

# Computer Systems and Architecture

## Data Representation

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# Outline

Positive Numbers

Negative Numbers

Floating point Numbers

Exercises

## Base 10 to base 2

**Example.** Convert  $(23.375)_{10}$  to base 2.

## Base 10 to base 2 (II)

Step 1. Remainder method: convert  $(23)_{10}$  to base 2

$23 / 2 = 11$	Remainder	<b>1</b>	LSB
$11 / 2 = 5$		<b>1</b>	
$5 / 2 = 2$		<b>1</b>	
$2 / 2 = 1$		<b>0</b>	
$1 / 2 = 0$		<b>1</b>	MSB

$$(23)_{10} = (10111)_2$$

## Base 10 to base 2 (III)

Step 2. Multiplication method: convert  $(.375)_{10}$  to base 2

$$.375 \times 2 = \mathbf{0} \ .75$$

$$.75 \times 2 = \mathbf{1} \ .5$$

$$.5 \times 2 = \mathbf{1} \ .0$$

$$(.375)_{10} = (.011)_2$$

Step 3. Total:  $(23.375)_{10} = (10111.011)_2$

# Base 2 to base 10

▶ Number in base 2:  $b_n b_{n-1} \dots b_1 b_0 . b_{-1} b_{-2} \dots b_{-m}$

▶ Value in base 10 =  $\sum_{i=-m}^{n-1} b_i \cdot 2^i$

▶ **Weighted position code**

▶ Polynomial method

## Base 2 to base 10 (II)

**Example.** Convert  $(1010.01)_2$  to base 10.

$$\begin{aligned}(1010.01)_2 &= (1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2})_{10} \\ &= (8 + 0 + 2 + 0 + 0 + .25)_{10} \\ &= (10.25)_{10}\end{aligned}$$

# Extensible to other bases

Conversion between base  $k$  and base 10:

- ▶ **Remainder method.** Divide by  $k$ .
- ▶ **Multiplication method.** Multiply by  $k$ .

- ▶ **Polynomial method.** 
$$\sum_{i=-m}^{n-1} b_i \cdot k^i$$



## Bases 2, 4, 8, 16

**Example.** Convert  $(10110110)_2$  to base 4, 8 and 16.

$$(10110110)_2$$

$$= (10)_2(11)_2(01)_2(10)_2 = (2)_4(3)_4(1)_4(2)_4 = (2312)_4$$

$$= (010)_2(110)_2(110)_2 = (2)_8(6)_8(6)_8 = (266)_8$$

$$= (1011)_2(0110)_2 = (B)_{16}(6)_{16} = (B6)_{16}$$

# Negative numbers

- ▶ Signed magnitude
- ▶ One's complement
- ▶ Two's complement
- ▶ Excess (biased)

# Signed magnitude

- ▶ First bit as sign
- ▶ **Example.**
  - ▶  $(+12)_{10} = (00001100)_2$
  - ▶  $(-12)_{10} = (10001100)_2$

# One's complement

- ▶ Convert all zero's in 1's and 1's in zero's.
- ▶ **Example.**
  - ▶  $(+12)_{10} = (00001100)_2$
  - ▶  $(-12)_{10} = (11110011)_2$

# Two's complement

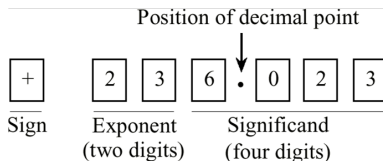
- ▶ Take the one's complement, and add 1.
- ▶ Only 1 representation for 0 !
- ▶ **Example.**
  - ▶  $(+12)_{10} = (00001100)_2$
  - ▶  $(-12)_{10} = (11110100)_2$

## Excess (biased)

- ▶ Add *bias* to every number and convert it as a positive number.
- ▶ **Example:** Excess 128 (*bias* = 128)
  - ▶  $12 + 128 = 140$   
 $(+12)_{10} = (10001100)_2$
  - ▶  $-12 + 128 = 116$   
 $(-12)_{10} = (01110100)_2$

# Floating point

- ▶ Wide range of numbers in a limited number of bits.
- ▶ **Example.**  $+6.023 \cdot 10^{23}$



# Normalization

- ▶ Different representations for the same number.
- ▶ **Example.**  $+254 \cdot 10^0$ 
  - ▶  $25.4 \cdot 10^1$
  - ▶  $0.0254 \cdot 10^4$
  - ▶  $254000 \cdot 10^{-3}$
- ▶ Normalization: the dot is placed right of the leftmost digit.
  - ▶  $2.54 \cdot 10^2$
- ▶ Hidden bit: in base 2 the leftmost bit can be discarded
  - ▶  $1.1010 \rightarrow 1010$



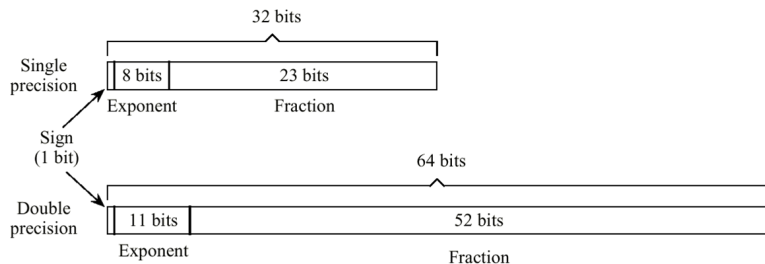
# Conversion

**Example.** Convert  $(9.375 \cdot 10^{-2})_{10}$  to base 2 (scientific notation).

- ▶ Fixed point:  $(.09375)_{10}$
- ▶ Multiplication method:  $(.00011)_2$
- ▶ Normalize:  $.00011 = .00011 \cdot 2^0 = 1.1 \cdot 2^{-4}$

# IEEE-754

- ▶ Sign bit
- ▶ Exponent: Excess 127 - 1023
- ▶ Number representation with hidden bit



## IEEE-754 (II)

**Example.** Represent  $(-12.625)_{10}$  in IEEE-754 Single precision format.

**Step 1.** Convert to base 2:  $(-12.625)_{10} = (-1100.101)_2$

**Step 2.** Normalize:  $(-1100.101)_2 = (-1.100101)_2 \cdot 2^3$

**Step 3.** Calculate exponent in excess 127:

$$3 + 127 = 130 \rightarrow (10000010)_2$$

1 1000 0010 100 1010 0000 0000 0000 0000

# IEEE-754 (III)

Special cases (Single precision):

▶ +0 :

0 0000 0000 000 0000 0000 0000 0000 0000

▶  $+\infty$  :

0 1111 1111 000 0000 0000 0000 0000 0000

▶ + NaN :

0 1111 1111 001 0010 0000 0100 1000 0000

▶  $2^{-128}$  (Denormalized) :

0 0000 0000 010 0000 0000 0000 0000 0000

# Error

- ▶ Distance between 2 consecutive representable numbers
- ▶ Notation:
  - ▶  $b$  = base
  - ▶  $s$  = # significante digits
  - ▶  $M$  = grootste exponent
  - ▶  $m$  = kleinste exponent

## Error (II)

- ▶ Number of representable bits (hidden bit):  
 $2 \cdot ((M - m) + 1) \cdot b^s + 1$
- ▶ Biggest representable number:  
 $b^M \cdot (1 - b^{-s})$
- ▶ Smallest representable number:  
 $b^m \cdot b^{-1}$
- ▶ Biggest gap:  
 $b^M \cdot b^{-s}$
- ▶ Smallest gap:  
 $b^m \cdot b^{-s}$

# Exercises

- ▶ <http://msdl.cs.mcgill.ca/people/hv/teaching/ComputerSystemsArchitecture/#CS4>