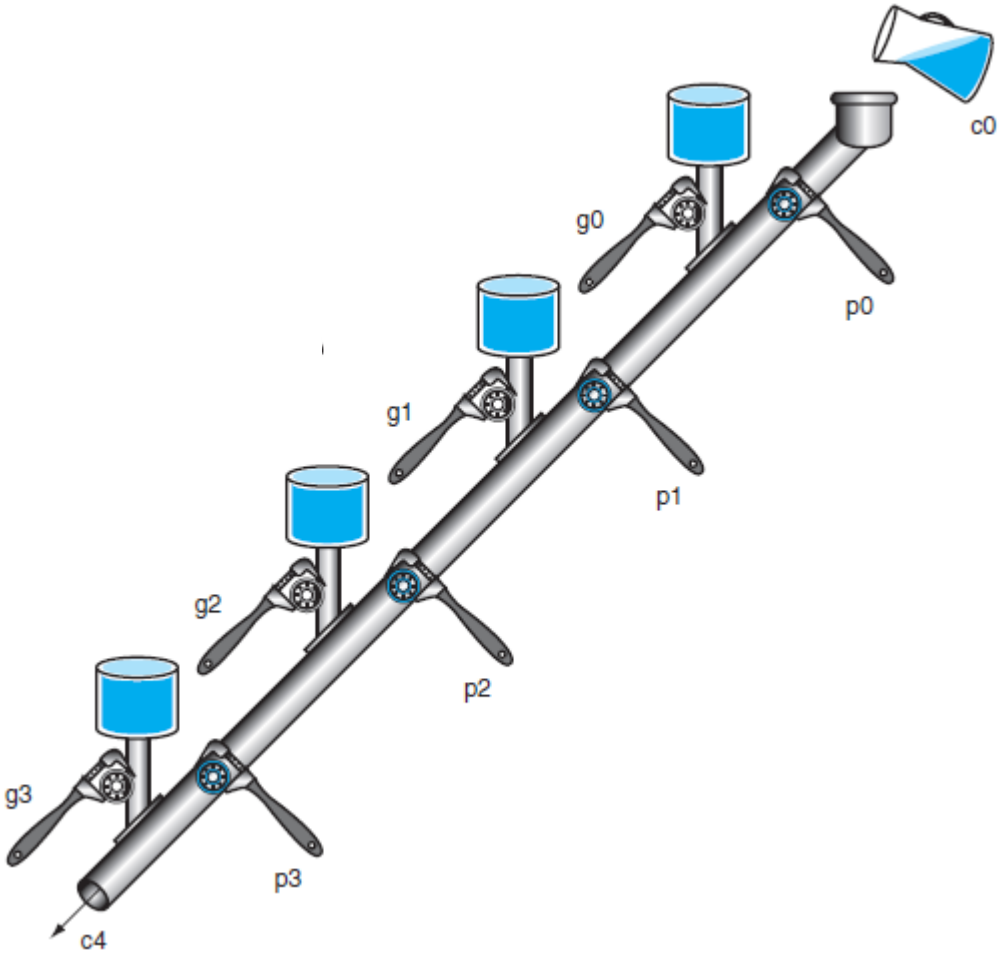


Carry lookahead adder

- $c_2 = g_1 + (p_1 \cdot g_0) + (p_1 \cdot p_0 \cdot c_0)$



Carry lookahead adder



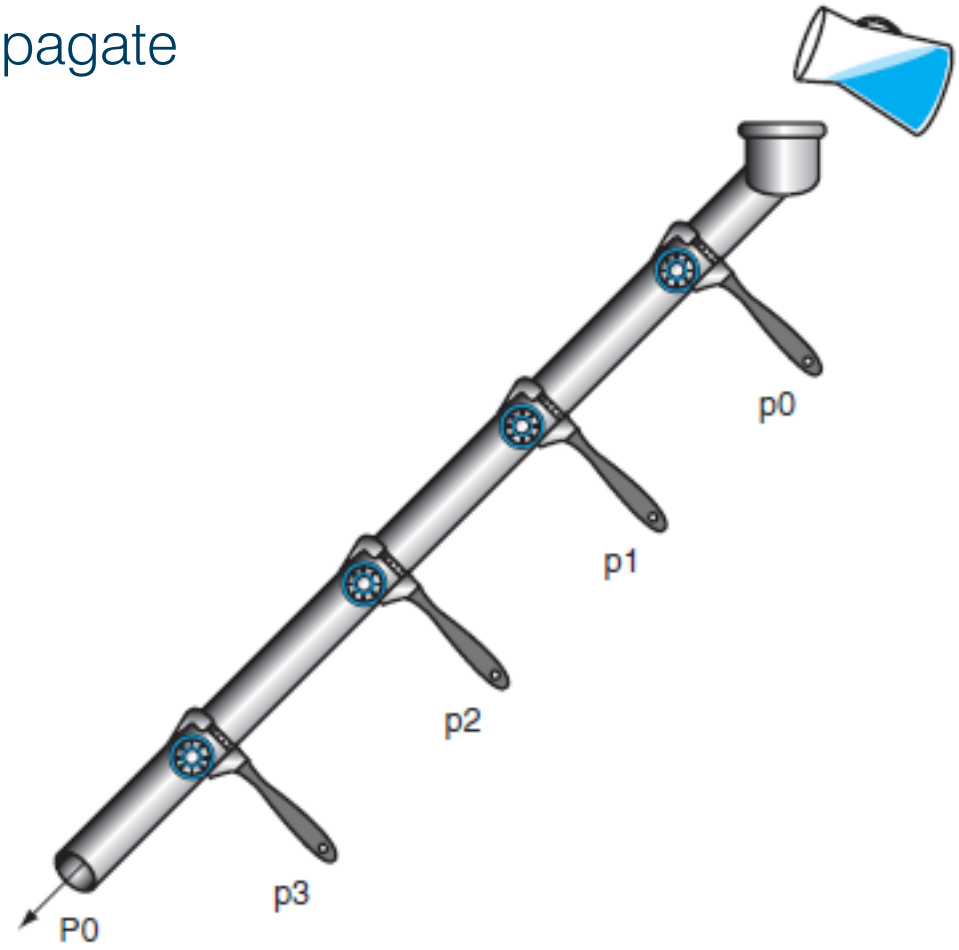
Carry lookahead adder

- Carry "lookahead"
- Grouping of generates and propagates
  - super generates and super propagates
- Example: group adders in groups of 4

$$P_1 = p_3 \cdot p_2 \cdot p_1 \cdot p_0$$
$$P_2 = p_7 \cdot p_6 \cdot p_5 \cdot p_4$$
$$P_3 = p_{11} \cdot p_{10} \cdot p_9 \cdot p_8$$
$$P_4 = p_{15} \cdot p_{14} \cdot p_{13} \cdot p_{12}$$

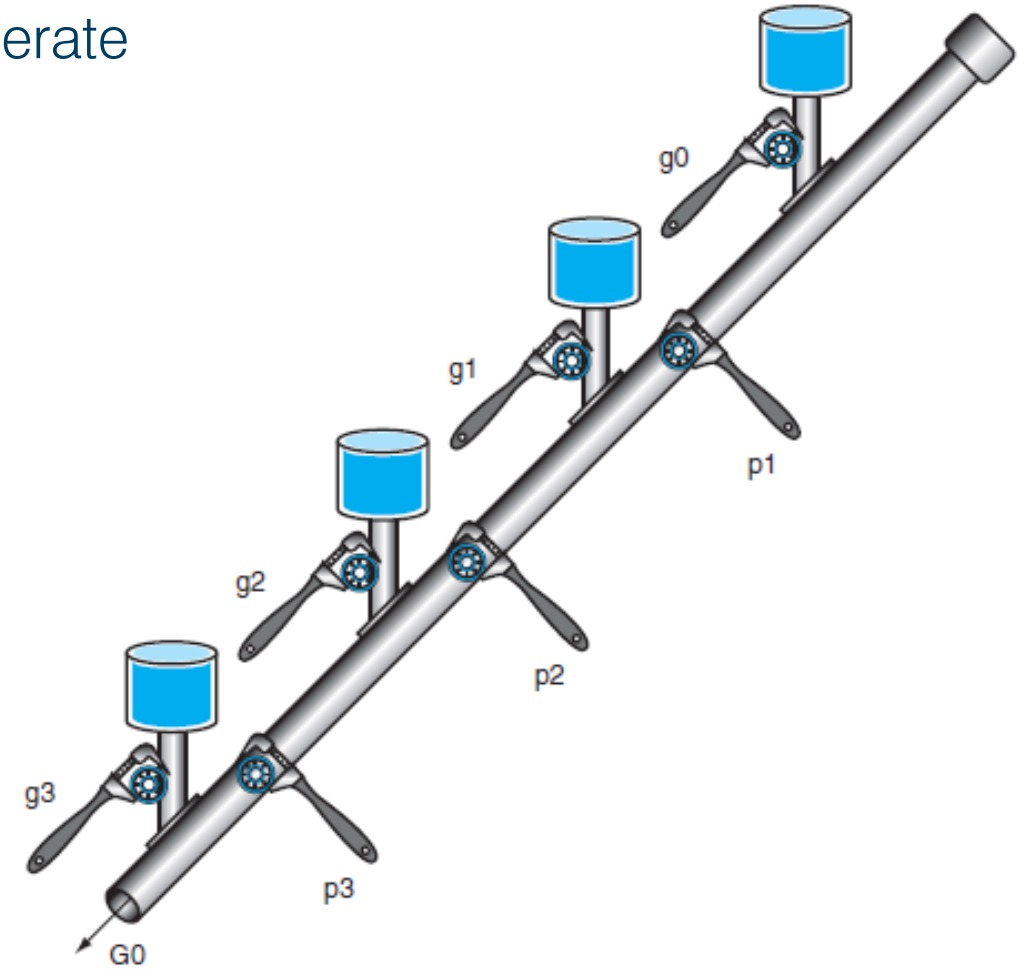
Carry lookahead adder

- Super propagate



Carry lookahead adder

– Super generate



Carry lookahead adder

$$G_0 = g_3 + (p_3 \cdot g_2) + (p_3 \cdot p_2 \cdot g_1) + (p_3 \cdot p_2 \cdot p_1 \cdot g_0)$$
$$G_1 = g_7 + (p_7 \cdot g_6) + (p_7 \cdot p_6 \cdot g_5) + (p_7 \cdot p_6 \cdot p_5 \cdot g_4)$$
$$G_2 = g_{11} + (p_{11} \cdot g_{10}) + (p_{11} \cdot p_{10} \cdot g_9) + (p_{11} \cdot p_{10} \cdot p_9 \cdot g_8)$$
$$G_3 = g_{15} + (p_{15} \cdot g_{14}) + (p_{15} \cdot p_{14} \cdot g_{13}) + (p_{15} \cdot p_{14} \cdot p_{13} \cdot g_{12})$$

– Super carries

$$C_1 = G_0 + (P_0 \cdot c_0)$$
$$C_2 = G_1 + (P_1 \cdot G_0) + (P_1 \cdot P_0 \cdot c_0)$$
$$C_3 = G_2 + (P_2 \cdot G_1) + (P_2 \cdot P_1 \cdot G_0) + (P_2 \cdot P_1 \cdot P_0 \cdot c_0)$$
$$C_4 = G_3 + (P_3 \cdot G_2) + (P_3 \cdot P_2 \cdot G_1) + (P_3 \cdot P_2 \cdot P_1 \cdot G_0) + (P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot c_0)$$