# The Modelling and Simulation Process

1. History of Modelling and Simulation
2. Modelling and Simulation Concepts
3. Levels of Abstraction
4. Experimental Frame
5. Validation
6. Studying a mass-spring system
7. The Modelling and Simulation Process

# Modelling and simulation: past

(1950–): Numerical simulations: numerical analysis, statistical analysis, simulation languages (CSSL, discrete-event world views).
focus: performance, accuracy

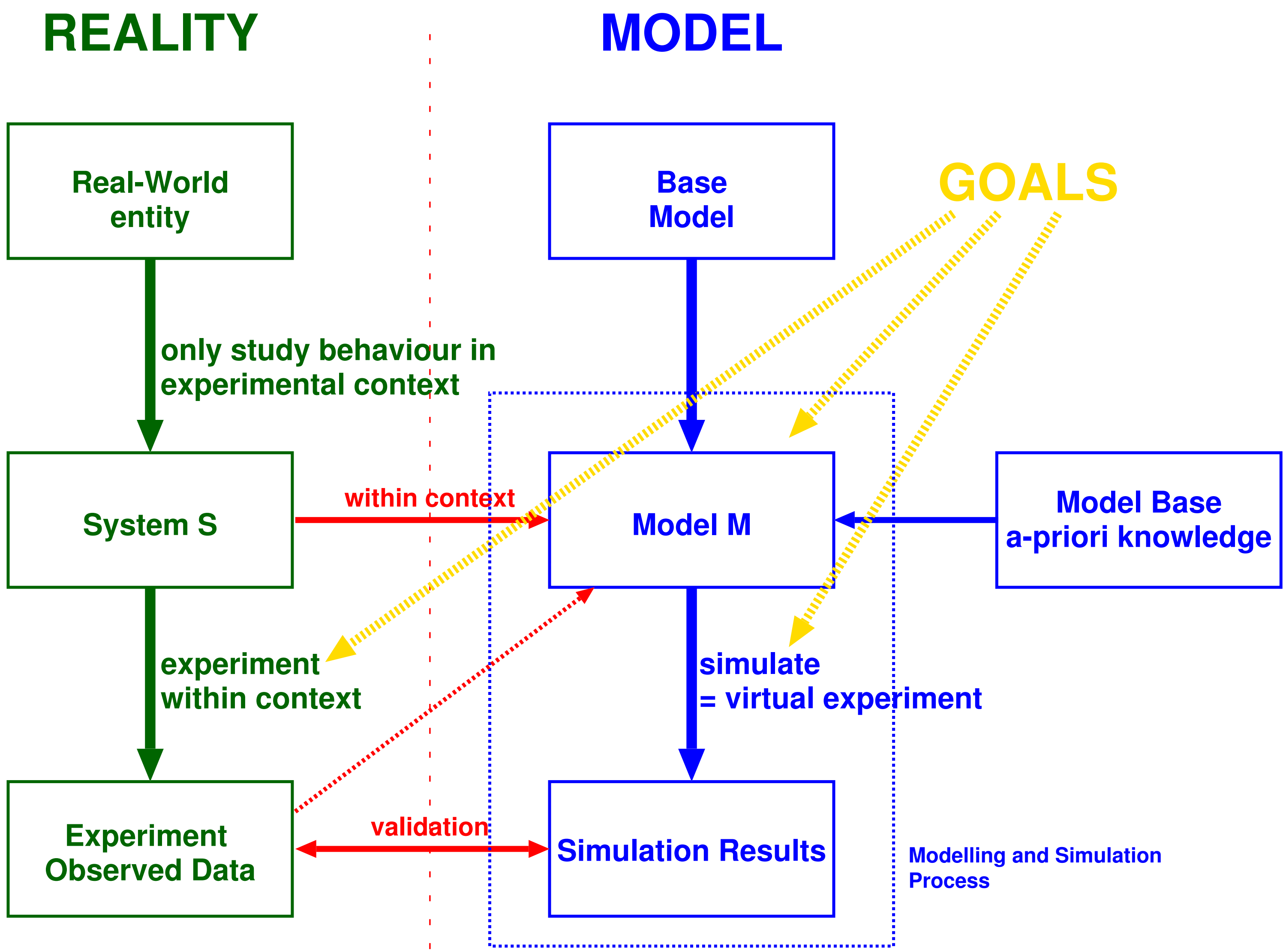
(1981–): Artificial Intelligence: model = knowledge representation
Use AI techniques in modelling, AI uses simulation ("deep" knowledge)
focus: knowledge

(1988–): Object-oriented modelling and simulation
focus: object orientation, later "agents", non-causal modelling

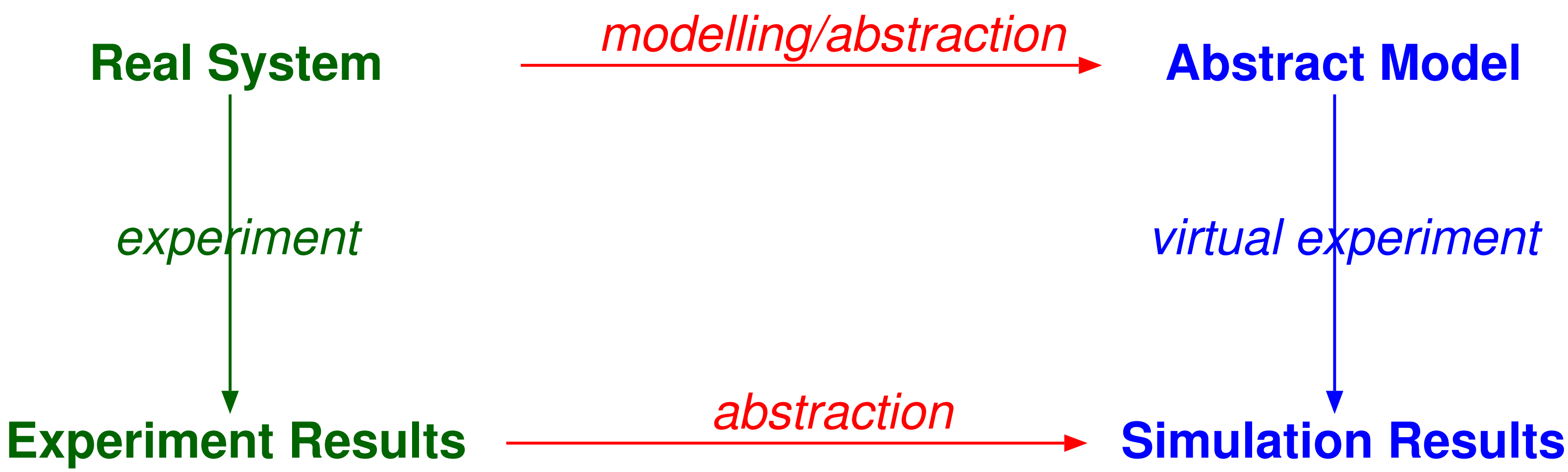
# Modelling and simulation: past, present, future

(1993–): Multi-formalism, Multi-paradigm (2001 –)

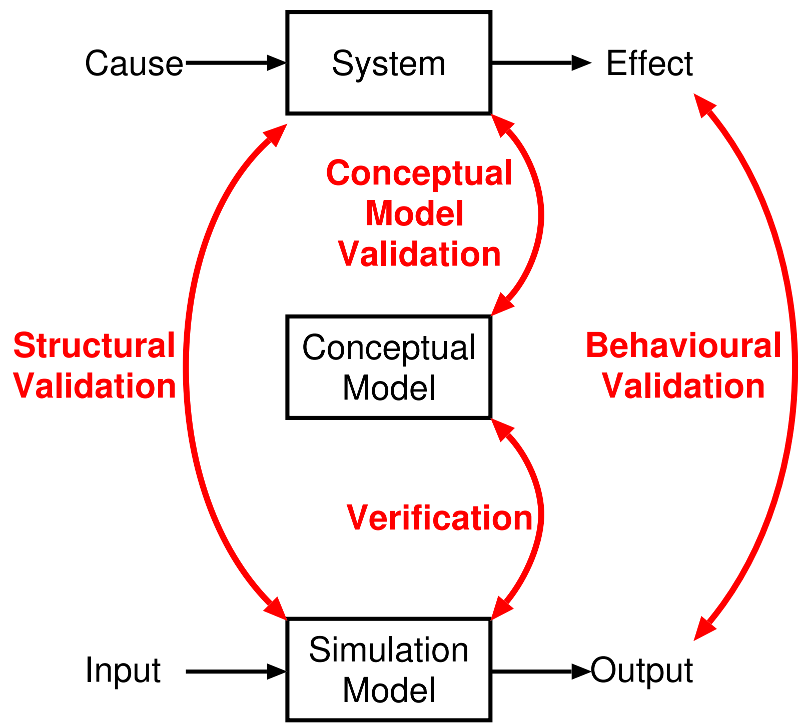
1. Do it right (optimally) the first time (market pressure)
2. Complex systems: **multi-formalism**
3. Hybrid: continuous-discrete, hardware/software
4. **Exchange** (between humans/tools) and **re-use** (validated model)
5. User focus: do not expect user to know details
   (software: glueing of components), need for **tools**


REALITY
MODEL
Real-World entity
Base Model
GOALS
only study behaviour in experimental context
System S
within context
Model M
Model Base a-priori knowledge
experiment within context
simulate = virtual experiment
Experiment Observed Data
validation
Simulation Results
Modelling and Simulation Process

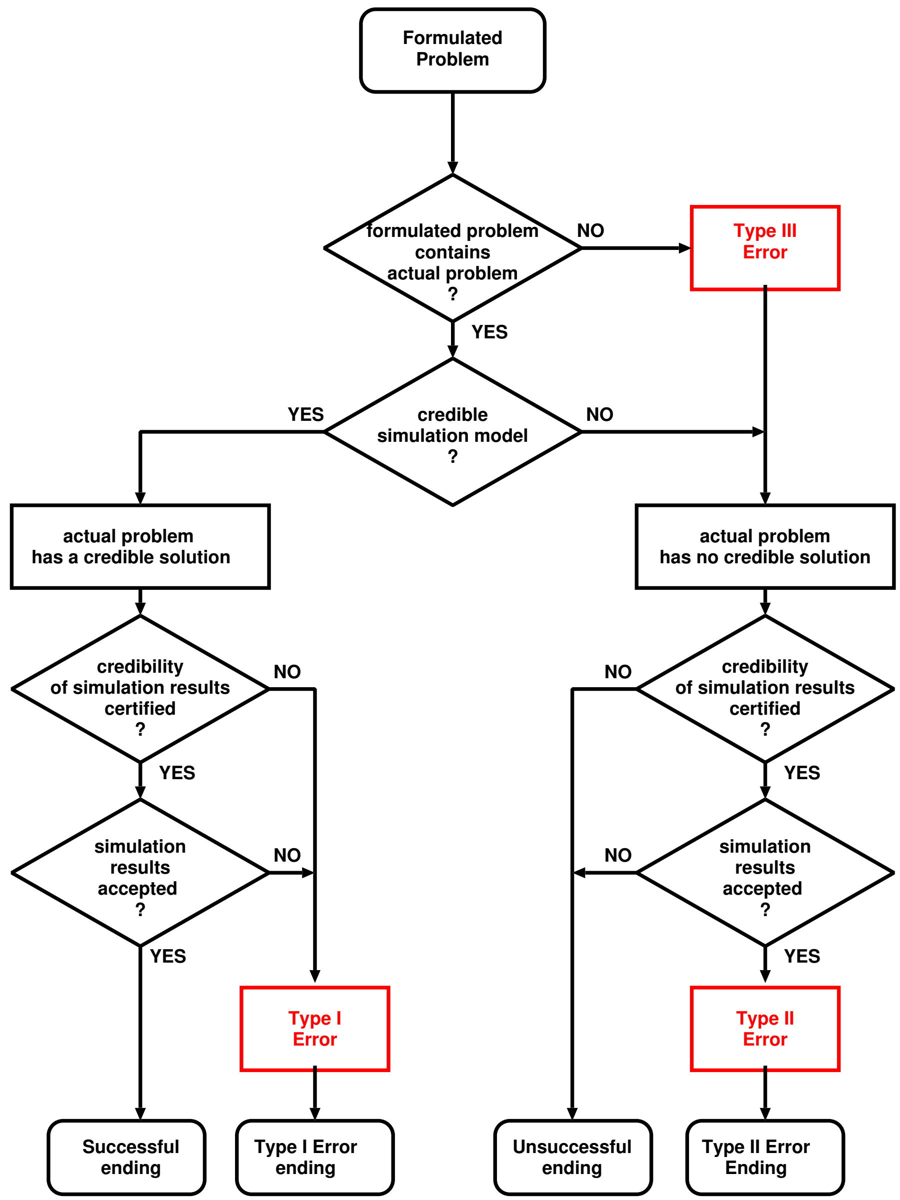
# Behaviour (homo)morphism



# Verification and Validation




Carl Popper: Falsification, Confidence

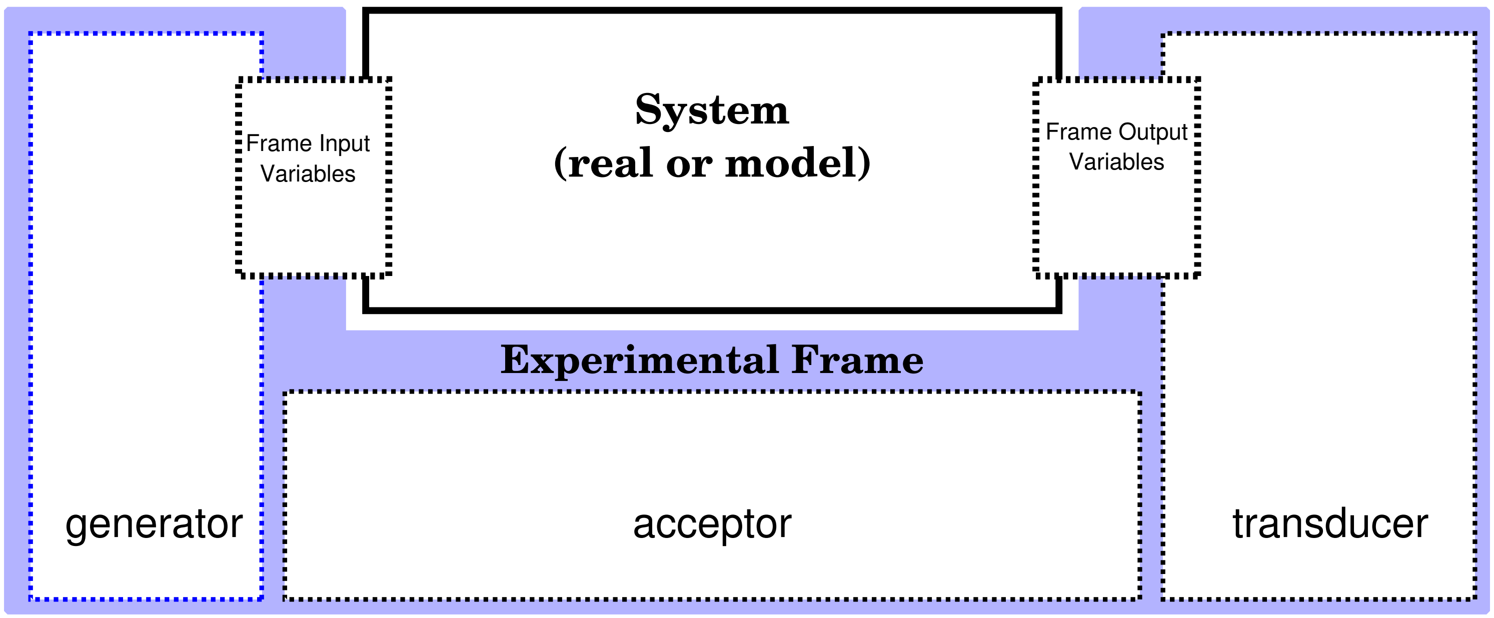

Formulated Problem
formulated problem contains actual problem ?
NO
Type III Error
YES
credible simulation model ?
YES
NO
actual problem has a credible solution
actual problem has no credible solution
credibility of simulation results certified ?
NO
YES
simulation results accepted ?
NO
YES
credibility of simulation results certified ?
NO
YES
simulation results accepted ?
NO
YES
Type I Error
Type II Error
Successful ending
Type I Error ending
Unsuccessful ending
Type II Error Ending

# System, Base Model, Lumped Model

$$D_{BaseModel} \equiv D_{RealSystem}$$

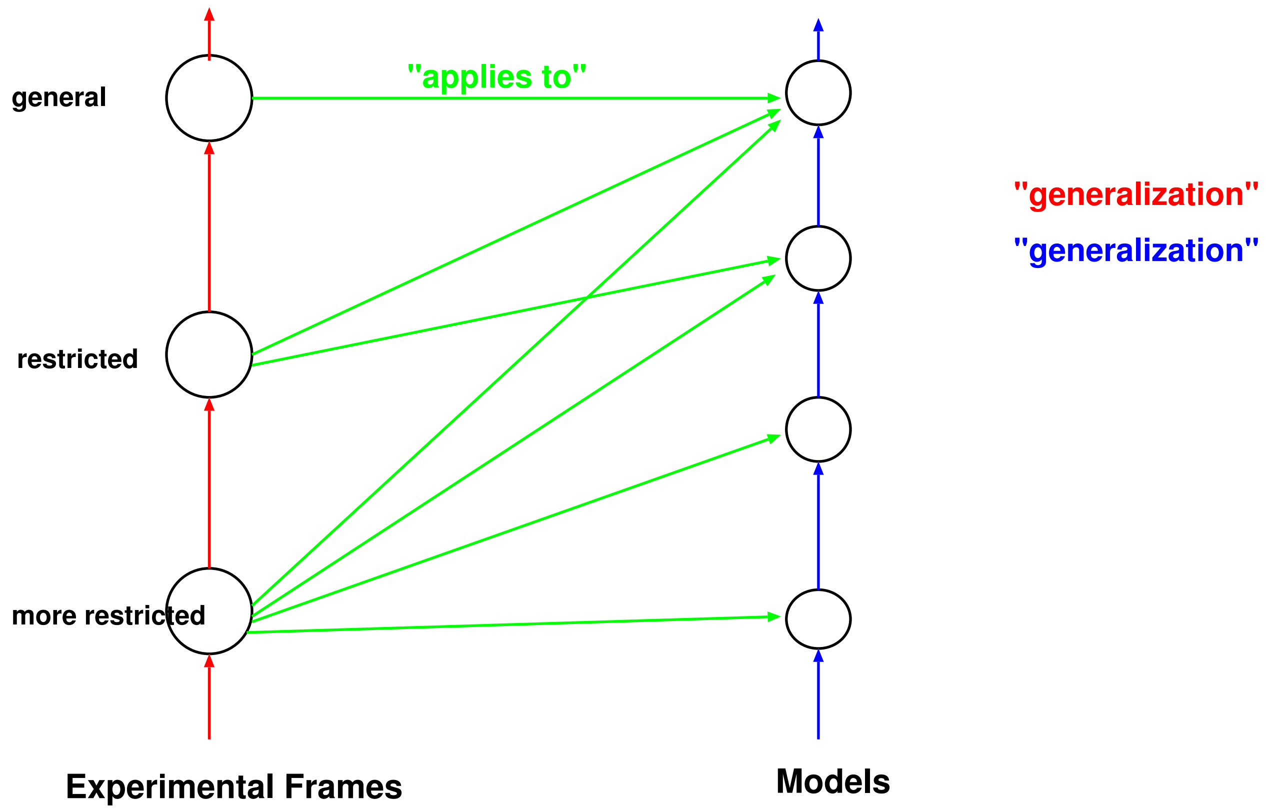
$$D_{LumpedModel} || E \equiv D_{RealSystem} || E$$

# Experimental Frame Structure




$\sim$ Programming Language Types, Pre/Post-conditions

# Models and matching Experimental Frames



# Experimental Frame and Validity

Replicative Validity ($\equiv$: within accuracy bounds):

$$D_{LumpedModel} \| E \equiv D_{BaseModel} \| E$$

Predictive Validity:

$$F_{LumpedModel} \| E \subseteq F_{BaseModel} \| E$$

Structural Validity (morphism $\stackrel{\triangle}{=}$):

$$LumpedModel \| E \stackrel{\triangle}{=} BaseModel \| E$$

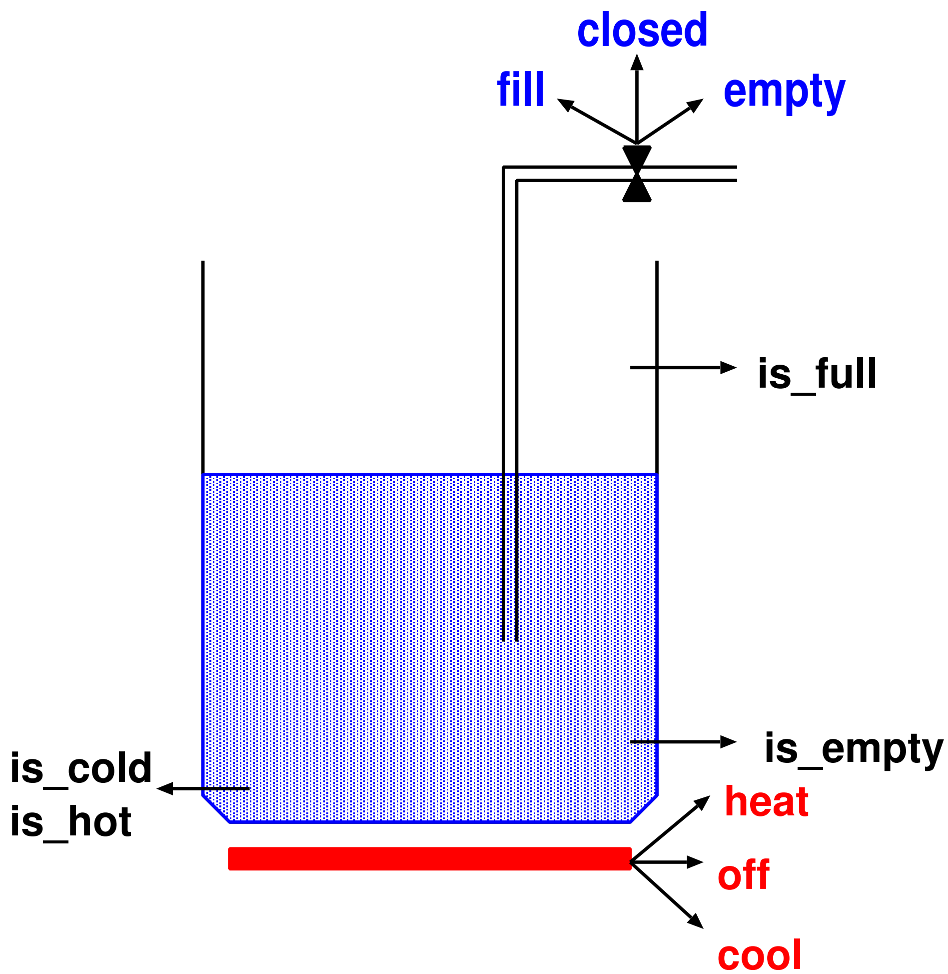
Simulator Verification:

$$D_{Simulator} \equiv D_{LumpedModel}$$

# Modelling (and Simulation) Choices

1. System Boundaries and Constraints: Experimental Frame (EF)
2. Level of Abstraction
3. Formalism(s)
4. Level of Accuracy

# System under study: $T, l$ controlled liquid



# System Boundaries (Experimental Frame)

- Inputs: liquid flow rate, heating/cooling rate
- Outputs: observed level, temperature
- Contraints: no overflow/underflow, one phase only (no boiling)

# Abstraction: detailed (continuous) view, ALG + ODE formalism

Inputs (discontinuous $\rightarrow$ hybrid model):

- Emptying, filling flow rate $\phi$
- Temperature of inflowing liquid $T_{in}$
- Rate of adding/removing heat $W$

Parameters:

- Cross-section surface of vessel $A$
- Specific heat of liquid $c$
- Density of liquid $\rho$

State variables:

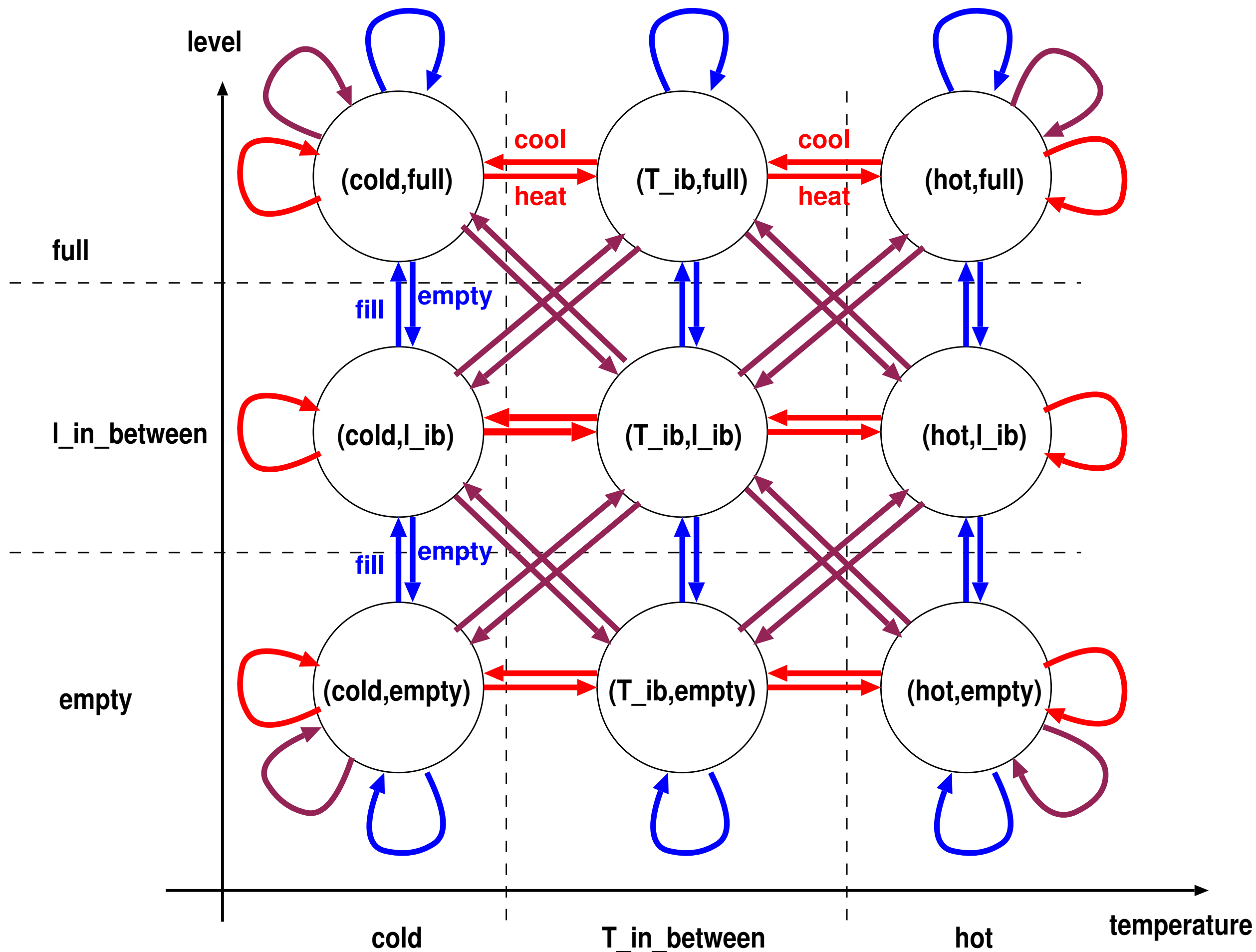
- Temperature $T$
- Level of liquid $l$

Outputs (sensors):

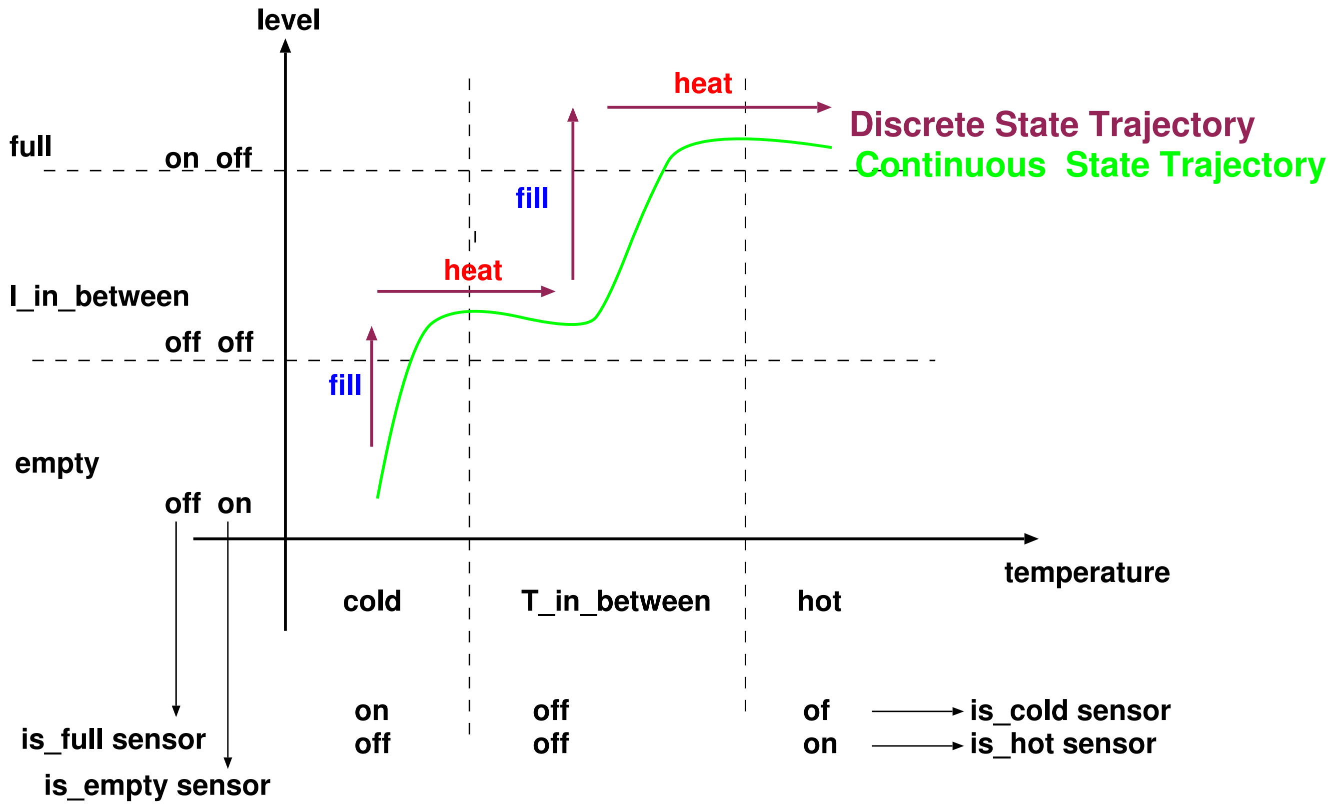
- $is\_low, is\_high, is\_cold, is\_hot$

$$\left\{ \begin{array}{rcl} \frac{dT}{dt} & = & \frac{1}{l}\left[\frac{W}{c\rho A} - \phi(T - T_{in})\right] \\ \frac{dl}{dt} & = & \phi \\ is\_low & = & (l < l_{low}) \\ is\_high & = & (l > l_{high}) \\ is\_cold & = & (T < T_{cold}) \\ is\_hot & = & (T > T_{hot}) \end{array} \right.$$

# Abstraction: high-level (discrete) view, FSA formalism



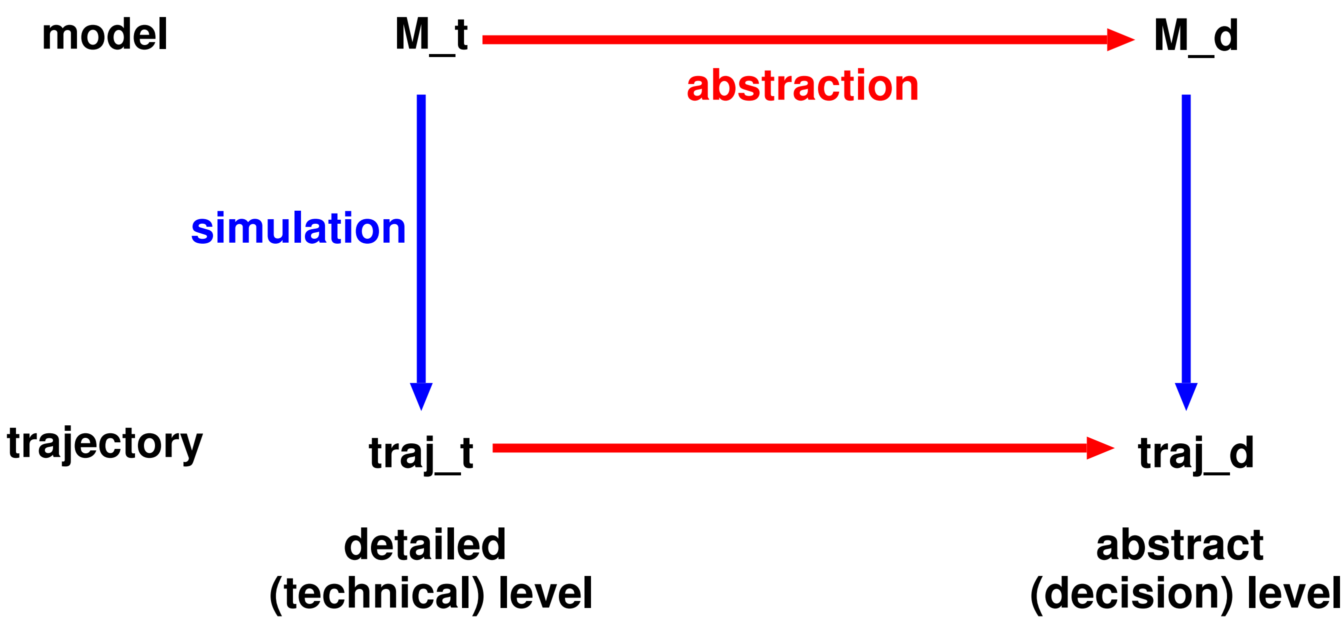
# Levels of abstraction: trajectories (behaviour)



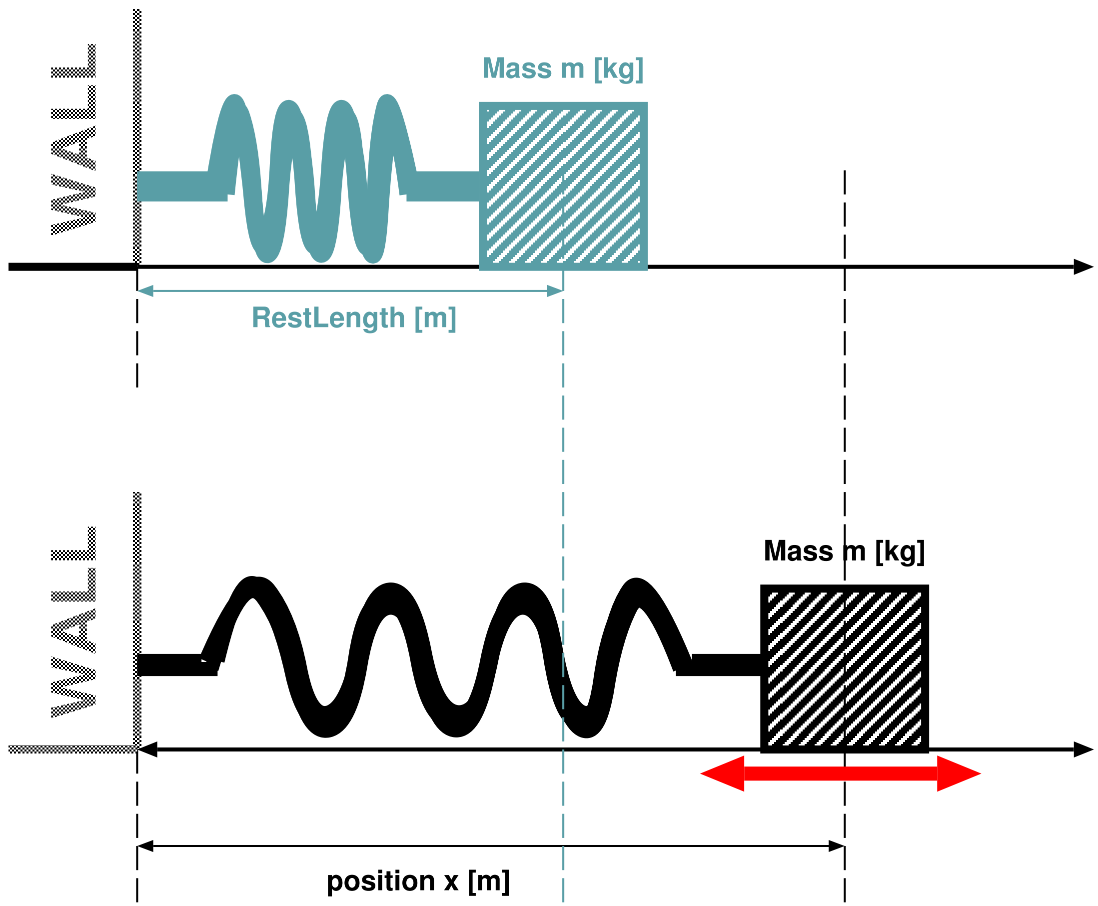
# Levels of accuracy

- Depends on “equality” metric (definition of accuracy)
- Depends on choice of formalism
- Depends on choice of numerical approximation

# Levels of abstraction: behaviour morphism



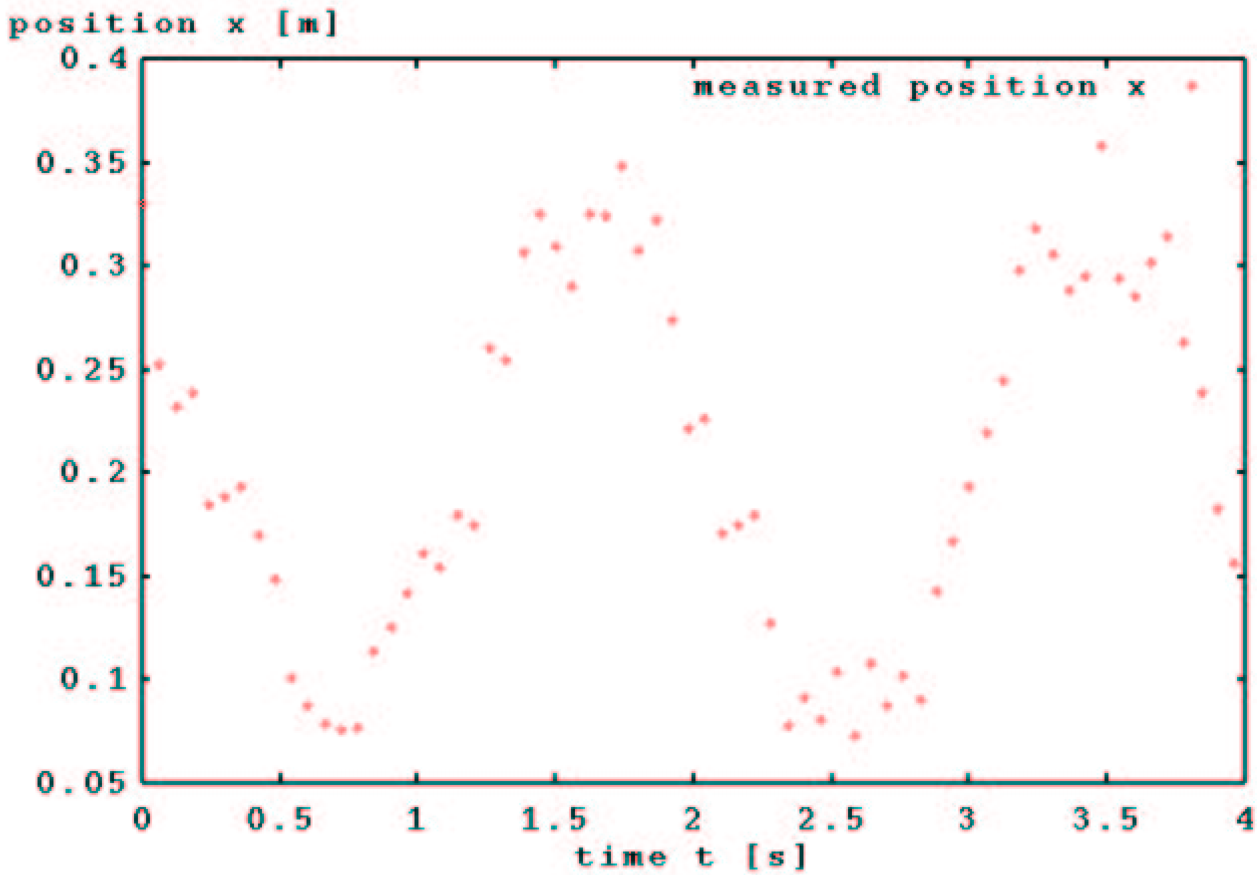
# A Modelling and Simulation Exercise: the Mass-Spring system



# Knowledge Sources

- A Priori Knowledge: Laws of Physics
- Goals, Intentions: Predict trajectory given Initial Conditions, “optimise” behaviour, . . .
    1. Analysis
    2. Design
    3. Control
- Measurement Data

# Measured Data



# Experimental Frame

- Room Temperature, normal humidity, . . .
- Frictionless, Ideal Spring, . . .
- Apply deviation from rest position
- Observe position as function of time

# Structure Characterisation

- $n-1$-order polynomial will perfectly fit $n$ data points
- Ideal Spring: *Feature* = maximum amplitude constant
- Spring with Damping: *Feature* = amplitute decreases

$\Rightarrow$ Ideal Spring

# Building the model from a-priori knowledge

Newton's Law

$$F = M\frac{d^2\Delta x}{dt}$$

Ideal Spring

$$F = -K\Delta x$$

$$\downarrow$$

$$\frac{d^2\Delta x}{dt^2} = -\frac{K}{M}\Delta x$$

# Model representation

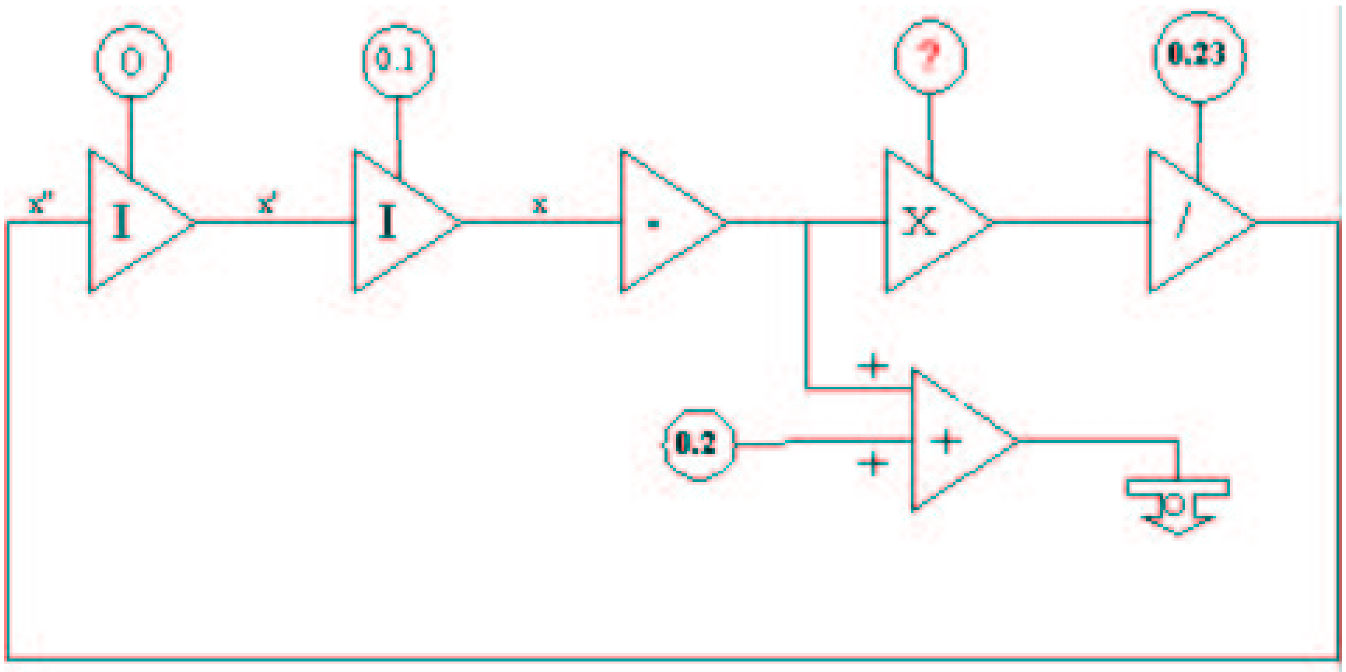
```
CLASS Spring "Ideal Spring": DAEmodel :=
{
 OBJ F_left: ForceTerminal,
 OBJ F_right: ForceTerminal,

 OBJ RestLength: LengthParameter,
 OBJ SpringConstant: SCParameter,

 OBJ x: LengthState,
 OBJ v: SpeedState,

 F_left - F_right = - SpringConstant * (x - RestLength),
 DERIV([ x, [t,] ]) = v,

 EF_assert( x - RestLenght < RestLength/100),
},
```

# From Model to Simulation

Block-diagrams

analog computers, Continuous System Modelling Program (CSMP)

- From (algebraic) equation to Block Diagram
- Higher order differential equations

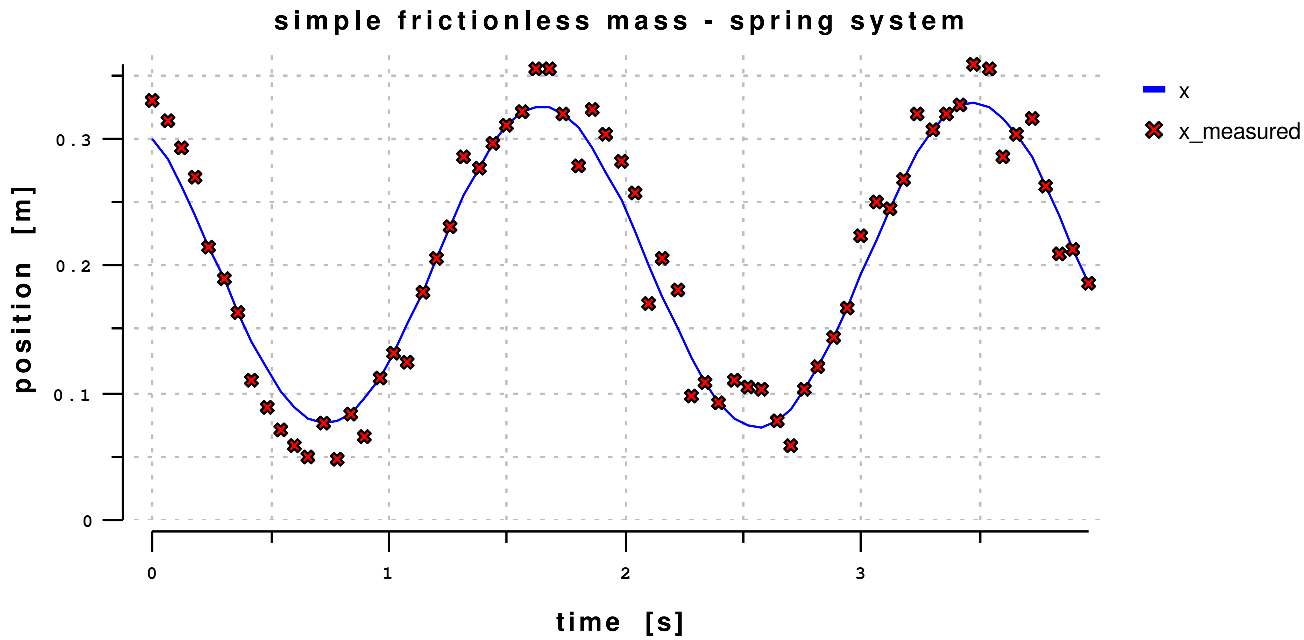


# Time-slicing Simulator

# Experimentation

1. Model
2. Parameters (constant for each simulation run)
3. Initial Conditions
4. Input (file, interactive, real system)
5. Output (file, plot, real system)
6. Solver Configuration
7. Experiment type
   (simulation, optimization, parameter estimation $=$ model calibration)

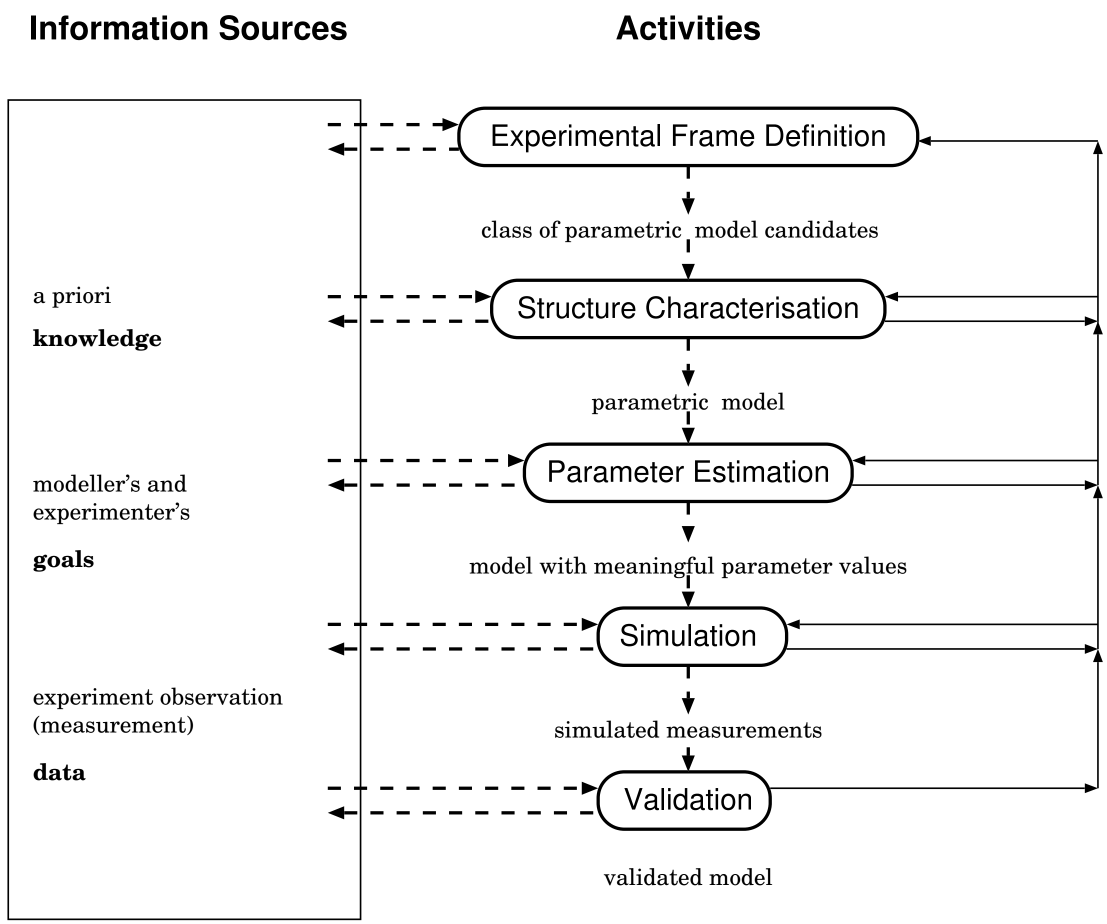
# Model Calibration: Parameter Fit

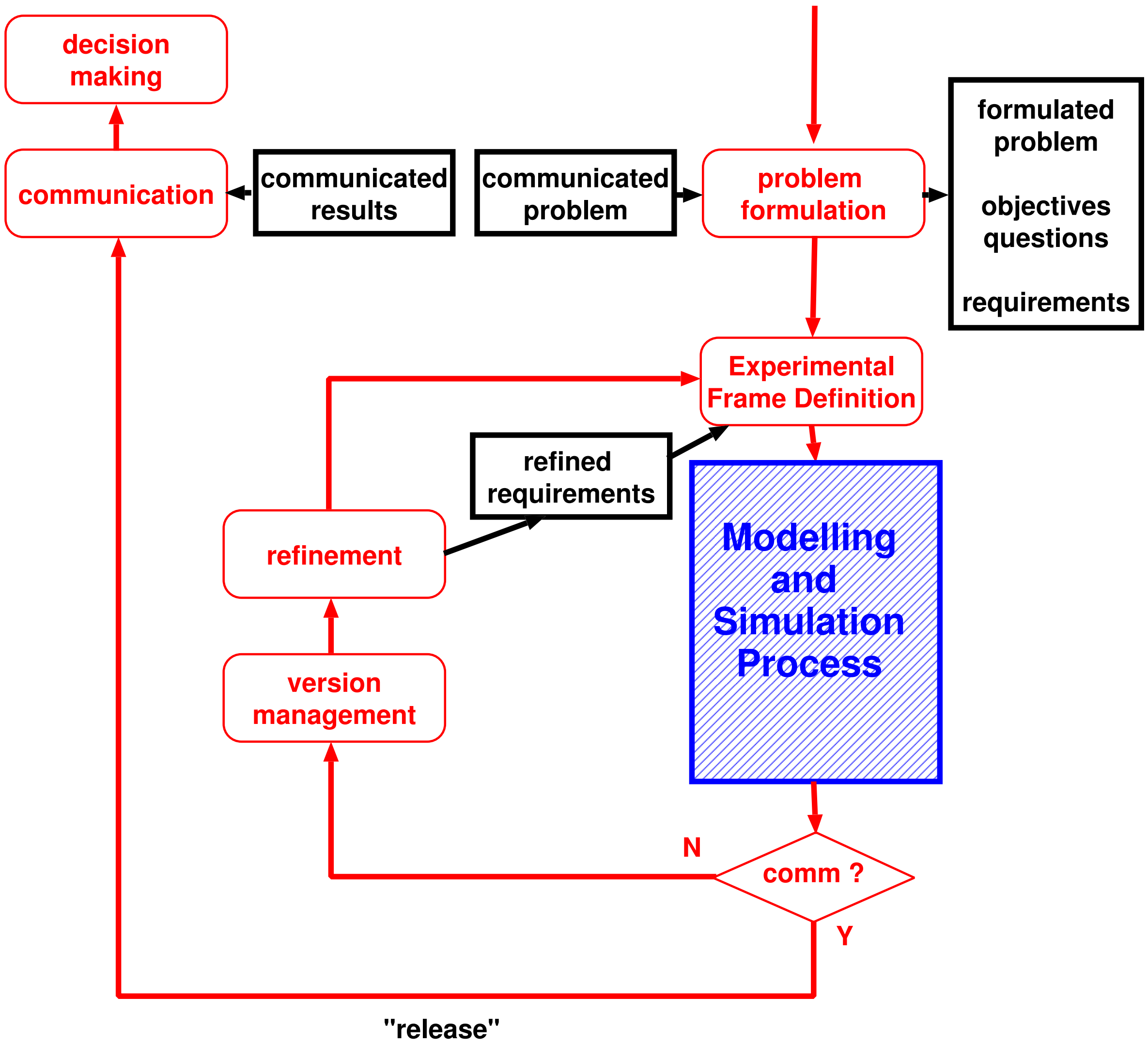


# From Here On . . .

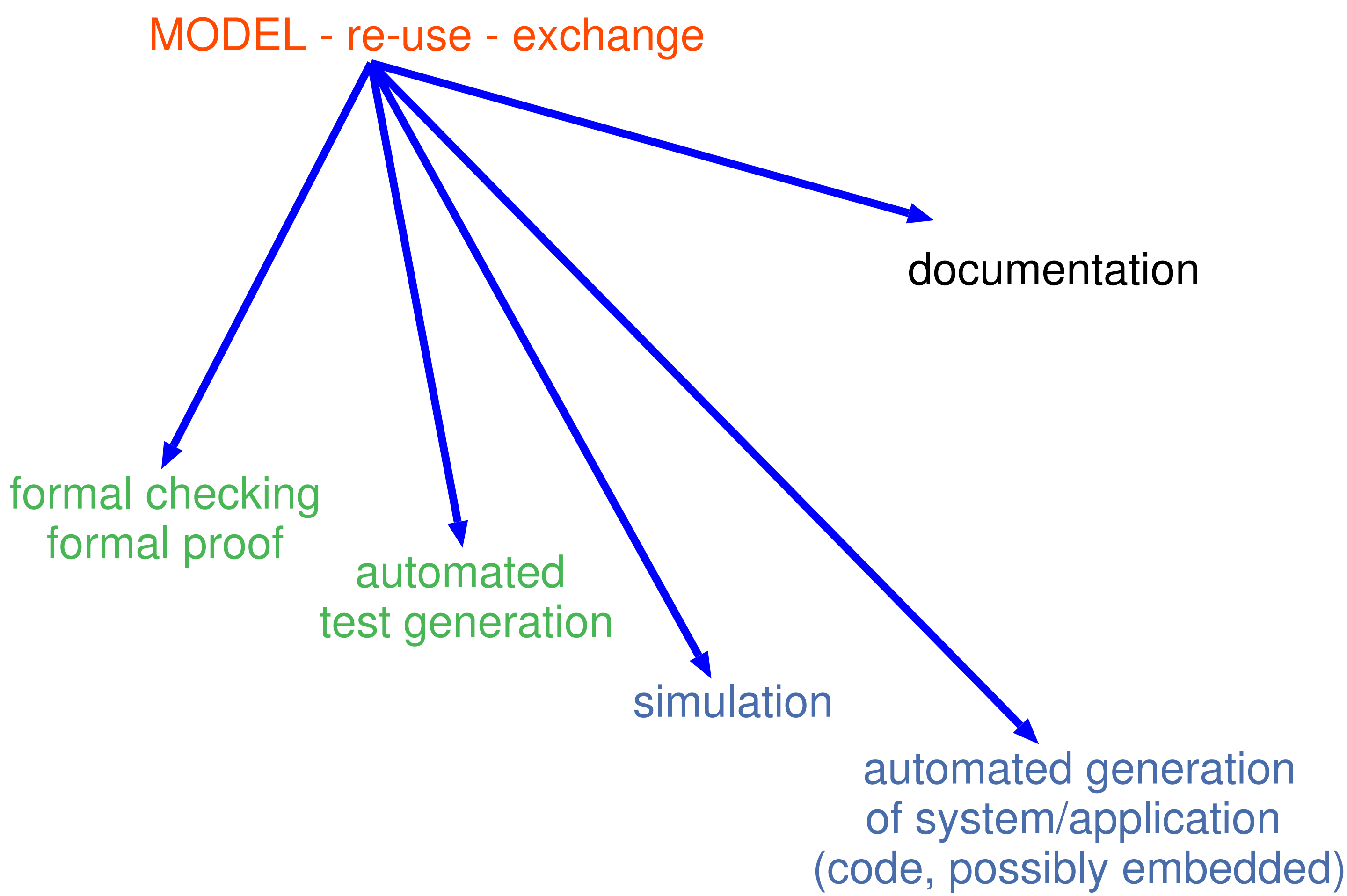
- Virtual Experiments: simulation, optimisation, what-if, . . .
- Validation/Falsification

# The Modelling and Simulation *Process*




decision
making
communication
communicated
results
communicated
problem
problem
formulation
formulated
problem
objectives
questions
requirements
Experimental
Frame Definition
refined
requirements
refinement
version
management
Modelling
and
Simulation
Process
N
comm ?
Y
"release"

# Model uses


MODEL - re-use - exchange
documentation
formal checking
formal proof
automated
test generation
simulation
automated generation
of system/application
(code, possibly embedded)