The International Association for the Properties of Water and Steam

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Supplementary Release on Backward Equations for Pressure as a Function of Enthalpy and Entropy *p*(*h*,*s*) to the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam

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The backward equations for pressure as a function of enthalpy and entropy $p(h,s)$ provided in this release are recommended to supplement the "IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" (IAPWS-IF97) [1], [2]. Further details about the equations $p(h,s)$ can be found in the corresponding article by H.-J. Kretzschmar et al. [3].

Further information about this supplementary release, the IAPWS-IF97 and other releases issued by IAPWS can be obtained from the Executive Secretary of IAPWS or from http://www.iapws.org .

Contents

1 Nomenclature

Thermodynamic quantities:

f	Specific Helmholtz free energy
g	Specific Gibbs free energy
h	Specific enthalpy
p	Pressure
s	Specific entropy
T	Absolute temperature [a]
Δ	Difference in any quantity
η	Reduced enthalpy, $\eta = h/h^*$
π	Reduced pressure, $\pi = p/p^*$
σ	Reduced entropy, $\sigma = s/s^*$

Root-mean-square value:

$$\Delta x_{\mathrm{RMS}} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\Delta x_n)^2}$$

where Δx_n can be either absolute or percentage difference between the corresponding quantities x; N is the number of Δx_n values (100 million random points over the range of validity).

Superscripts:

97	Quantity or equation of IAPWS-IF97
*	Reducing quantity
"	Saturated vapor state

Subscripts:

1	Region 1
2	Region 2
2a	Subregion 2a
2b	Subregion 2b
2c	Subregion 2c
2ab	Boundary between subregions 2a and 2b
2bc	Boundary between subregions 2b and 2c
3	Region 3
5	Region 5
B23	Boundary between regions 2 and 3
max	Maximum value of a quantity
RMS	Root-mean-square value of a quantity
sat	Saturation state
tol	Tolerance of a quantity

[a] Note: T denotes absolute temperature on the International Temperature Scale of 1990 (ITS-90).

2 Background

The Industrial Formulation IAPWS-IF97 for the thermodynamic properties of water and steam [1], [2] contains basic equations, saturation equations and equations for the most often used backward functions $T^{97}(p,h)$ and $T^{97}(p,s)$ valid in the liquid region 1 and the vapor region 2; see Figure 1.

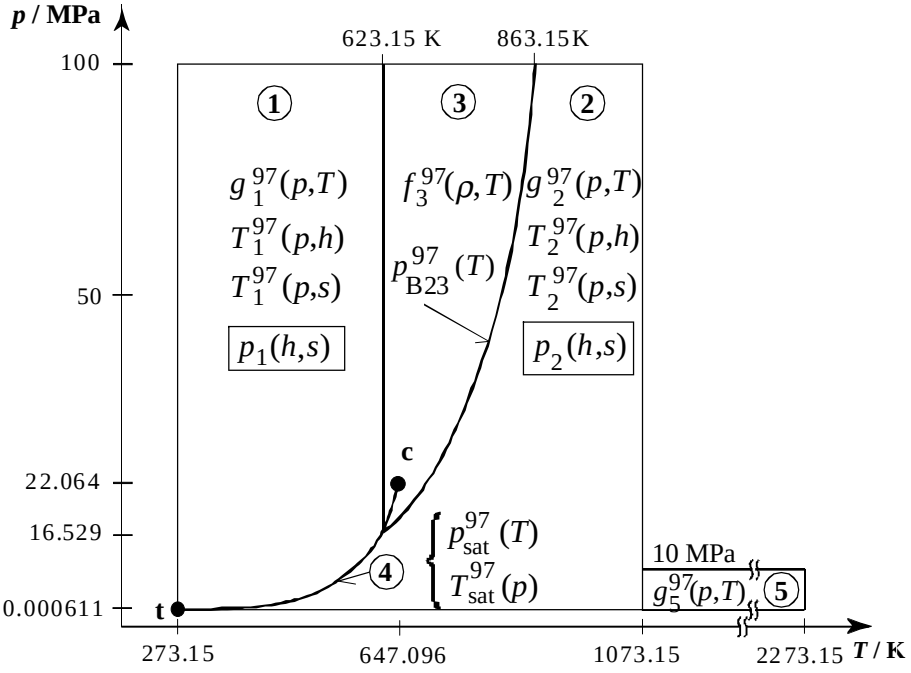

Figure 1. Equations of the IAPWS-IF97 and backward equations $p(h,s)$

In thermodynamic process modelling, the backward functions $p(h,s)$ and $T(h,s)$ for water and steam are also used. The calculations of these functions from the IAPWS-IF97 Formulation are time consuming, because they require two-dimensional iterations from $h^{97}(p,T)$ and $s^{97}(p,T)$, where $h^{97}(p,T)$ and $s^{97}(p,T)$ are derivatives of the IAPWS-IF97 basic equations $g^{97}(p,T)$ of regions 1 and 2. Although the functions $p(h,s)$ and $T(h,s)$ may not often be used in process modelling, the computing time to calculate them is very high in comparison with the times for other property functions.

In order to avoid such iterations, this release provides equations for the backward function $p(h,s)$ valid in regions 1 and 2; see Figure 1. With the pressure $p(h,s)$, the temperature $T(h,s)$ can be calculated by using the IAPWS-IF97 backward equations $T^{97}(p,h)$[1)]. The numerical consistencies of calculated p and T to the IAPWS-IF97 equations $g^{97}(p,T)$ are sufficient for most applications in heat cycle and steam turbine calculations. For applications where the demands on numerical consistency are extremely high, iterations using the IAPWS-IF97 equations may be necessary. In these cases, the equations $p(h,s)$ can be used for

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1) The alternative use of the IAPWS-IF97 backward equations $T^{97}(p,s)$ leads to lower numerical consistency.

calculating very accurate starting values.

The backward equations $p(h,s)$ can only be used in their ranges of validity described in Sections 5 and 6. They should not be used for determining any thermodynamic derivatives.

In any case, depending on the application, a conscious decision is required whether to use the backward equations $p(h,s)$ or to calculate the corresponding values $p(h,s)$ by iterations from the basic equations of IAPWS-IF97.

3 Numerical Consistency Requirements

The permissible values Δp_{tol} for the numerical consistency for the equations $p(h,s)$ can be estimated from the total differential

$$\Delta p_{\mathrm{tol}} = \left(\frac{\partial p}{\partial h}\right)_s \Delta h_{\mathrm{tol}} + \left(\frac{\partial p}{\partial s}\right)_h \Delta s_{\mathrm{tol}} \ ,$$

where Δh_{tol} and Δs_{tol} are values determined by IAPWS [4] as a result of an international survey. Table 1 shows these values and the resulting numerical consistencies Δp_{tol} for regions 1 and 2. Because of the weak dependence on pressure, it is suitable to divide the liquid region 1 into two ranges by using the pressure line p = 2.5 MPa. In the vapor region 2, the numerical consistency requirement is higher for entropies greater than or equal to $5.85\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$.

For the numerical consistency ΔT_{tol} of the backward function $T(h,s)$, the values that IAPWS had determined for the backward equations $T^{97}(p,h)$ and $T^{97}(p,s)$ [5] have been taken.

Table 1. Numerical consistency values $|\Delta h|_{\mathrm{tol}}$ and $|\Delta s|_{\mathrm{tol}}$ of [4], resulting values $|\Delta p|_{\mathrm{tol}}$ required for $p(h,s)$, and values $|\Delta T|_{\mathrm{tol}}$ of [5] required for $T(h,s)$

Region		$\lvert\Delta h\rvert_{\mathrm{tol}}$	$\lvert\Delta s\rvert_{\mathrm{tol}}$	$\lvert\Delta p\rvert_{\mathrm{tol}}$		$\lvert\Delta T\rvert_{\mathrm{tol}}$
1		$80\ \mathrm{J\ kg^{-1}}$	$0.10\ \mathrm{J\ kg^{-1}\ K^{-1}}$	$p \le 2.5$ MPa $p > 2.5$ MPa	0.6 % 15 kPa	25 mK
2	$s < 5.85\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$	$80\ \mathrm{J\ kg^{-1}}$	$0.10\ \mathrm{J\ kg^{-1}\ K^{-1}}$	0.0088 %		25 mK
	$s \ge 5.85\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$	$32\ \mathrm{J\ kg^{-1}}$	$0.04\ \mathrm{J\ kg^{-1}\ K^{-1}}$	0.0035 %		10 mK

4 Structure of the Equation Set

The equation set consists of backward equations $p(h,s)$ for the IAPWS-IF97 regions 1 and 2, see Figure 1.

Region 1 is defined by:

$$p_{\mathrm{sat}}^{97}(T) \le p \le 100\ \mathrm{MPa} \quad \text{and} \quad 273.15\ \mathrm{K} \le T \le 623.15\ \mathrm{K}$$

where $p_{\mathrm{sat}}^{97}(T)$ is the saturation pressure equation of IAPWS-IF97.

Region 2 is defined by:

$$611.213\ \mathrm{Pa} \le p < p_{\mathrm{sat}}^{97}(T) \quad \text{for} \quad 273.15\ \mathrm{K} \le T \le 623.15\ \mathrm{K}$$

$$611.213\ \mathrm{Pa} \le p \le p_{\mathrm{B23}}^{97}(T) \quad \text{for} \quad 623.15\ \mathrm{K} < T \le 863.15\ \mathrm{K}$$

$$611.213\ \mathrm{Pa} \le p \le 100\ \mathrm{MPa} \quad \text{for} \quad 863.15\ \mathrm{K} < T \le 1073.15\ \mathrm{K}$$

where $p_{\mathrm{B23}}^{97}(T)$ represents the B23-equation of IAPWS-IF97. This equation roughly describes an isentropic line; the entropy values along this line are between $s = 5.047\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$ and $s = 5.261\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$.

5 Backward Equation *p*(*h*,*s*) for Region 1

5.1 The Equation

The backward equation $p(h,s)$ for region 1 has the following dimensionless form

$$\frac{p_1(h,s)}{p^*} = \pi(\eta,\sigma) = \sum_{i=1}^{19} n_i (\eta + 0.05)^{I_i} (\sigma + 0.05)^{J_i} \ , \tag{1}$$

where $\pi = p/p^*$, $\eta = h/h^*$, and $\sigma = s/s^*$ with $p^* = 100$ MPa, $h^* = 3400$ kJ kg^{-1}, and $s^* = 7.6$ kJ kg^{-1} K^{-1}. The coefficients n_i and exponents I_i and J_i of Eq. (1) are listed in Table 2.

Table 2. Coefficients and exponents of the backward equation $p_1(h,s)$ for region 1 in its dimensionless form, Eq. (1)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	$-0.691\,997\,014\,660\,582$	11	1	4	$-0.319\,947\,848\,334\,300 \times 10^{3}$
2	0	1	$-0.183\,612\,548\,787\,560 \times 10^{2}$	12	1	6	$-0.928\,354\,307\,043\,320 \times 10^{3}$
3	0	2	$-0.928\,332\,409\,297\,335 \times 10^{1}$	13	2	0	$0.303\,634\,537\,455\,249 \times 10^{2}$
4	0	4	$0.659\,639\,569\,909\,906 \times 10^{2}$	14	2	1	$-0.650\,540\,422\,444\,146 \times 10^{2}$
5	0	5	$-0.162\,060\,388\,912\,024 \times 10^{2}$	15	2	10	$-0.430\,991\,316\,516\,130 \times 10^{4}$
6	0	6	$0.450\,620\,017\,338\,667 \times 10^{3}$	16	3	4	$-0.747\,512\,324\,096\,068 \times 10^{3}$
7	0	8	$0.854\,680\,678\,224\,170 \times 10^{3}$	17	4	1	$0.730\,000\,345\,529\,245 \times 10^{3}$
8	0	14	$0.607\,523\,214\,001\,162 \times 10^{4}$	18	4	4	$0.114\,284\,032\,569\,021 \times 10^{4}$
9	1	0	$0.326\,487\,682\,621\,856 \times 10^{2}$	19	5	0	$-0.436\,407\,041\,874\,559 \times 10^{3}$
10	1	1	$-0.269\,408\,844\,582\,931 \times 10^{2}$				

Computer-program verification

To assist the user in computer-program verification of Eq. (1), Table 3 contains test values for the calculated pressure.

Table 3. Selected pressure values calculated from the backward equation $p_1(h,s)$, Eq. (1) [a]

$h / \left(\text{kJ kg}^{-1}\right)$	$s / \left(\text{kJ kg}^{-1}\ \text{K}^{-1}\right)$	$p_1(h,s)$ / MPa
0.001	0	$9.800\,980\,612 \times 10^{-4}$
90	0	$9.192\,954\,727 \times 10^{1}$
1500	3.4	$5.868\,294\,423 \times 10^{1}$

[a] It is recommended to verify programmed functions using 8 byte real values for all variables.

5.2 Numerical Consistency with the Basic Equation of IAPWS-IF97

The maximum pressure differences and related root-mean-square differences between $p_1(h,s)$, Eq. (1), and the IAPWS-IF97 basic equation $g_1^{97}(p,T)$ for pressures less than or equal to 2.5 MPa or greater than this value are listed in Table 4.

Table 4. Maximum differences and root-mean-square differences between pressures calculated from Eq. (1) and from the IAPWS-IF97 basic equation $g_1^{97}(p,T)$ in comparison with the permissible differences

	$\lvert\Delta p\rvert_{\text{tol}}$	$\lvert\Delta p\rvert_{\text{max}}$	$(\Delta p)_{\text{RMS}}$
$p \le 2.5$ MPa	0.60 %	0.55 %	0.11 %
$p > 2.5$ MPa	15 kPa	14 kPa	6 kPa

6 Backward Equations $p(h,s)$ for Region 2

6.1 Subregions

Region 2 is covered by three $p(h,s)$ equations, corresponding to the three $T_2^{97}(p,h)$ backward equations or to the three $T_2^{97}(p,s)$ backward equations for the IAPWS-IF97. Figure 2 shows the way in which region 2 is divided into the three subregions 2a, 2b and 2c for the backward equations.

The boundary between subregions 2a and 2b corresponds to the isobar p = 4 MPa. In order to decide which $p(h,s)$ equation, 2a or 2b, must be used for given values of h and s, the boundary equation $h_{2ab}(s)$, Eq. (2), has to be used. This equation is a polynomial of the third degree and reads

$$\frac{h_{2ab}(s)}{h^*} = \eta(\sigma) = n_1 + n_2\,\sigma + n_3\,\sigma^2 + n_4\,\sigma^3\ , \qquad (2)$$

where $\eta = h/h^*$ and $\sigma = s/s^*$ with $h^* = 1$ kJ kg^{-1} and $s^* = 1$ kJ kg^{-1} K^{-1}. The coefficients n_1 to n_4 of Eq. (2) are listed in Table 5.

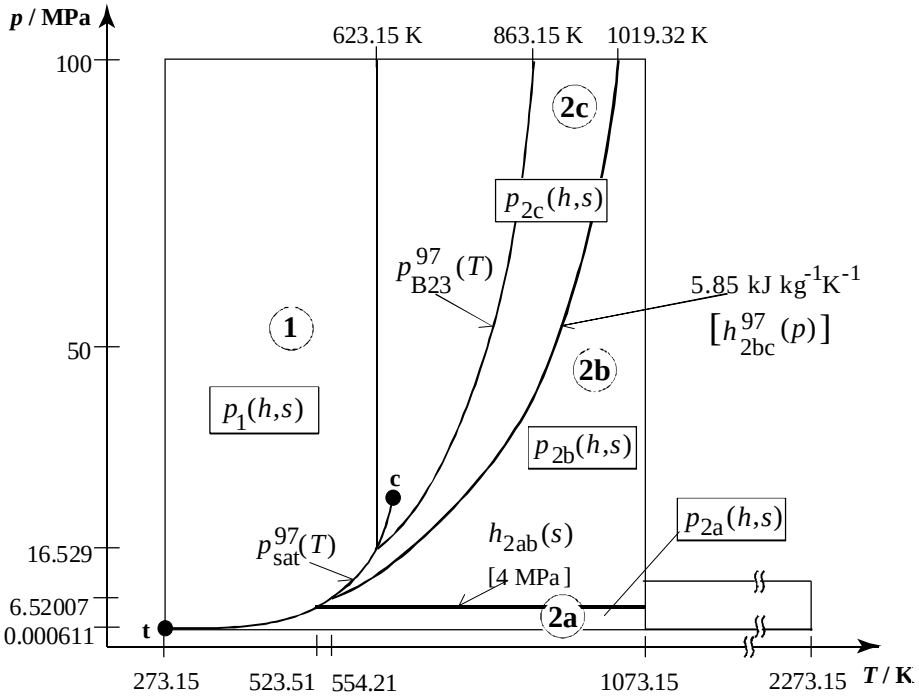

Figure 2. Division of region 2 into three subregions 2a, 2b, 2c for the backward equations $p(h,s)$

The range of equation $h_{2ab}(s)$ is from $s''(p = 4\,\text{MPa})$ on the saturated vapor line to $s_2^{97}(p = 4\,\text{MPa}, T = 1073.15\,\text{K})$; see Figure 2. Based on its simple form, Eq. (2) does not exactly describe the isobaric line $p = 4$ MPa. The maximum pressure deviation was determined as

$$|\Delta p_{2ab}|_{max} = |\, p_2^{97}(h_{2ab}(s_2^{97}), s_2^{97}) - 4\ \text{MPa}\,| = 22\ \text{kPa},$$

where p_2^{97} was obtained by iteration and $s_2^{97}(p = 4\,\text{MPa}, T)$.

Table 5. Numerical values of the coefficients of the equation $h_{2ab}(s)$ in its dimensionless form, Eq. (2), for defining the boundary between subregions 2a and 2b

i	n_i	i	n_i
1	$-0.349\,898\,083\,432\,139 \times 10^{4}$	3	$-0.421\,073\,558\,227\,969 \times 10^{3}$
2	$0.257\,560\,716\,905\,876 \times 10^{4}$	4	$0.276\,349\,063\,799\,944 \times 10^{2}$

If the given specific enthalpy h is greater than $h_{2ab}(s)$ calculated from the given specific entropy s, then the point of state to be calculated is situated in subregion 2b, otherwise it is in subregion 2a (see Figure 2).

The boundary between the subregions 2b and 2c is the entropy line $s = 5.85$ kJ kg^{-1} K^{-1}. Input points can be tested directly to identify the subregion since the specific entropy is an independent variable. If the given specific entropy s is greater than or equal

to 5.85 kJ kg^{-1} K^{-1}, then the state point to be calculated is located in subregion 2b; otherwise it is in subregion 2c.

For *computer-program verification*, Eq. (2) gives the following *s*-*h* point:
$s = 7$ kJ kg^{-1} K^{-1}, $h_{2ab} = 3376.437\,884$ kJ kg^{-1}.

6.2 The Equations

Subregion 2a

The backward equation $p_{2a}(h,s)$ for subregion 2a in its dimensionless form reads as follows:

$$\frac{p_{2a}(h,s)}{p^*} = \pi(\eta,\sigma) = \left[\sum_{i=1}^{29} n_i (\eta - 0.5)^{I_i} (\sigma - 1.2)^{J_i}\right]^4, \tag{3}$$

where $\pi = p/p^*$, $\eta = h/h^*$, and $\sigma = s/s^*$ with $p^* = 4$ MPa, $h^* = 4200$ kJ kg^{-1}, and $s^* = 12$ kJ kg^{-1} K^{-1}. The coefficients n_i and exponents I_i and J_i of Eq. (3) are listed in Table 6.

Table 6. Coefficients and exponents of the backward equation $p_{2a}(h,s)$ for subregion 2a in its dimensionless form, Eq. (3)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	1	$-0.182\,575\,361\,923\,032 \times 10^{-1}$	16	1	22	$0.431\,757\,846\,408\,006 \times 10^{4}$
2	0	3	$-0.125\,229\,548\,799\,536$	17	2	3	$0.112\,894\,040\,802\,650 \times 10^{1}$
3	0	6	$0.592\,290\,437\,320\,145$	18	2	16	$0.197\,409\,186\,206\,319 \times 10^{4}$
4	0	16	$0.604\,769\,706\,185\,122 \times 10^{1}$	19	2	20	$0.151\,612\,444\,706\,087 \times 10^{4}$
5	0	20	$0.238\,624\,965\,444\,474 \times 10^{3}$	20	3	0	$0.141\,324\,451\,421\,235 \times 10^{-1}$
6	0	22	$-0.298\,639\,090\,222\,922 \times 10^{3}$	21	3	2	$0.585\,501\,282\,219\,601$
7	1	0	$0.512\,250\,813\,040\,750 \times 10^{-1}$	22	3	3	$-0.297\,258\,075\,863\,012 \times 10^{1}$
8	1	1	$-0.437\,266\,515\,606\,486$	23	3	6	$0.594\,567\,314\,847\,319 \times 10^{1}$
9	1	2	$0.413\,336\,902\,999\,504$	24	3	16	$-0.623\,656\,565\,798\,905 \times 10^{4}$
10	1	3	$-0.516\,468\,254\,574\,773 \times 10^{1}$	25	4	16	$0.965\,986\,235\,133\,332 \times 10^{4}$
11	1	5	$-0.557\,014\,838\,445\,711 \times 10^{1}$	26	5	3	$0.681\,500\,934\,948\,134 \times 10^{1}$
12	1	6	$0.128\,555\,037\,824\,478 \times 10^{2}$	27	5	16	$-0.633\,207\,286\,824\,489 \times 10^{4}$
13	1	10	$0.114\,144\,108\,953\,290 \times 10^{2}$	28	6	3	$-0.558\,919\,224\,465\,760 \times 10^{1}$
14	1	16	$-0.119\,504\,225\,652\,714 \times 10^{3}$	29	7	1	$0.400\,645\,798\,472\,063 \times 10^{-1}$
15	1	20	$-0.284\,777\,985\,961\,560 \times 10^{4}$				

Subregion 2b

The backward equation $p_{2b}(h,s)$ for subregion 2b in its dimensionless form reads as follows:

$$\frac{p_{2b}(h,s)}{p^*} = \pi(\eta,\sigma) = \left[\sum_{i=1}^{33} n_i (\eta - 0.6)^{I_i} (\sigma - 1.01)^{J_i}\right]^4, \tag{4}$$

where $\pi = p/p^*$, $\eta = h/h^*$, and $\sigma = s/s^*$ with $p^* = 100$ MPa, $h^* = 4100$ kJ kg^{-1}, and $s^* = 7.9$ kJ kg^{-1} K^{-1}. The coefficients n_i and exponents I_i and J_i of Eq. (4) are listed in Table 7.

Table 7. Coefficients and exponents of the backward equation $p_{2b}(h,s)$ for subregion 2b in its dimensionless form, Eq. (4)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	$0.801\,496\,989\,929\,495 \times 10^{-1}$	18	3	12	$0.336\,972\,380\,095\,287 \times 10^{8}$
2	0	1	$-0.543\,862\,807\,146\,111$	19	4	1	$-0.586\,634\,196\,762\,720 \times 10^{3}$
3	0	2	$0.337\,455\,597\,421\,283$	20	4	16	$-0.221\,403\,224\,769\,889 \times 10^{11}$
4	0	4	$0.890\,555\,451\,157\,450 \times 10^{1}$	21	5	1	$0.171\,606\,668\,708\,389 \times 10^{4}$
5	0	8	$0.313\,840\,736\,431\,485 \times 10^{3}$	22	5	12	$-0.570\,817\,595\,806\,302 \times 10^{9}$
6	1	0	$0.797\,367\,065\,977\,789$	23	6	1	$-0.312\,109\,693\,178\,482 \times 10^{4}$
7	1	1	$-0.121\,616\,973\,556\,240 \times 10^{1}$	24	6	8	$-0.207\,841\,384\,633\,010 \times 10^{7}$
8	1	2	$0.872\,803\,386\,937\,477 \times 10^{1}$	25	6	18	$0.305\,605\,946\,157\,786 \times 10^{13}$
9	1	3	$-0.169\,769\,781\,757\,602 \times 10^{2}$	26	7	1	$0.322\,157\,004\,314\,333 \times 10^{4}$
10	1	5	$-0.186\,552\,827\,328\,416 \times 10^{3}$	27	7	16	$0.326\,810\,259\,797\,295 \times 10^{12}$
11	1	12	$0.951\,159\,274\,344\,237 \times 10^{5}$	28	8	1	$-0.144\,104\,158\,934\,487 \times 10^{4}$
12	2	1	$-0.189\,168\,510\,120\,494 \times 10^{2}$	29	8	3	$0.410\,694\,867\,802\,691 \times 10^{3}$
13	2	6	$-0.433\,407\,037\,194\,840 \times 10^{4}$	30	8	14	$0.109\,077\,066\,873\,024 \times 10^{12}$
14	2	18	$0.543\,212\,633\,012\,715 \times 10^{9}$	31	8	18	$-0.247\,964\,654\,258\,893 \times 10^{14}$
15	3	0	$0.144\,793\,408\,386\,013$	32	12	10	$0.188\,801\,906\,865\,134 \times 10^{10}$
16	3	1	$0.128\,024\,559\,637\,516 \times 10^{3}$	33	14	16	$-0.123\,651\,009\,018\,773 \times 10^{15}$
17	3	7	$-0.672\,309\,534\,071\,268 \times 10^{5}$				

Subregion 2c

The backward equation $p_{2c}(h,s)$ for subregion 2c in its dimensionless form reads as follows:

$$\frac{p_{2c}(h,s)}{p^*} = \pi(\eta,\sigma) = \left[\sum_{i=1}^{31} n_i (\eta - 0.7)^{I_i} (\sigma - 1.1)^{J_i}\right]^4, \tag{5}$$

where $\pi = p/p^*$, $\eta = h/h^*$, and $\sigma = s/s^*$ with $p^* = 100$ MPa, $h^* = 3500$ kJ kg^{-1}, and $s^* = 5.9$ kJ kg^{-1} K^{-1}. The coefficients n_i and exponents I_i and J_i of Eq. (5) are listed in Table 8.

Computer-program verification

To assist the user in computer-program verification of Eqs. (3), (4) and (5), Table 9 contains test values for the calculated pressure.

6.3 Numerical Consistency with the Basic Equation of IAPWS-IF97

The maximum percentage deviations for pressure and related root-mean-square values of the Eqs. (3), (4) and (5) from the IAPWS-IF97 basic equation $g_2^{97}(p,T)$ in comparison with the permissible differences are listed in Table 10.

Table 8. Coefficients and exponents of the backward equation $p_{2c}(h,s)$ for subregion 2c in its dimensionless form, Eq. (5)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	0.112 225 607 199 012	17	3	0	0.772 465 073 604 171
2	0	1	$-0.339\,005\,953\,606\,712 \times 10^{1}$	18	3	5	$0.463\,929\,973\,837\,746 \times 10^{5}$
3	0	2	$-0.320\,503\,911\,730\,094 \times 10^{2}$	19	3	8	$-0.137\,317\,885\,134\,128 \times 10^{8}$
4	0	3	$-0.197\,597\,305\,104\,900 \times 10^{3}$	20	3	16	$0.170\,470\,392\,630\,512 \times 10^{13}$
5	0	4	$-0.407\,693\,861\,553\,446 \times 10^{3}$	21	3	18	$-0.251\,104\,628\,187\,308 \times 10^{14}$
6	0	8	$0.132\,943\,775\,222\,331 \times 10^{5}$	22	4	18	$0.317\,748\,830\,835\,520 \times 10^{14}$
7	1	0	$0.170\,846\,839\,774\,007 \times 10^{1}$	23	5	1	$0.538\,685\,623\,675\,312 \times 10^{2}$
8	1	2	$0.373\,694\,198\,142\,245 \times 10^{2}$	24	5	4	$-0.553\,089\,094\,625\,169 \times 10^{5}$
9	1	5	$0.358\,144\,365\,815\,434 \times 10^{4}$	25	5	6	$-0.102\,861\,522\,421\,405 \times 10^{7}$
10	1	8	$0.423\,014\,446\,424\,664 \times 10^{6}$	26	5	14	$0.204\,249\,418\,756\,234 \times 10^{13}$
11	1	14	$-0.751\,071\,025\,760\,063 \times 10^{9}$	27	6	8	$0.273\,918\,446\,626\,977 \times 10^{9}$
12	2	2	$0.523\,446\,127\,607\,898 \times 10^{2}$	28	6	18	$-0.263\,963\,146\,312\,685 \times 10^{16}$
13	2	3	$-0.228\,351\,290\,812\,417 \times 10^{3}$	29	10	7	$-0.107\,890\,854\,108\,088 \times 10^{10}$
14	2	7	$-0.960\,652\,417\,056\,937 \times 10^{6}$	30	12	7	$-0.296\,492\,620\,980\,124 \times 10^{11}$
15	2	10	$-0.807\,059\,292\,526\,074 \times 10^{8}$	31	16	10	$-0.111\,754\,907\,323\,424 \times 10^{16}$
16	2	18	$0.162\,698\,017\,225\,669 \times 10^{13}$				

Table 9. Selected pressure values calculated from Eqs. (3), (4), and (5) [a]

Equation	h / $\left(\mathrm{kJ\ kg^{-1}}\right)$	s / $\left(\mathrm{kJ\ kg^{-1}\ K^{-1}}\right)$	$p(h,s)$ / MPa
$p_{2a}(h,s)$, Eq.(3)	2800	6.5	1.371 012 767
	2800	9.5	$1.879\,743\,844 \times 10^{-3}$
	4100	9.5	$1.024\,788\,997 \times 10^{-1}$
$p_{2b}(h,s)$, Eq.(4)	2800	6	4.793 911 442
	3600	6	$8.395\,519\,209 \times 10^{1}$
	3600	7	7.527 161 441
$p_{2c}(h,s)$, Eq.(5)	2800	5.1	$9.439\,202\,060 \times 10^{1}$
	2800	5.8	8.414 574 124
	3400	5.8	$8.376\,903\,879 \times 10^{1}$

[a] It is recommended to verify programmed functions using 8 byte real values for all variables.

Table 10. Maximum differences and root-mean-square differences between pressures calculated from Eqs. (3), (4) and (5) and from the IAPWS-IF97 basic equation $g_2^{97}(p,T)$ in comparison with the permissible differences

Subregion	Equation	$\lvert\Delta p/p\rvert_{\mathrm{tol}}$	$\lvert\Delta p/p\rvert_{\mathrm{max}}$	$(\Delta p/p)_{\mathrm{RMS}}$
2a	(3)	0.0035 %	0.0029 %	0.0013 %
2b	(4)	0.0035 %	0.0034 %	0.0005 %
2c	(5)	0.0088 %	0.0063 %	0.0010 %

6.4 Consistency at Boundaries between Subregions

The relative pressure differences between the two backward equations $p(h,s)$ of the adjacent subregions are smaller than the numerical consistencies of these equations with the IAPWS-IF97 basic equations.

At the boundary equation $h_{2ab}(s)$, Eq. (2), between subregions 2a and 2b (see Fig. 2), the maximum difference between the corresponding equations was determined as:

$$|\Delta p/p|_{max} = |p_{2a}(h_{2ab},s) - p_{2b}(h_{2ab},s)| / p_{2b}(h_{2ab},s) = 0.0022\ \% .$$

At the boundary line $s = 5.85\ \mathrm{kJ\,kg^{-1}\,K^{-1}}$ between subregions 2c and 2b, the maximum difference is :

$$|\Delta p/p|_{max} = |p_{2c}(h,s) - p_{2b}(h,s)| / p_{2b}(h,s) = 0.0033\ \% .$$

7 Backward Functions $T(h,s)$ for Regions 1 and 2

7.1 Calculation of the Backward Functions $T(h,s)$

The $p(h,s)$ equations described in Sections 5 and 6 together with the backward equations $T^{97}(p,h)$ of IAPWS-IF97 make it possible to determine T from h and s without iteration.

Liquid Region 1

For calculating the temperature T from given specific enthalpy h and entropy s for region 1, the following steps should be made:

First, the pressure p is calculated using the equation $p_1(h,s)$, Eq. (1).

Second, the temperature T can be calculated using the IAPWS-IF97 equation $T_1^{97}(p,h)$ (see Fig. 1), where p is the pressure previously calculated.

Vapor Region 2

The calculations of the backward function $T_2(h,s)$ for the entire vapor region 2 should be made as follows:

First, using the equation $h_{2ab}(s)$, Eq. (2), and the entropy line $s = 5.85\ \mathrm{kJ\,kg^{-1}\,K^{-1}}$ (see Fig. 2), search to identify the subregion (2a, 2b or 2c) for the given values of h and s. Then, the pressure p for the subregion can be calculated using the equations $p_{2a}(h,s)$, Eq. (3) or $p_{2b}(h,s)$, Eq. (4) or $p_{2c}(h,s)$, Eq. (5).

Second, using the IAPWS-IF97 equation $h_{2bc}^{97}(p)$ and pressure line $p = 4$ MPa, search to identify the IAPWS-IF97 subregion (2a, 2b or 2c) for the given value of h and the calculated value of p. Then, the temperature T can be calculated for the subregion using the IAPWS-IF97 backward equations $T_{2a}^{97}(p,h)$, $T_{2b}^{97}(p,h)$ or $T_{2c}^{97}(p,h)$.

7.2 Numerical Consistency with the Basic Equations of IAPWS-IF97

The maximum temperature differences and related root-mean-square values between the calculated temperature and the IAPWS-IF97 basic equations $g_1^{97}(p,T)$ and $g_2^{97}(p,T)$ of regions 1 and 2 in comparison with the permissible differences are listed in Table 11. The temperature differences were calculated using the equation $\Delta T_1 = (T_1^{97}(p_1(h_1^{97},s_1^{97}),h_1^{97}) - T)$ for region 1 and $\Delta T_{2a} = (T_2^{97}(p_{2a}(h_2^{97},s_2^{97}),h_2^{97}) - T)$ for subregion 2a and analogous relations for subregions 2b and 2c. The function T_2^{97} represents the calculation of $T(p,h)$ using the IAPWS-IF97 backward equations of region 2 including the determination in which subregion (2a, 2b, or 2c) the point is located.

Table 11. Maximum differences and root-mean-square differences between calculated temperatures and IAPWS-IF97 basic equations $g_1^{97}(p,T)$ and $g_2^{97}(p,T)$ in comparison to the permissible differences

Region/Subregion	$\lvert\Delta T\rvert_{tol}$	$\lvert\Delta T\rvert_{max}$	$(\Delta T)_{RMS}$
1	25 mK	24.0 mK	13.4 mK
2a	10 mK	9.7 mK	3.0 mK
2b	10 mK	9.8 mK	4.0 mK
2c	25 mK	24.9 mK	10.3 mK

7.3 Consistency at Boundaries Between Subregions

The temperature differences between the two backward equations of the adjacent subregions have the following values.

Between subregions 2a and 2b, the following maximum differences were determined.

Along the boundary equation $h_{2ab}(s)$, Eq. (2):

$|\Delta T|_{max} = |T_2^{97}(p_{2a}(h_{2ab},s), h_{2ab}) - T_2^{97}(p_{2b}(h_{2ab},s), h_{2ab})| = 6.7$ mK ,

where the function T_2^{97} represents the calculation of $T(p,h)$ using the IAPWS-IF97 backward equations of region 2 including the determination of the subregion (2a, 2b, or 2c).

Along the boundary line $p = 4$ MPa:

$|\Delta T|_{max} = |T_{2a}^{97}(p_2(h_2^{97},s_2^{97}),h_2^{97}) - T_{2b}^{97}(p_2(h_2^{97},s_2^{97}),h_2^{97})| = 8.7$ mK ,

where $h_2^{97}(p=4\,\text{MPa},T)$ and $s_2^{97}(p=4\,\text{MPa},T)$. The function p_2 represents the calculation of $p(h,s)$ using the backward equations of region 2 and includes the determination of the subregion (2a, 2b, or 2c).

Between subregions 2b and 2c, the following maximum differences were determined.

Along the boundary line $s = 5.85$ kJ kg^{-1} K^{-1}:

$$|\Delta T|_{\max} = |T_2^{97}(p_{2c}(h,s), h) - T_2^{97}(p_{2b}(h,s), h)| = 2.7 \text{ mK} .$$

Along the IAPWS-IF97 boundary equation $h_{2bc}^{97}(p)$:

$$|\Delta T|_{\max} = |T_{2c}^{97}(p_2(h_{2bc}^{97}, s_2^{97}), h_{2bc}^{97}) - T_{2b}^{97}(p_2(h_{2bc}^{97}, s_2^{97}), h_{2bc}^{97})| = 21.8 \text{ mK} ,$$

where s_2^{97} was iteratively calculated from p and h_{2bc}^{97}. Thus, the temperature differences between the two backward functions $T(h,s)$ of the adjacent subregions calculated as shown above are smaller than the numerical consistencies with the IAPWS-IF97 equations.

8 Computing Time in Relation to IAPWS-IF97

A very important motivation for the development of the backward equations $p(h,s)$ was reducing the computing time to obtain the values of p and T from given values of h and s. In IAPWS-IF97, time consuming iterative processes, e. g. the 2-dimensional Newton method, is required. Using the $p(h,s)$ equations combined with IAPWS-IF97 backward equations $T^{97}(p,h)$, the calculation speed is between 20 and 30 times faster than that of the 2-dimensional Newton method. The numerical consistency of p and T obtained in this way is sufficient for most heat cycle calculations.

For users not satisfied with the numerical consistency of the backward equations, the equations are still recommended to give good starting points for the iterative process. It will significantly reduce the time to reach the convergence criteria of the iteration.

9 References

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