Computer Systems and Architecture

Data Representation

Stephen Pauwels

University of Antwerp

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Outline

Base 10 to base 2

Example. Convert $(23.375)_{10}$ to base 2.

Base 10 to base 2 (II)

Step 1. Remainder method: convert $(23)_{10}$ to base 2

			Remainder	
$23\ /2$	$=$	11	**1**	LSB
$11\ /2$	$=$	5	**1**	
$5\ /2$	$=$	2	**1**	
$2\ /2$	$=$	1	**0**	
$1\ /2$	$=$	0	**1**	MSB

$$(23)_{10} = (10111)_2$$

Base 10 to base 2 (III)

Step 2. Multiplication method: convert $(.375)_{10}$ to base 2

$$\begin{array}{rcll} .375 & \times 2 = & \mathbf{0} & .75 \\ .75 & \times 2 = & \mathbf{1} & .5 \\ .5 & \times 2 = & \mathbf{1} & .0 \end{array}$$

$$(.375)_{10} = (.011)_2$$

Step 3. Total: $(23.375)_{10} = (10111.011)_2$

Base 2 to base 10

- Number in base 2: $b_n b_{n-1} ... b_1 b_0 . b_{-1} b_{-2} ... b_{-m}$
- Value in base 10 $= \sum_{i=-m}^{n-1} b_i \cdot 2^i$
- **Weighted position code**
- Polynomial method

Base 2 to base 10 (II)

Example. Convert $(1010.01)_2$ to base 10.

$$\begin{aligned}(1010.01)_2 &= (1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2})_{10} \\ &= (8 + 0 + 2 + 0 + 0 + .25)_{10} \\ &= (10.25)_{10}\end{aligned}$$

Extensible to other bases

Conversion between base k and base 10:

- Remainder method. Divide by k.
- Multiplication method. Multiply by k.
- Polynomial method. $\sum_{i=-m}^{n-1} b_i \cdot \mathbf{k}^i$

Bases 2, 4, 8, 16

Example. Convert $(10110110)_2$ to base 4, 8 and 16.

$(10110110)_2$

$= (10)_2(11)_2(01)_2(10)_2 = (2)_4(3)_4(1)_4(2)_4 = (2312)_4$

$= (010)_2(110)_2(110)_2 = (2)_8(6)_8(6)_8 = (266)_8$

$= (1011)_2(0110)_2 = (B)_{16}(6)_{16} = (B6)_{16}$

Negative numbers

- Signed magnitude
- One's complement
- Two's complement
- Excess (biased)

Signed magnitude

- First bit as sign
- Example.
  - $(+12)_{10} = (00001100)_2$
  - $(-12)_{10} = (10001100)_2$

One's complement

- Convert all zero's in 1's and 1's in zero's.
- Example.
  - $(+12)_{10} = (00001100)_2$
  - $(-12)_{10} = (11110011)_2$

Two's complement

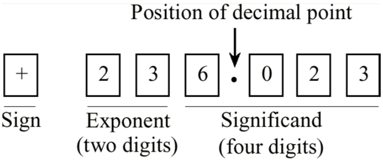
- Take the one's complement, and add 1.
- Only 1 representation for 0 !
- Example.
  - $(+12)_{10} = (00001100)_2$
  - $(-12)_{10} = (11110100)_2$

Excess (biased)

- Add *bias* to every number and convert it as a positive number.
- Example: Excess 128 (*bias* = 128)
  - $12 + 128 = 140$
    $(+12)_{10} = (10001100)_2$
  - $-12 + 128 = 116$
    $(-12)_{10} = (01110100)_2$

Floating point

- Wide range of numbers in a limited number of bits.
- Example. $+6.023 \cdot 10^{23}$



Normalization

- Different representations for the same number.
- Example. $+254 \cdot 10^0$
  - $25.4 \cdot 10^1$
  - $0.0254 \cdot 10^4$
  - $254000 \cdot 10^{-3}$
- Normalization: the dot is placed right of the leftmost digit.
  - $2.54 \cdot 10^2$
- Hidden bit: in base 2 the leftmost bit can be discarded
  - $1.1010 \rightarrow 1010$

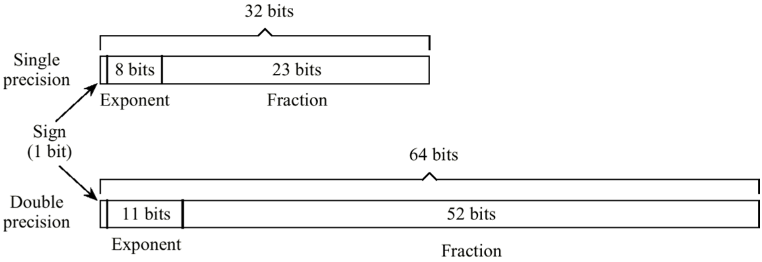
Conversion

Example. Convert $(9.375 \cdot 10^{-2})_{10}$ to base 2 (scientific notation).

- Fixed point: $(.09375)_{10}$
- Multiplication method: $(.00011)_2$
- Normalize: $.00011 = .00011 \cdot 2^0 = 1.1 \cdot 2^{-4}$

IEEE-754

- Sign bit
- Exponent: Excess 127 - 1023
- Number representation with hidden bit



IEEE-754 (II)

Example. Represent $(-12.625)_{10}$ in IEEE-754 Single precision format.

Step 1. Convert to base 2: $(-12.625)_{10} = (-1100.101)_2$

Step 2. Normalize: $(-1100.101)_2 = (-1.100101)_2 \cdot 2^3$

Step 3. Calculate exponent in excess 127:
$3 + 127 = 130 \rightarrow (10000010)_2$

1 1000 0010 100 1010 0000 0000 0000 0000

IEEE-754 (III)

Special cases (Single precision):

- $+0$:
  0 0000 0000 000 0000 0000 0000 0000 0000
- $+\infty$:
  0 1111 1111 000 0000 0000 0000 0000 0000
- $+$ NaN :
  0 1111 1111 001 0010 0000 0100 1000 0000
- 2^{-128} (Denormalized) :
  0 0000 0000 010 0000 0000 0000 0000 0000

Error

- Distance between 2 consecutive representable numbers
- Notation:
  - b = base
  - s = # significante digits
  - M = grootste exponent
  - m = kleinste exponent

Error (II)

- Number of representable bits (hidden bit):
  $2 \cdot ((M - m) + 1) \cdot b^s + 1$
- Biggest representable number:
  $b^M \cdot (1 - b^{-s})$
- Smallest representable number:
  $b^m \cdot b^{-1}$
- Biggest gap:
  $b^M \cdot b^{-s}$
- Smallest gap:
  $b^m \cdot b^{-s}$

Exercises

- http://msdl.cs.mcgill.ca/people/hv/teaching/ComputerSystemsArchitecture/#CS4