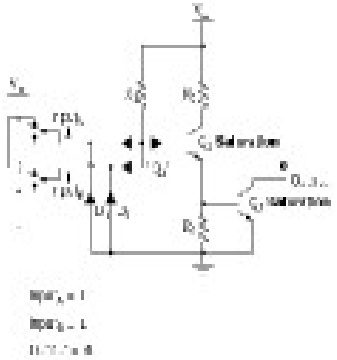
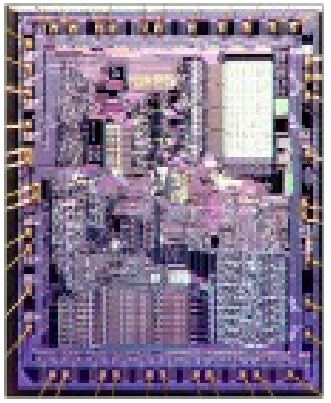
Computer Systemen en Computer Architectuur

Dinsdag 31 oktober 2023 – getalvoorstellingen ((mee)looples)

Hans Vangheluwe

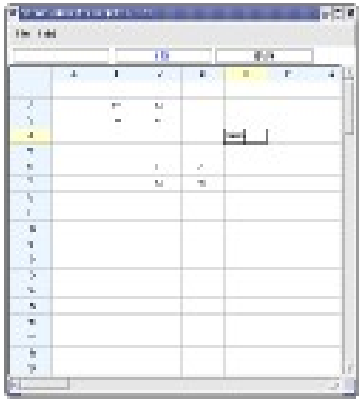




```
strcpy:
    addi $sp, $sp, -4
    sw   $s0, 0($sp)
    add  $s0, $zero, $zero
L1: add  $t1, $s0, $a1
    lbu  $t2, 0($t1)
    add  $t3, $s0, $a0
    sb   $t2, 0($t3)
    beq  $t2, $zero, L2
    addi $s0, $s0, 1
    j    L1
L2: lw   $s0, 0($sp)
    addi $sp, $sp, 4
    jr   $ra
```

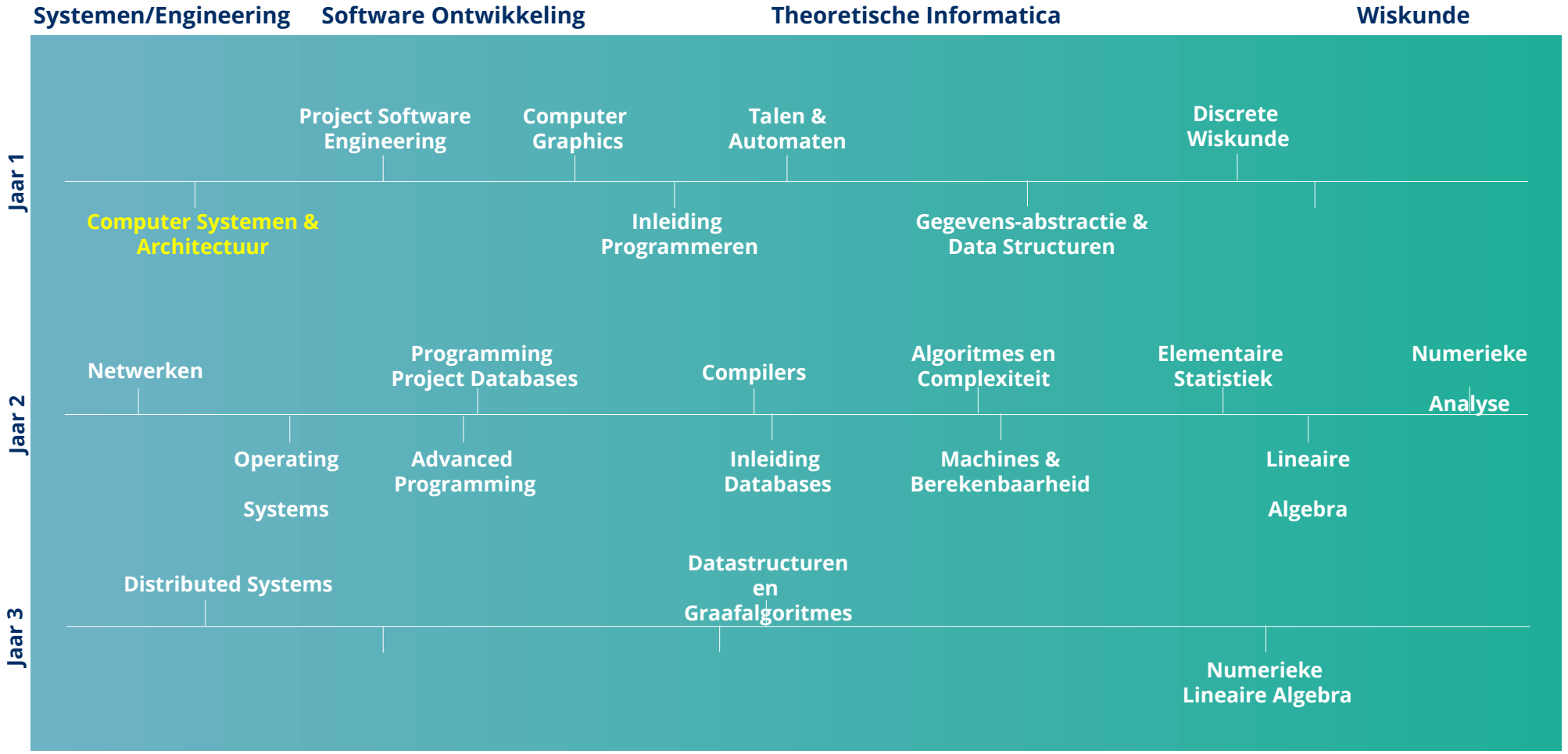
```
k = 1
xPowerK = x
Sign = 1; s = 0

while k <= N :
 term = sign*xPowerK/
        factorial(k)
 s = s + term
 k = k + 2
 xPowerK =
  xPowerK * xSquare
 sign = -sign
```

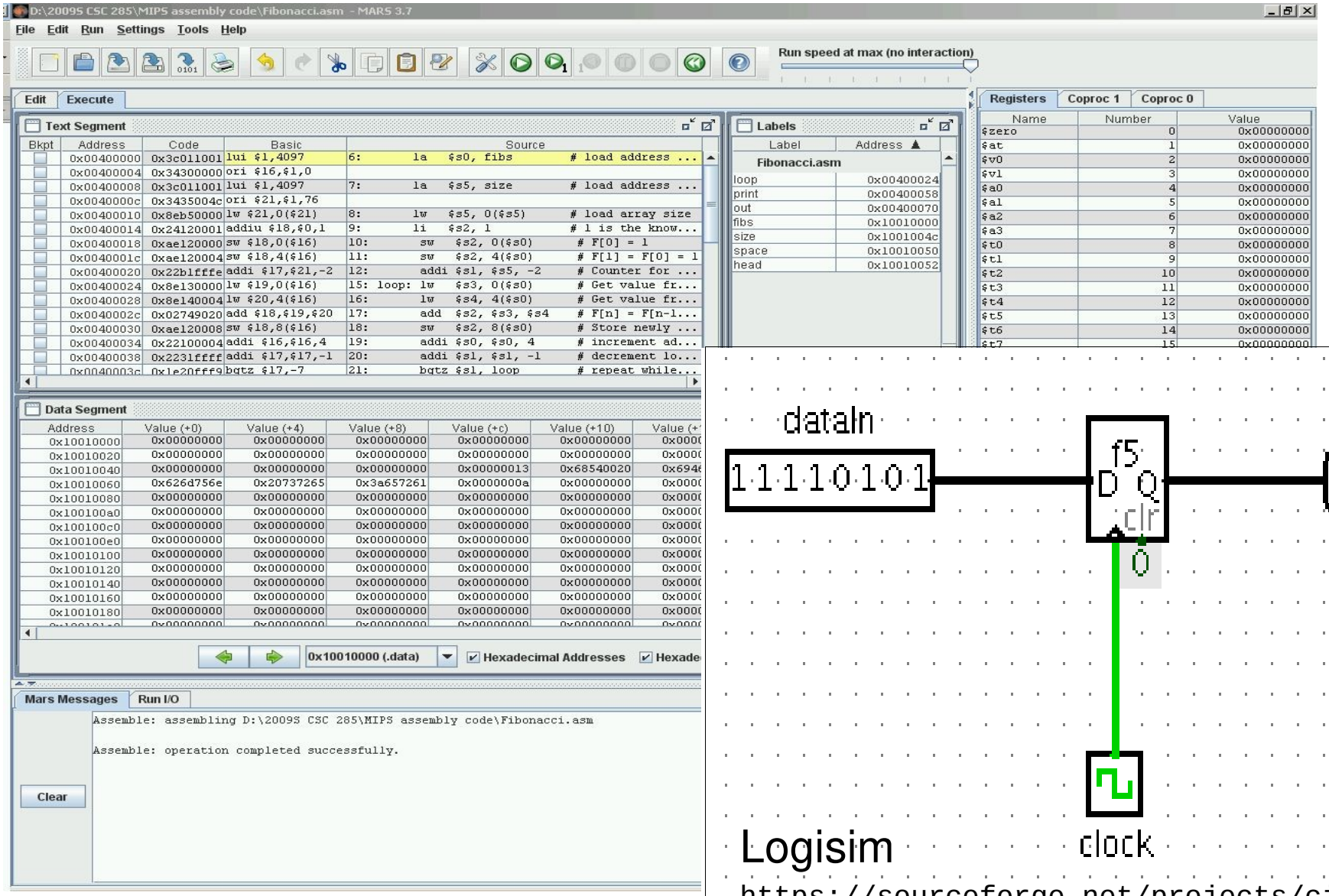


physics digital electronics — computer architecture / systems — HL progr. languages operating systems — complex SW applications

http://msdl.uantwerpen.be/people/hv/teaching/ComputerSystemsArchitecture/meelooples.pdf


Systemen/Engineering
Software Ontwikkeling
Theoretische Informatica
Wiskunde
Jaar 1
Jaar 2
Jaar 3
Project Software Engineering
Computer Graphics
Talen & Automaten
Discrete Wiskunde
Computer Systemen & Architectuur
Inleiding Programmeren
Gegevens-abstractie & Data Structuren
Netwerken
Programming Project Databases
Compilers
Algoritmes en Complexiteit
Elementaire Statistiek
Numerieke Analyse
Operating Systems
Advanced Programming
Inleiding Databases
Machines & Berekenbaarheid
Lineaire Algebra
Distributed Systems
Datastructuren en Graafalgoritmes
Numerieke Lineaire Algebra

Oefeningen ... al doende ...



MARS

http://courses.missouristate.edu/KenVollmar/MARS/

Data Representation

(binary) Data Representation

- Numbers

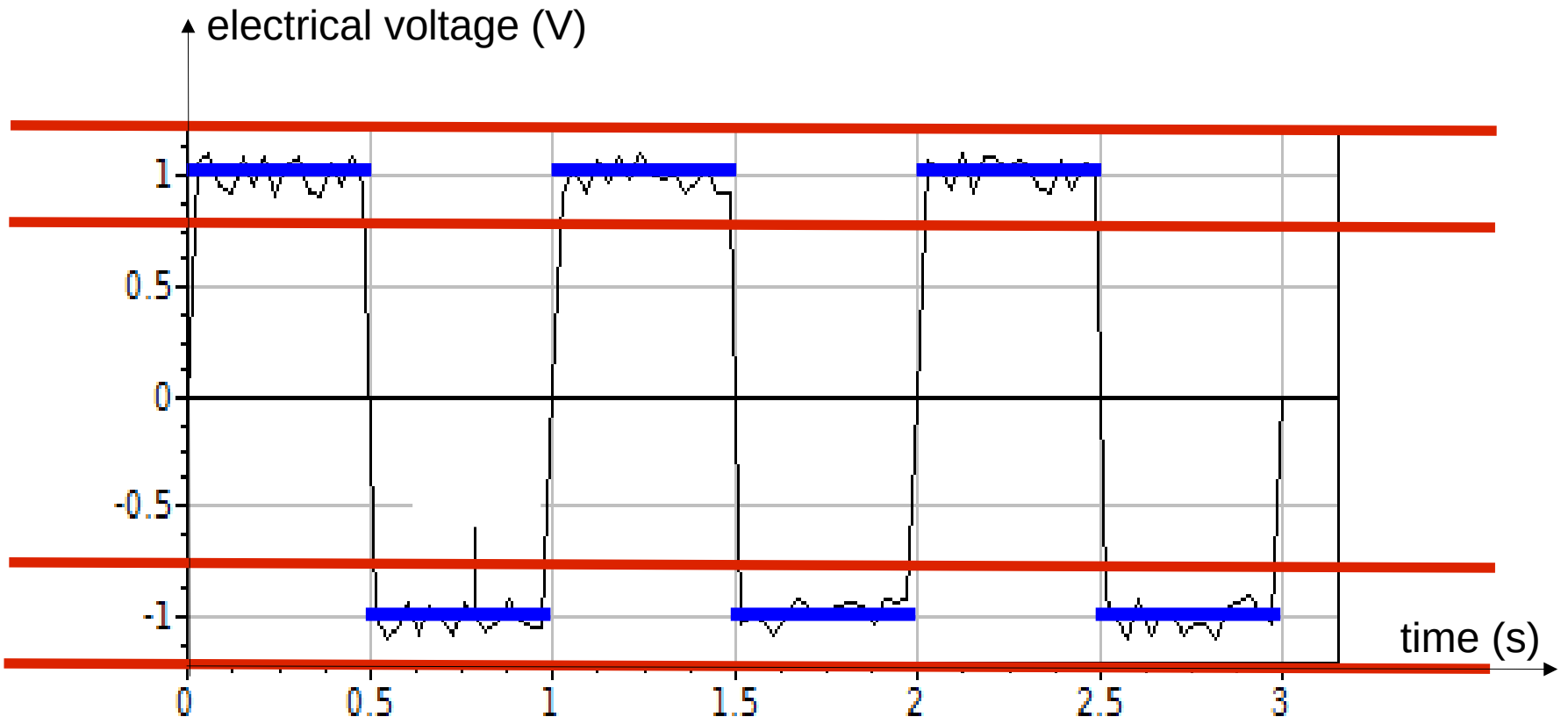
  $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$

- Letters (and “strings”)

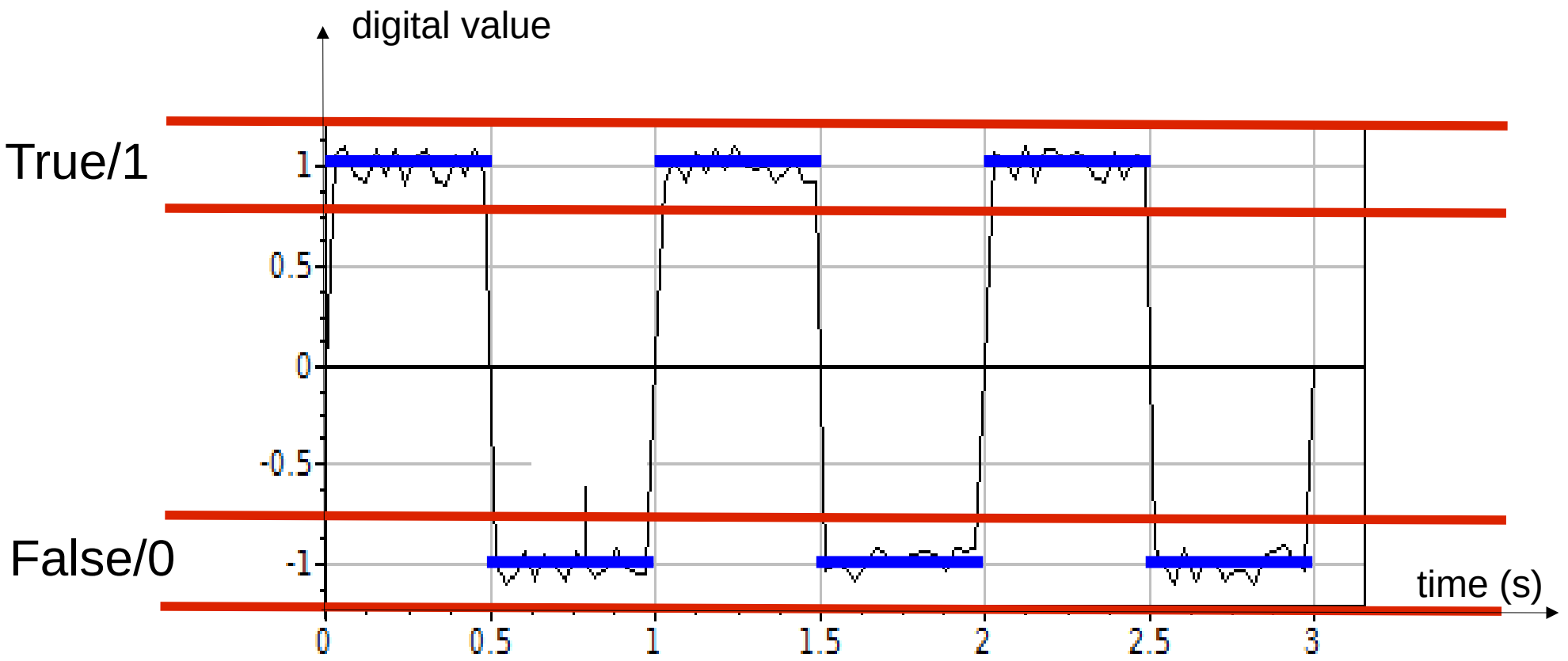
- Instructions

- ....

from Analog ...



… to Digital



logical 0/1 – Boolean True/False – asserted/de-asserted – high/low

Binary representation

- in computing and telecommunications
- a **bit** is a basic *unit of information storage*
- “**bi**nary digi**t**”
- the maximum amount of **information** that can be stored in only two distinct states

`0` or `1`

Bit sequences
(bit) “string” (vs. char string)

```
                0   1 bit    bit
             0111   4 bits   nibble
         01100001   8 bits   byte
0101010110110111  16 bits   half-word
              ...  32 bits   word
              ...  64 bits   word
```

French: “octet”

A **word** is a natural unit of data used by a particular processor design (historically: 8, 16, 24, 32, or 64 bits)

Bit is abbreviated as “b”. e.g., k**b**ps (kilo bits per second)
Byte is abbreviated as “B”. e.g., k**B** (kilo Bytes)

Binary representation ++

With 1 bit, can **represent** 2 **distinct entities**

With 2 bits, can represent 4 distinct entities

...

With **N** bits, can represent $\mathbf{2^N}$ distinct entities

Example: {Red, Green, Blue} encoded as {00, 01, 10}

1 Bit	2 Bits	3 Bits	4 Bits	5 Bits
0	00	000	0000	00000
1	01	001	0001	00001
	10	010	0010	00010
	11	011	0011	00011
		100	0100	00100
		101	0101	00101
		110	0110	00110
		111	0111	00111
			1000	01000
			1001	01001
			1010	01010
			1011	01011
			1100	01100
			1101	01101
			1110	01110
			1111	01111
				10000
				10001
				10010
				10011
				10100
				10101
				10110
				10111
				11000
				11001
				11010
				11011
				11100
				11101
				11110
				11111

Binary representation/approximation of numbers

Binary representation/encoding of **Unsigned Integers** ($\mathbb{N}$)

Binary Coded Decimal (BCD) representation/encoding of **Unsigned Integer** ($\mathbb{N}$) **/ Real** numbers ($\mathbb{R}$)

Binary representation/encoding of **Signed Integer Numbers** ($\mathbb{Z}$)

- Signed magnitude
- One's complement
- Two's complement
- Biased (Excess B)

Fixed-Point binary **approximation**/representation ($\mathbb{Q}$) of **Real** numbers ($\mathbb{R}$)

Floating Point binary **approximation**/representation of **Real** numbers ($\mathbb{R}$)

Binary representation/encoding of Unsigned Integers ($\mathbb{N}$)

"$x_{n-1}x_{n-2}...x_1x_0$" is an n-bit, **base** 2 (binary) encoding of value x

$$x = x_{n-1}2^{n-1} + x_{n-2}2^{n-2} + \cdots + x_1 2^1 + x_0 2^0$$

x_0 Least Significant Bit (**LSB**)

x_{n-1} Most Significant Bit (**MSB**)

- Range: $[0, +2^n - 1] \subseteq \mathbb{N}_0$
- Example

  $0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 1011_2$

  $= 0 + \ldots + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

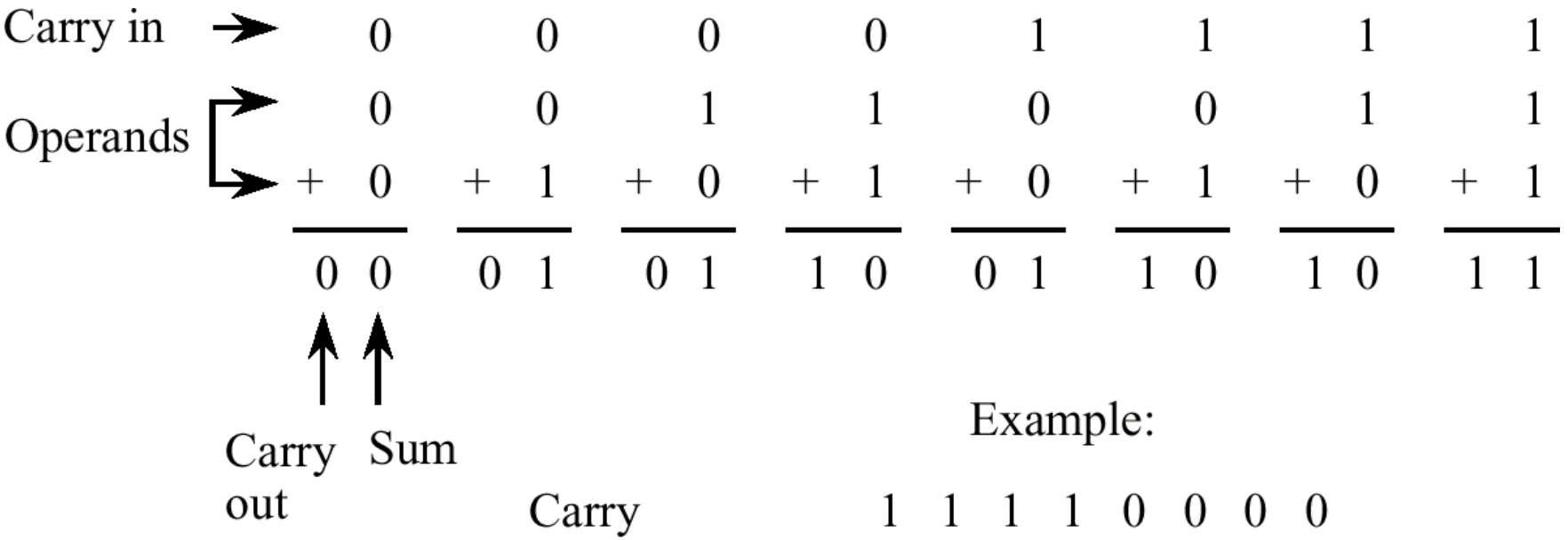
  $= 0 + \ldots + 8 + 0 + 2 + 1 = 11_{10}$

Powers of 2

N	2^N	binary encoding as unsigned integer
0	1	0000 0000 0000 0001
1	2	0000 0000 0000 0010
2	4	0000 0000 0000 0100
3	8	0000 0000 0000 1000
4	16	0000 0000 0001 0000
5	32	0000 0000 0010 0000
6	**6**4	0000 0000 0100 0000
7	128	0000 0000 1000 0000
8	256	0000 0001 0000 0000
9	512	0000 0010 0000 0000
10	**1**,024	0000 0100 0000 0000
11	2,048	0000 1000 0000 0000
12	4,096	0001 0000 0000 0000
13	8,192	0010 0000 0000 0000
14	16,384	0100 0000 0000 0000
15	32,768	1000 0000 0000 0000
16	65,536	1 0000 0000 0000 0000
…	…	…
32	4,294,967,296	1 0000 0000 0000 0000 0000 0000 0000 0000


10100010010100010101
11101100101100101010
0101 THERE ARE 101
001011 ONLY 10 10010
10010 TYPES OF 0010
101110 PEOPLE: 01011
0010 THOSE WHO 000
100 UNDERSTAND 011
110101 BINARY 10101
0001 AND THOSE 000
1010 WHO DON'T 000
10101110110011010101
11001010101010101000
10100100101001010000
10101110110011010101

(intermezzo) Binary addition of Unsigned Integers (ℕ)



Example:

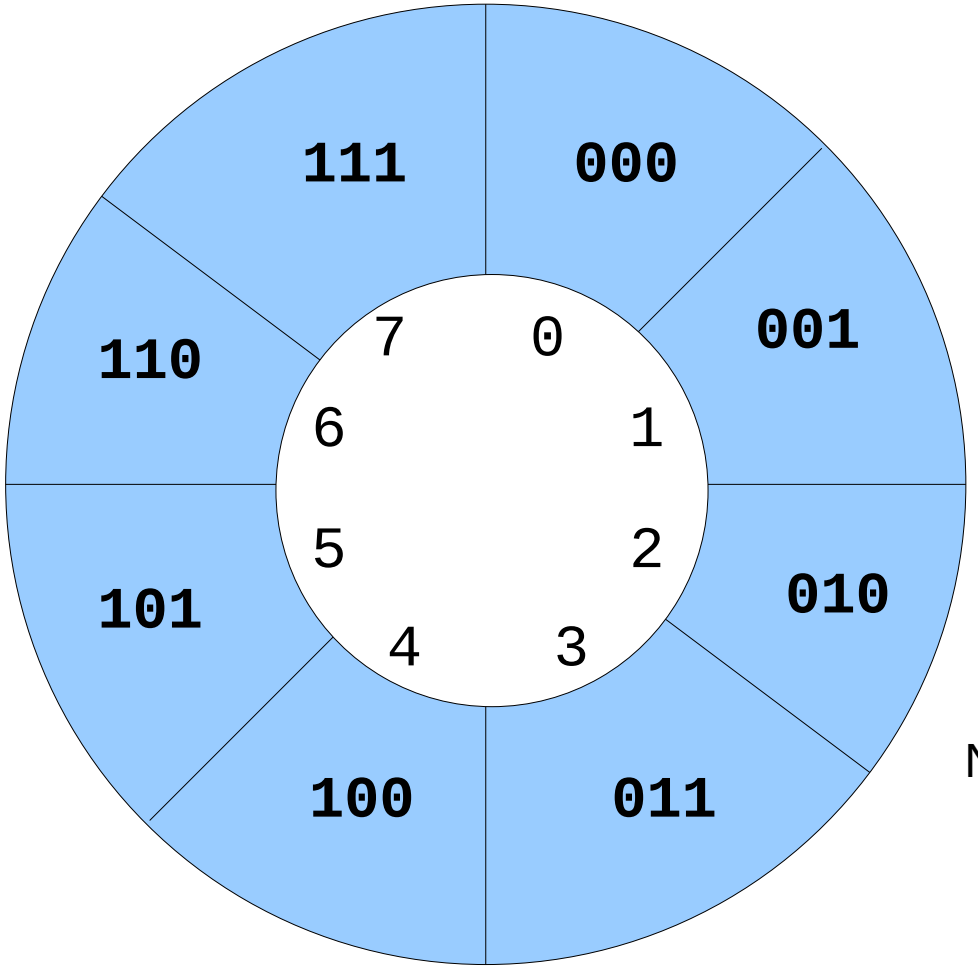
Carry		1	1	1	1	0	0	0	0	
Addend: A		0	1	1	1	1	1	0	0	$(124)_{10}$
Augend: B	+	0	1	0	1	1	0	1	0	$(90)_{10}$
Sum		1	1	0	1	0	1	1	0	$(214)_{10}$

For (much) more detail: see Computer Architecture (slides in presentation.logicDesign.pdf)

unsigned “modulo” arithmetic

$$x = x_{n-1}2^{n-1} + x_{n-2}2^{n-2} + \cdots + x_1 2^1 + x_0 2^0$$

+1 (unsigned): carry gets discarded, no “overflow”



000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

N bits: $[[2^N]] = (2^N) \text{ MOD } 2^N = 0;$
$(2^N) \text{ DIV } 2^N = 1$
$[[2^N+k]] = (2^N+k) \text{ MOD } 2^N = k$

used in for example address calculation (PC)

Binary Coded Decimal (BCD)
representation of unsigned Int (ℕ) / Real (ℝ)

```
Decimal:    0     1     2     3     4     5     6     7     8     9
BCD:     0000  0001  0010  0011  0100  0101  0110  0111  1000  1001
```

Every Decimal digit gets represented by 4 bits (Binary digits)

```
Decimal:             127  :     1      2      7
BCD:       000100100111  :  0001   0010   0111
```

Used mostly in

- Mainframes/servers (financial applications)
- Embedded microcontrollers/small processors (computation <<<)
- Only digital logic (no processor), display (7-segment)

Word size often 10 or 12 decimal digits (*e.g.,* in calculators)

Binary Coded Decimal (BCD)

Advantages:

- Easy to print, display (e.g., 7-segment), ... thanks to **per-digit** conversion
- Numbers such as decimal `0.2` have
  - an **infinite** representation in binary
    (`0.00110011001`1...)
  - a **finite** representation in binary-coded decimal
    (`0000.0010`)
- Scaling by factors of 10 by **shifting** (clever compiler ...)
- 10-based **rounding** is easy

Disadvantages:

- More **complex** to implement +,-,*,/ (+: 15-20% more circuitry)
- Slower
- More storage space required

  e.g., 15_{10}: 1111_{binary} vs. 00010101_{BCD}