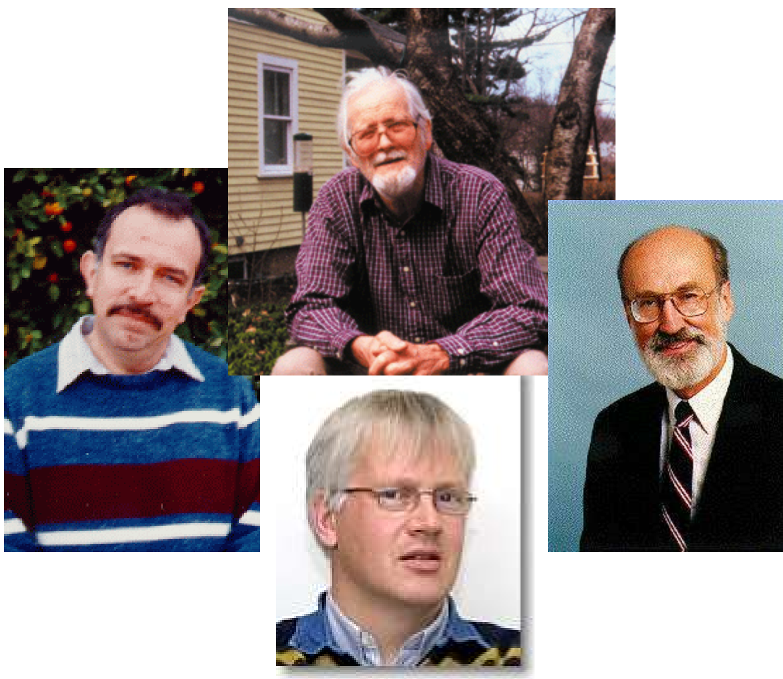
Bond-Graphs: A Formalism for Modeling Physical Systems



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The Ubiquity of Energy

- *Energy is the fundamental quantity that every physical system possess.*
- *Energy is the <u>potential for change</u>.*

System

Energy Efficient

Major Heat Energy Loss

Not So Energy Efficient (Falling)

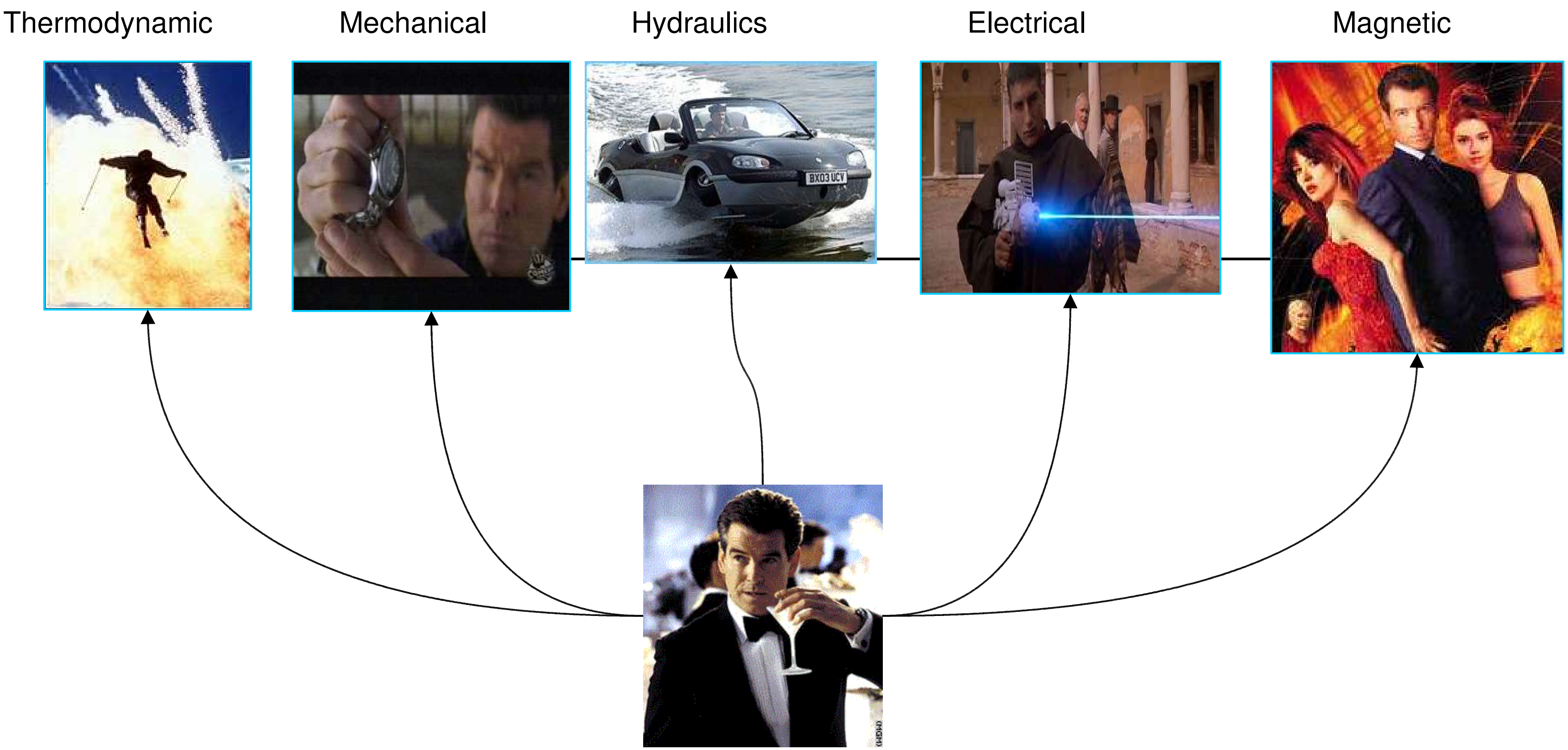
Back on your feet

Eventually

Since no inflow of free energy

Free Energy (Useful Energy): **DECREASES**
Heat Energy Released (Not so useful): **INCREASES**
Total Energy=Free Energy + Other Forms of Energy ,is conserved

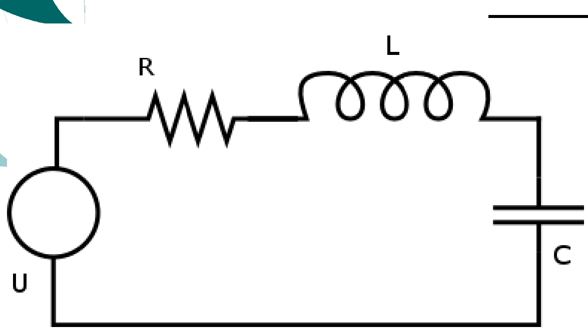
Bond Graphs: A Unifying Formalism



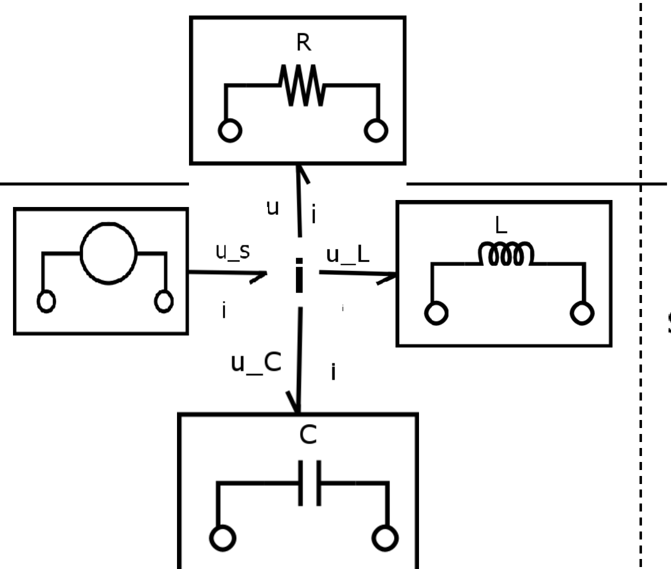
Why Bond-Graphs (BG)?

First Example: The RLC Circuit

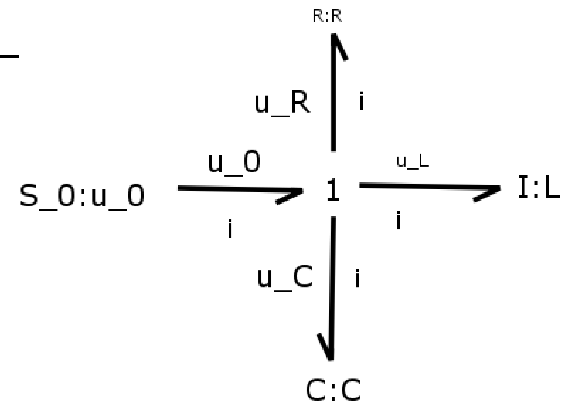
RLC Circuit



Symbolic BG

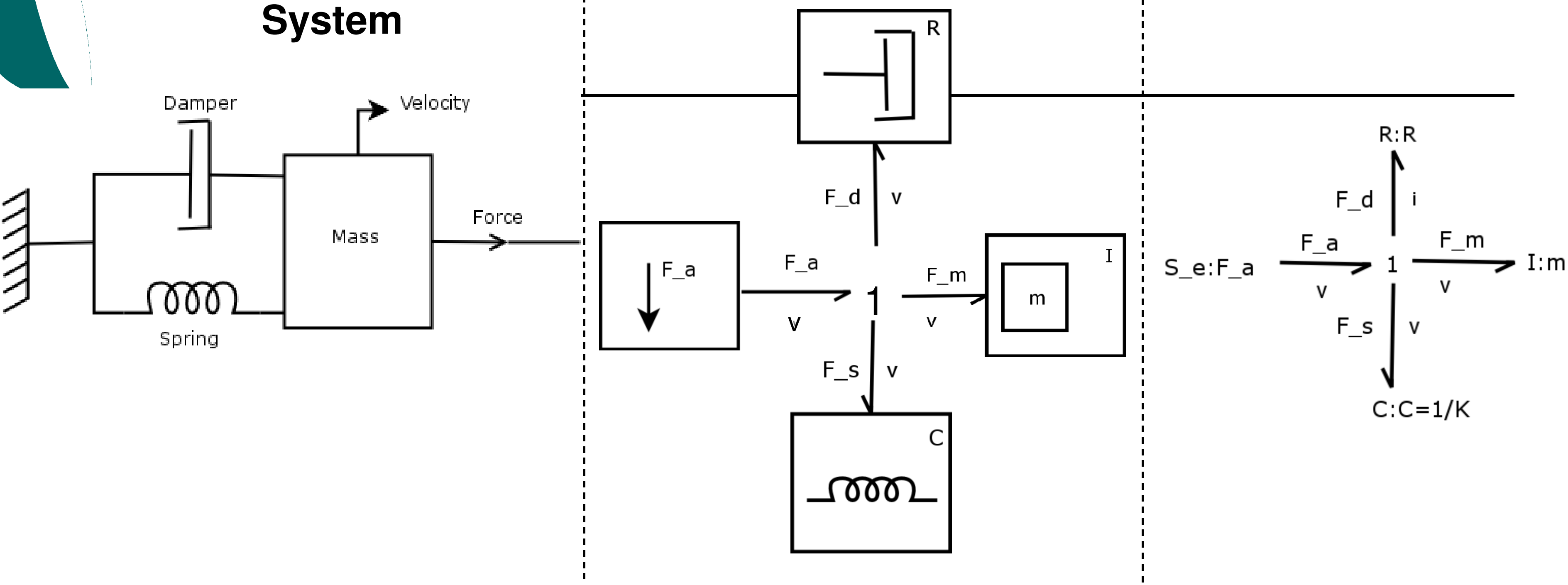


Standard BG



Second Example: Damped Mass Spring System

Damped Mass Spring System | **Symbolic BG** | **Standard BG**



Lets compare...

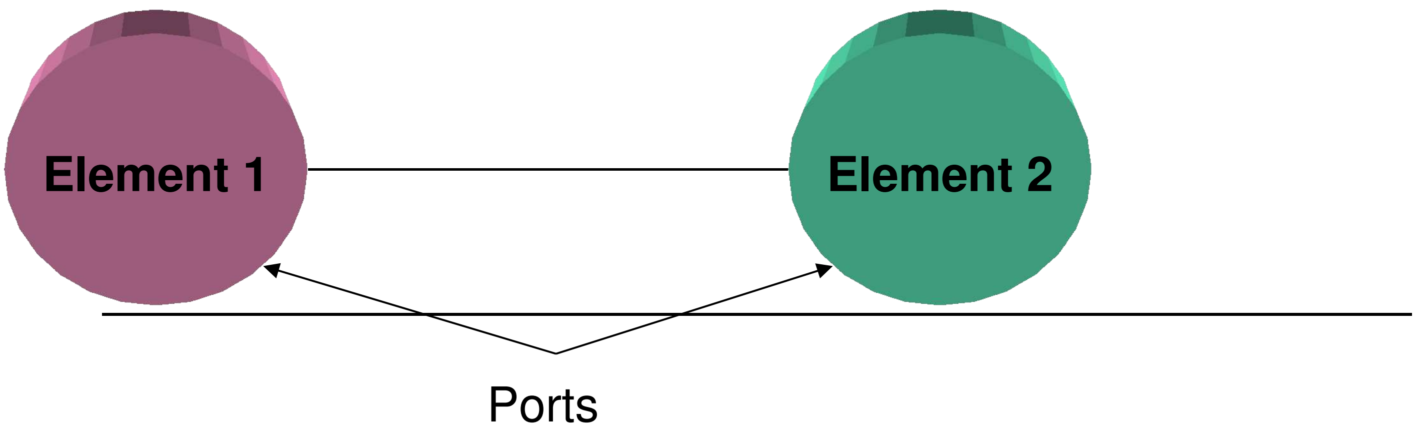
- The Damper is analogous to the Resistor
- The Spring is analogous to the Capacitor
- The Mass is analogous to the Inductor
- The Force is analogous to the voltage source
- The Common Velocity is analogous to the Loop Current

The Standard Bond-Graphs are pretty much Identical!!!

Common Bond Graph Elements

Symbol	Explanation	Examples
C	Storage element for a q-type variable	Capacitor (stores charges), Spring (stores displacement)
I	Storage element for a p-type variable	Inductor (stores flux linkage), mass (stores momentum)
R	Resistor dissipating free energy	Electric resistor, Mechanical friction
Se, Sf	Effort sources and Flow sources	Electric mains (voltage source), Gravity (force source), Pump (flow source)
TF	Transformer	Electric transformer, Toothed wheels, Lever
GY	Gyrator	Electromotor, Centrifugal Pump
0,1	0 and 1 Junctions	Ideal connection between two sub models

Closer Look at Bonds and Ports(1)



The energy flow along the bond has the physical dimension of power.

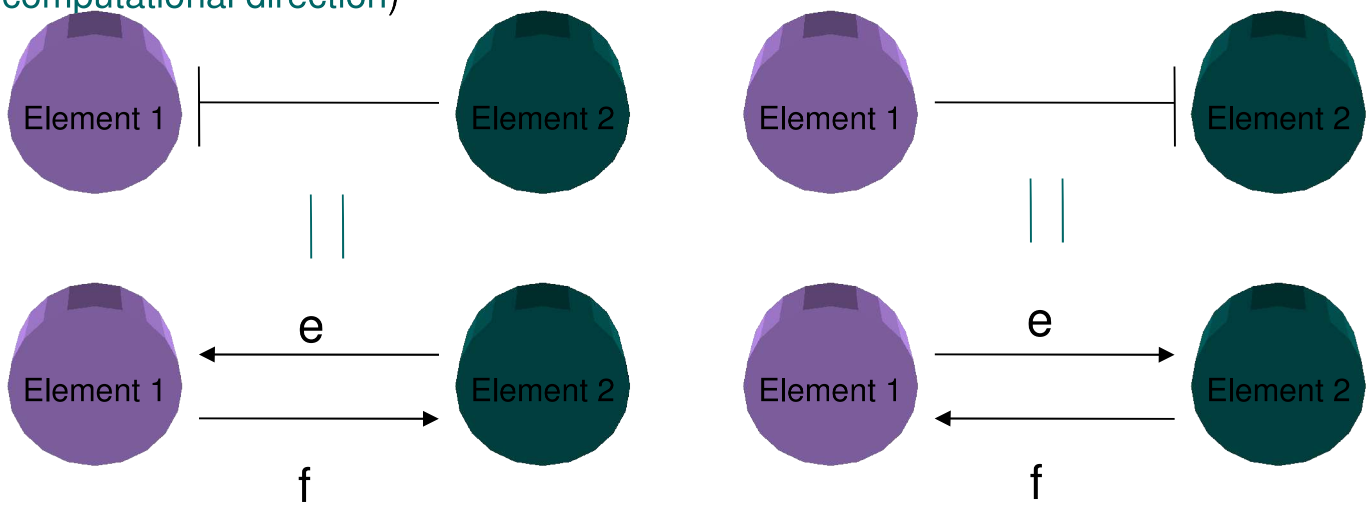
Domain	Effort	Flow	Power Expression
Electrical	Voltage (V)	Current (I)	P=VI
Mechanical: Translation	Force (F)	Velocity (v)	P=Fv
Mechanical: Rotation	Torque (T)	Angular Velocity (θ)	P=T θ
Hydraulics	Pressure (p)	Volume Flow (f)	P=pf
Thermodynamics	Temperature (T)	Entropy Flow (S)	P=TS

These pairs of variables are called (power-) conjugated variables

Closer Look at Bonds and Ports(2)

Two views for the interpretation of the bond

1. As an interaction of energy: The connected subsystems form a load to each other by the energy exchange. A physical quantity is exchanged over the power bond.
2. As a bilateral signal flow: Effort and Flow are flowing in opposite directions (determining the computational direction)



Element1.e=Element2.e

Element2.f=Element1.f

Element2.e=Element1.e

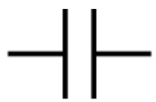
Element1.f=Element2.f

Why is the power direction not shown?

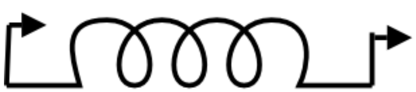
Bond Graph Elements (1): Storage Elements (C-element)

Storage elements store all kinds of free energy. C-elements accumulate net flow

Domain Specific Symbols



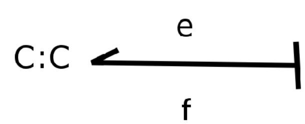
Capacitor



Translational Spring

Rotational Spring

Bond-graph Element



Equations

$$e = \frac{1}{C} q$$

$$q = \int f dt + q(0)$$

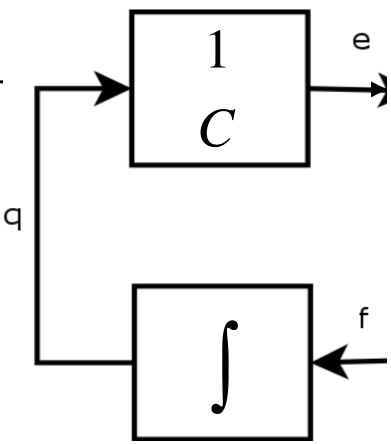
Eg.

C [F] is the capacitance

F=Kx=(1/C)x

K[N/m] is the stiffness and **C [m/N]** the Compliance

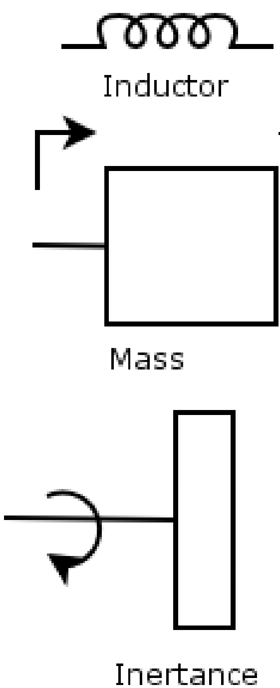
Block Diagram Representation

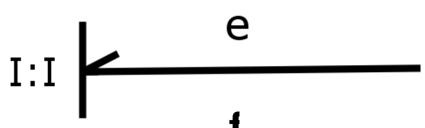


Bond Graph Elements (2):Storage Elements (I – element)

I-elements accumulate net effort

Domain Specific Symbols	Bond-graph Element	Equations	Block Diagram Representation





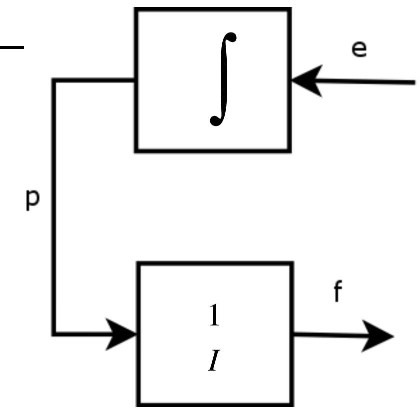
$$f = \frac{1}{I} p$$

$$p = \int e dt + p(0)$$

Eg.

L[H] is the inductance

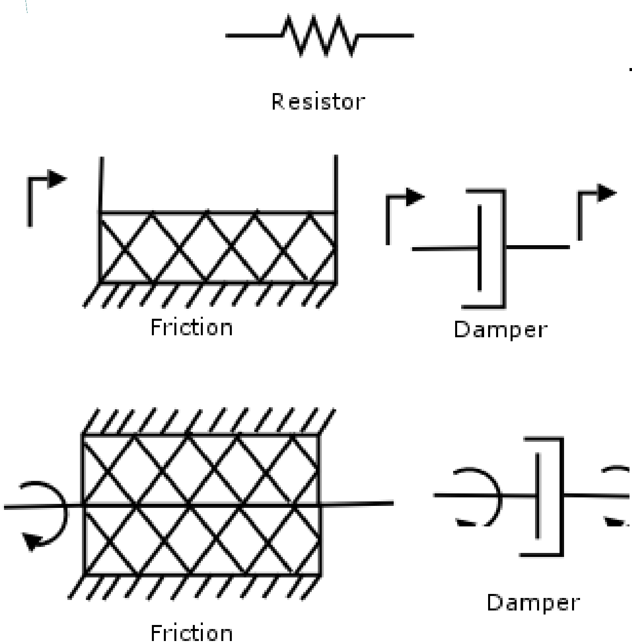
m [kg] is the mass



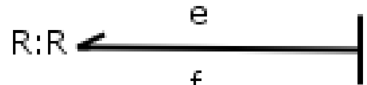
Bond Graph Elements (3):Resistors (R-element)

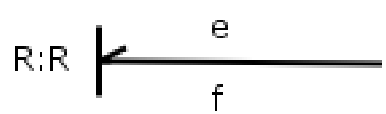
R-elements dissipate free energy

Domain Specific Symbols



Bond-graph Element





Equations

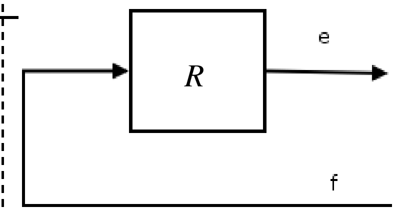
$$e = Rf$$

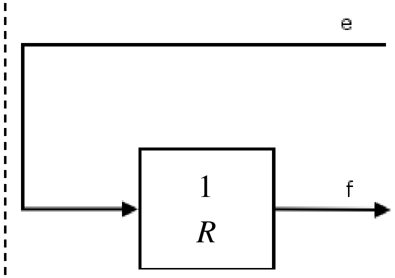
$$f = \frac{1}{R} e$$

Eg.

Electrical resistance **(ohms),** Viscous Friction **(Ns/m)**

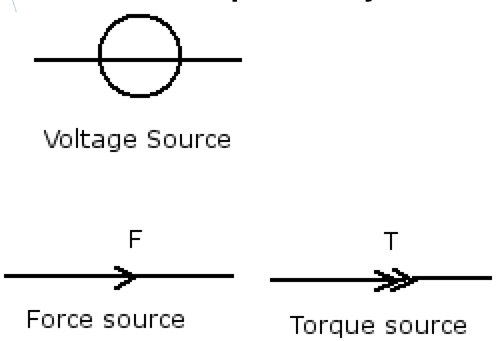
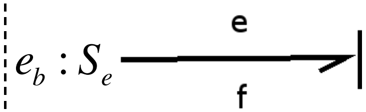
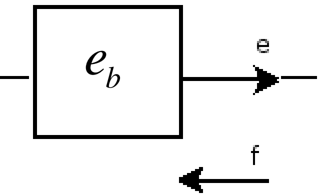
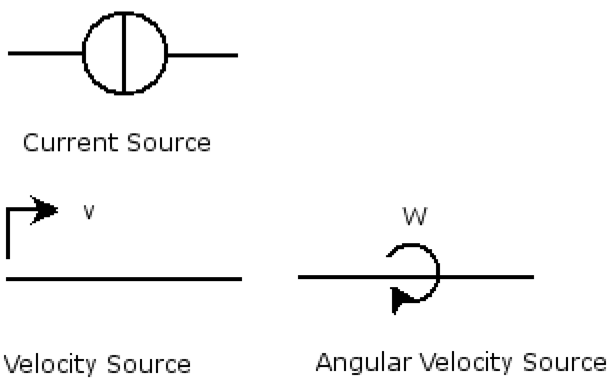
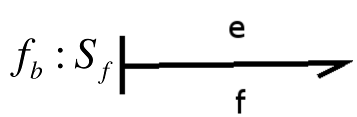
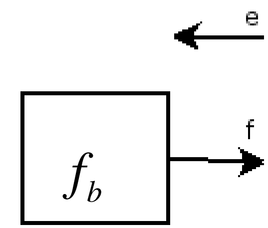
Block diagram expansion





Bond Graph Elements (4):Sources

Sources represent the interaction of a system with its environment

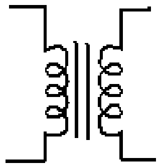
Domain Specific Symbols	Bond graph Element	Equations	Block diagram representation
		$e = e_b$	
		$f = f_b$	

We can also have modulated sources, resistors etc.

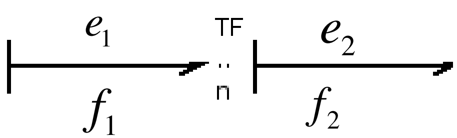
Bond Graph Elements (5): Transformers

Ideal transformers are power continuous, that is they do not dissipate any free energy. Efforts are transduced to efforts and flows to flows

Domain Specific Symbols | **Bond graph Element** | **Equations** | **Block diagram representation**

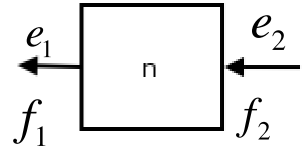


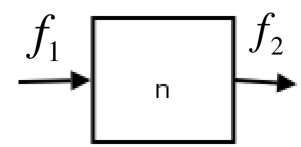
Transformer

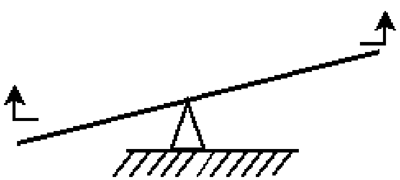


$$f_2 = n f_1$$

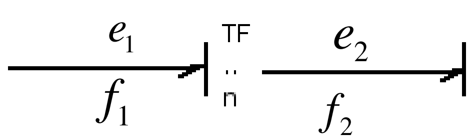
$$e_1 = n e_2$$





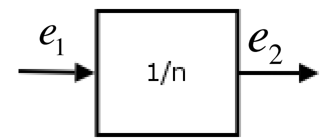


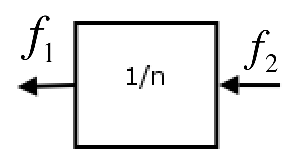
Cantilever

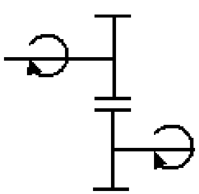


$$f_1 = \frac{f_2}{n}$$

$$e_2 = \frac{e_1}{n}$$







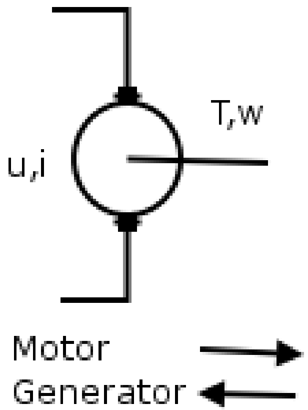
Mechanical Gear

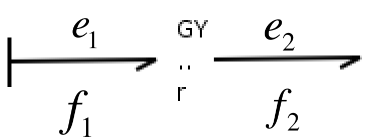
n is the transformer ratio

Bond Graph Elements (6):Gyrators

Ideal Gyrators are power continuous. Transducers representing domain transformation.

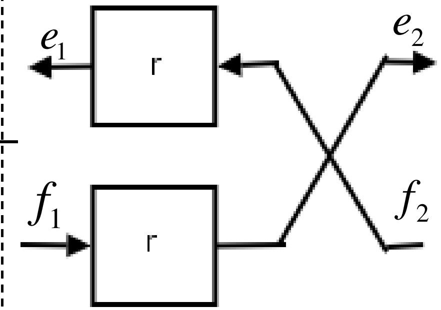
Domain Specific Symbols	Bond graph Element	Equations	Block diagram representation

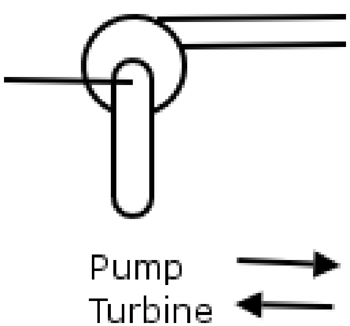


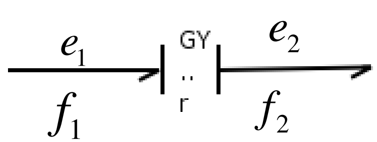


$$e_2 = rf_1$$

$$e_1 = rf_2$$



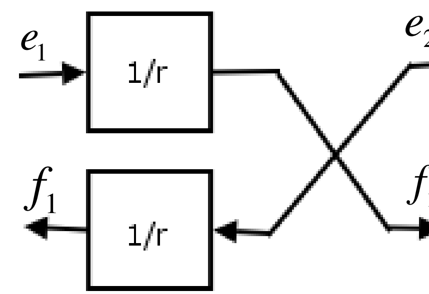




$$f_2 = \frac{e_1}{r}$$

$$f_1 = \frac{e_2}{r}$$

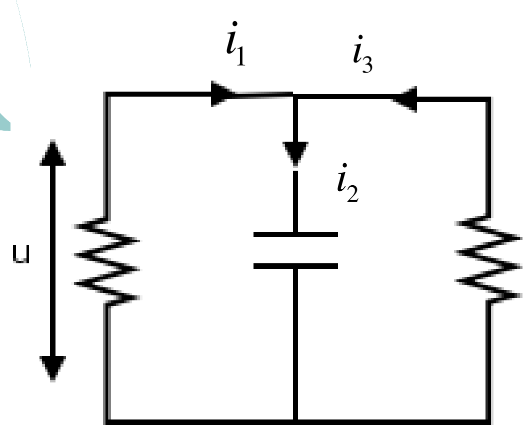
r is the gyrator ratio

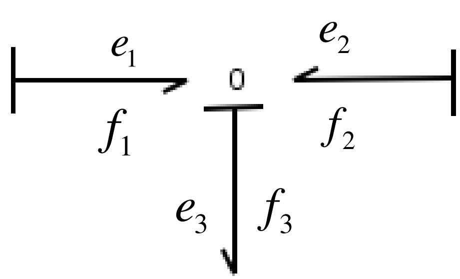


Bond Graph Elements (7):0-Junction

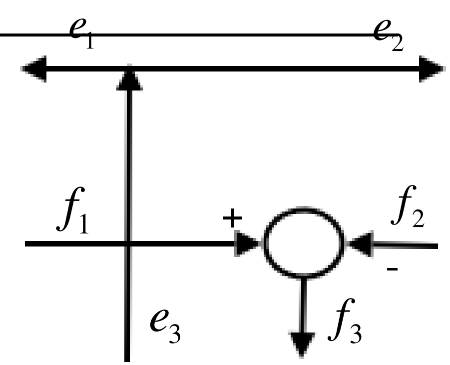
The 0-junction represents a node at which all efforts of the connecting bonds are equal

Domain Specific Symbols	**Bond graph Element**	**Equations**	**Block diagram representation**





$$e_1 = e_3$$
$$e_2 = e_3$$
$$f_3 = f_1 - f_2$$

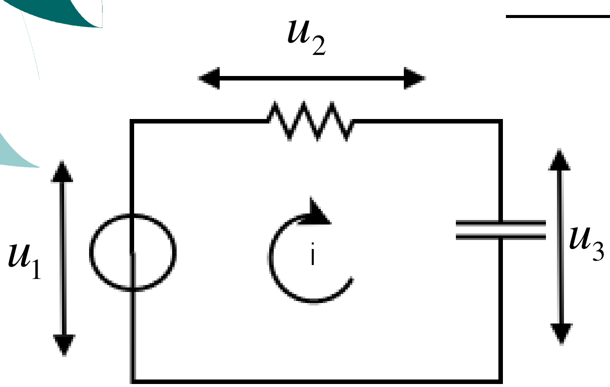


0-junction can be interpreted as the generalized Kirchoff's Current Law

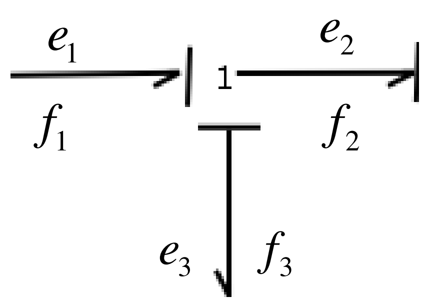
Bond Graph Elements (8):1-Junction

The 1-junction represents a node at which all flows of the connecting bonds are equal

Domain Specific Symbols



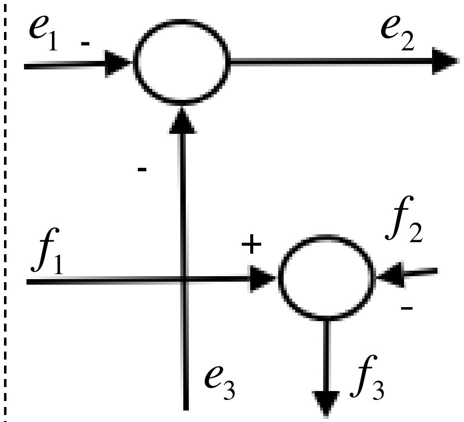
Bond graph Element



Equations

$$f_1 = f_2$$
$$f_3 = f_2$$
$$e_2 = e_1 - e_3$$

Block diagram representation



1-junction can be interpreted as the generalized Kirchoff's Voltage Law

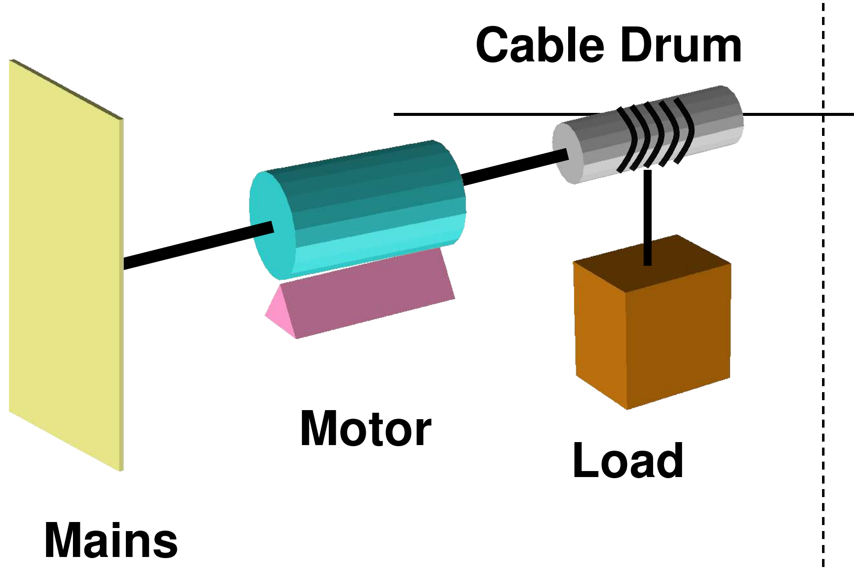
Some Misc. Stuff

- **Power direction:** The power is positive in the direction of the power bond. A port that has incoming power bond consumes power. Eg. R, C.
- ~~Transformers and Gyrators have one power bond coming~~ in and one going out.

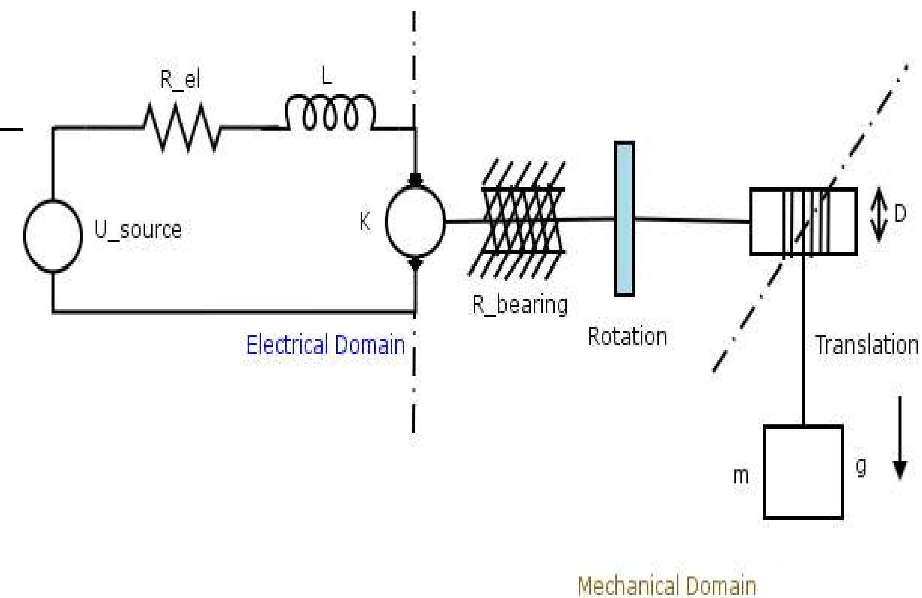
  These are ***constraints*** on the model!
- **Duality:** Two storage elements are each others dual form. The role of effort and flow are interchanged. A gyrator can be used to decompose an I-element to a GY and C element and vice versa.

Physical System to Acausal Bond Graph by Example (1): Hoisting Device

Sketch of a Hoisting Device

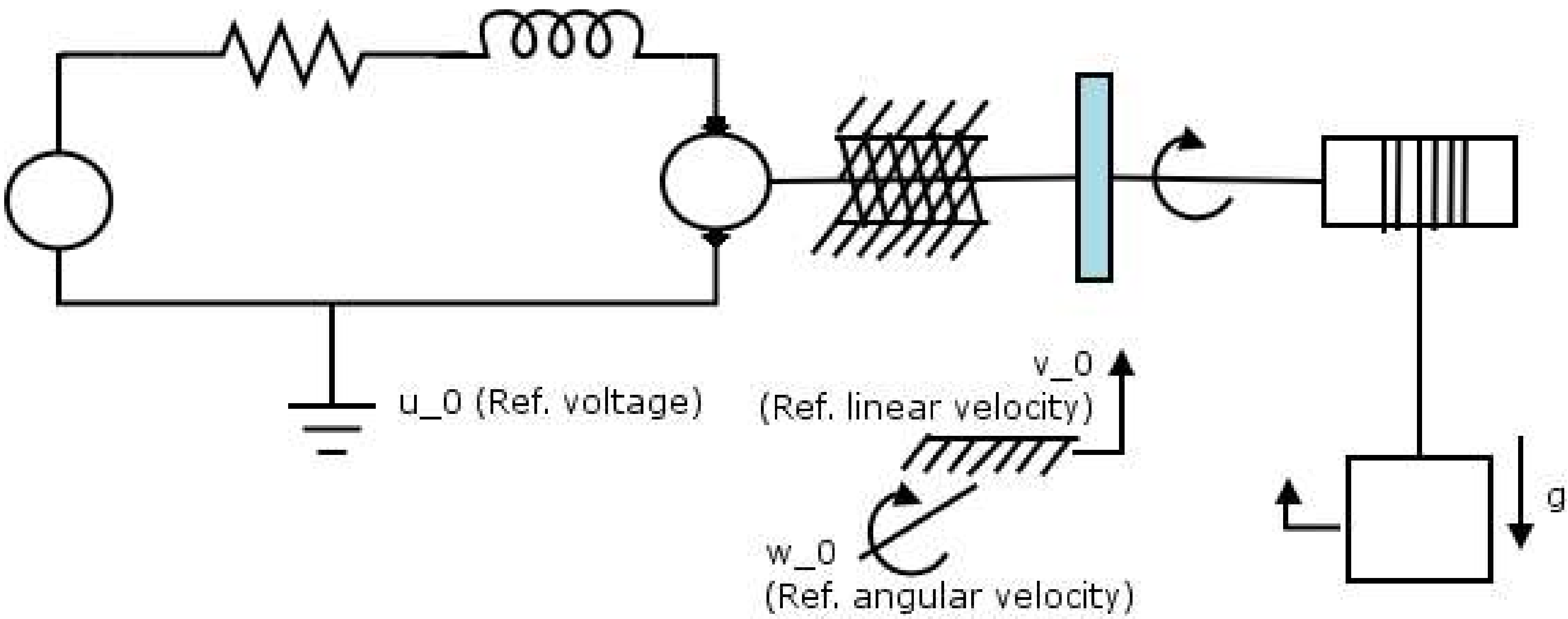


Ideal Physical Model with Domain Information (Step 1)



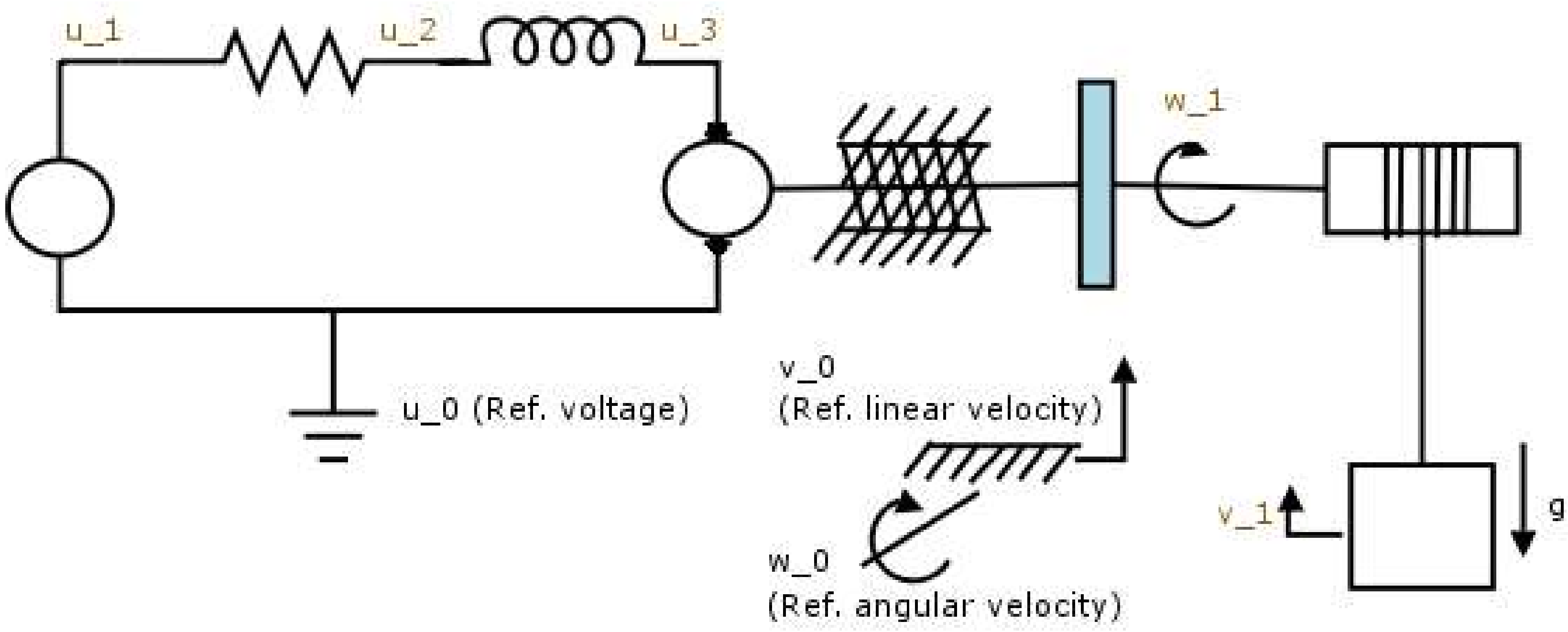
Step 1: Determine which *physical domains* exist in the system and identify all *basic elements* like C, I, R, Se, Sf, TF, GY

Physical System to Acausal Bond Graph by Example (2): Hoisting Device



Step 2: Identify the reference efforts in the physical model.

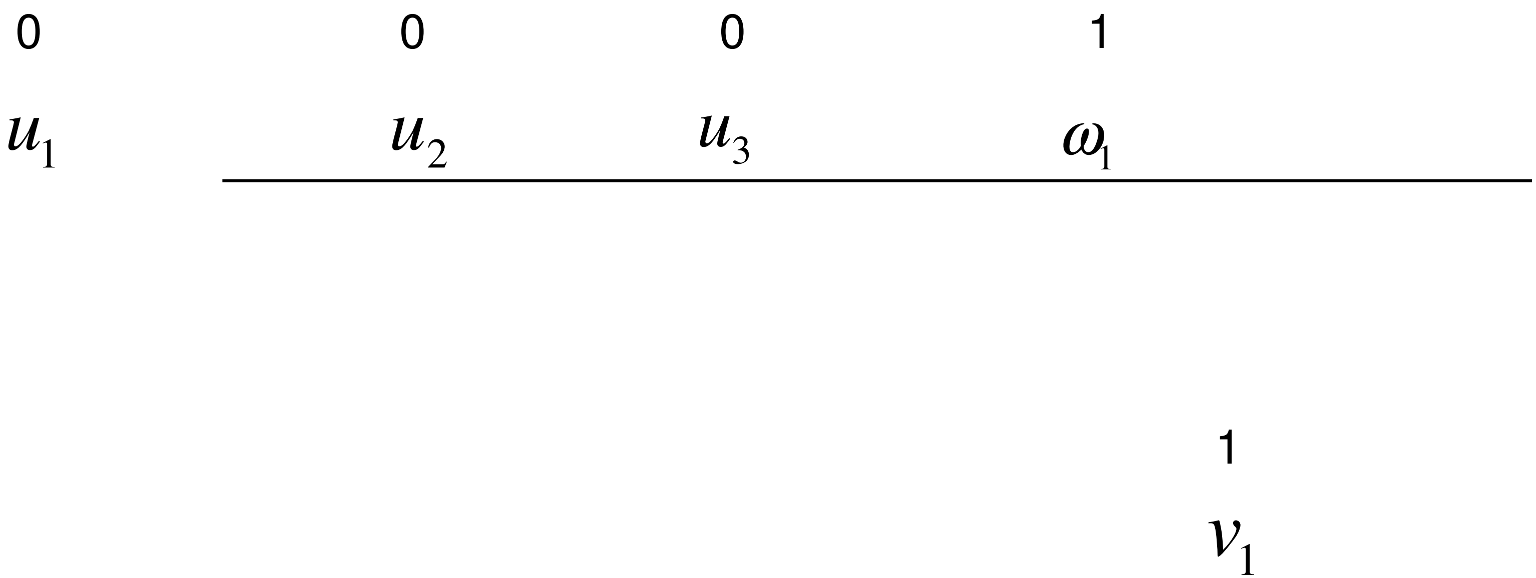
Physical System to Acausal Bond Graph by Example (3): Hoisting Device



Step 3: Identify other efforts and give them unique names

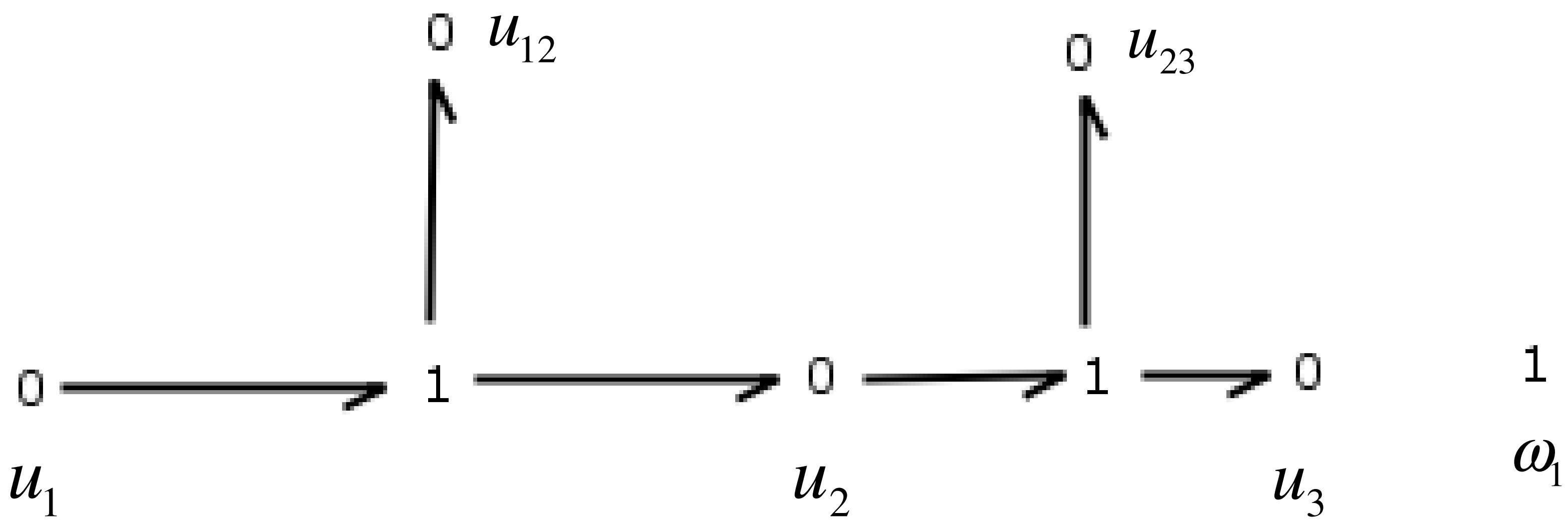
Physical System to Acausal Bond Graph by Example (4): Hoisting Device

Skeleton Bond Graph



Step 4: Draw the efforts (mechanical domain: velocity), and not references (references are usually zero), graphically by 0-junctions (mechanical 1-junction)

Physical System to Acausal Bond Graph by Example (5): Hoisting Device



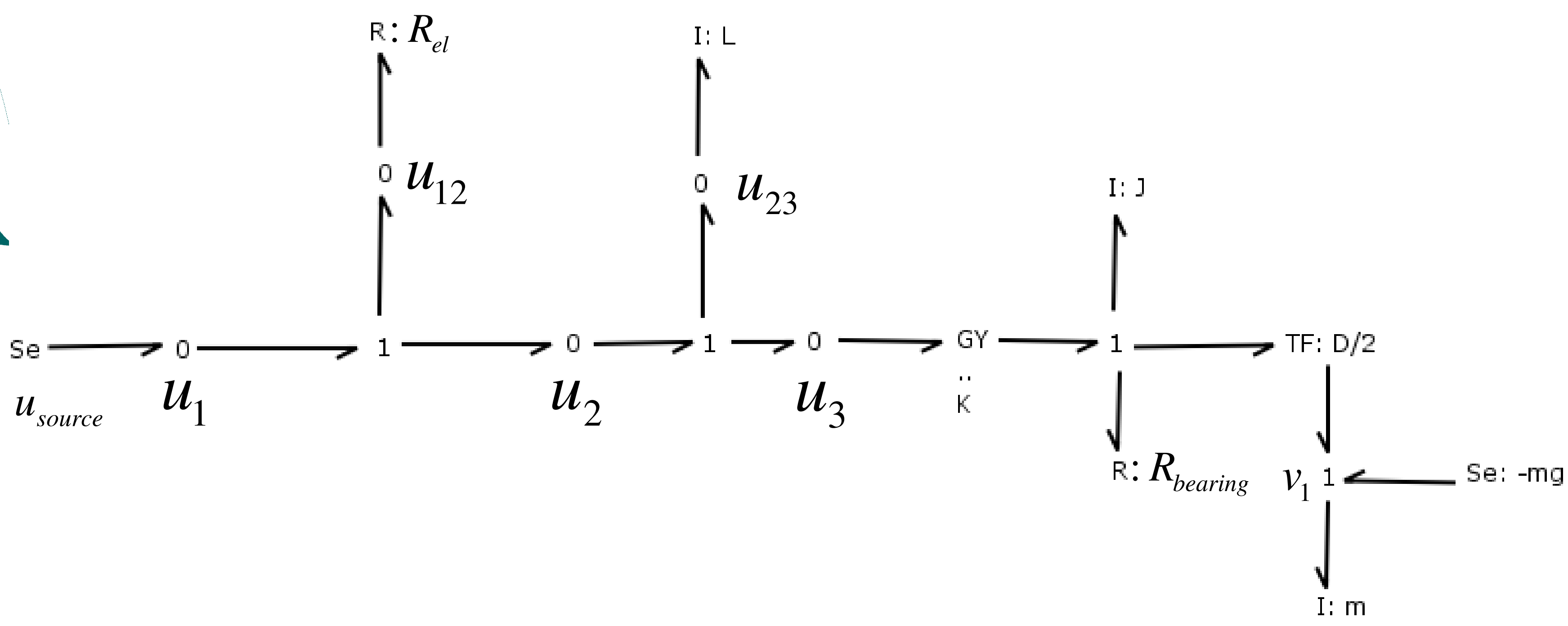
Step 5: Identify all *effort differences* (mechanical velocity(=flow) differences) needed to connect the ports of all elements enumerated in Step 1. Differences have a *unique name*.

1

v_1

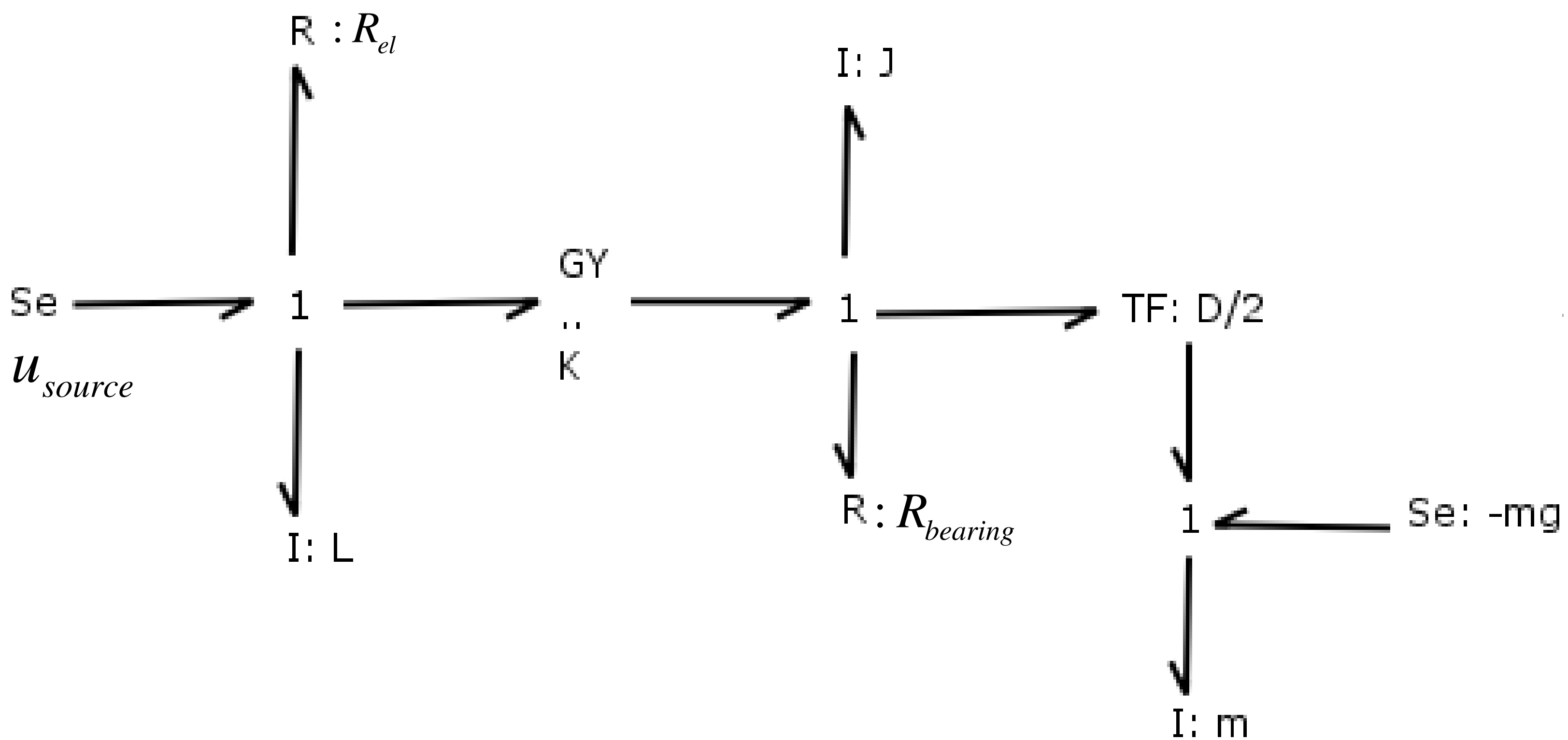
Step 6: Construct the effort differences using a *1-junction (mechanical: flow differences with 0-junctions)* and draw as such in the graph

Physical System to Acausal Bond Graph by Example (6): Hoisting Device



Step 7: Connect the port of all elements found at step 1 with 0-junctions of the corresponding efforts or effort differences (mechanical: 1-junctions of the corresponding flows or flow differences)

Physical System to Acausal Bond Graph by Example (7): Hoisting Device



Step 8: *Simplify* the graph by using the following simplification rules:

•A junction between two bonds can be left out, if the bonds have a through power direction (one incoming, one outgoing)

•A bond between two the same junctions can be left out, and the junctions can join into one junction.

•Two separately constructed identical effort or flow differences can join into one effort or flow difference.

Acausal to Causal Bond Graphs (1) : What is Causal Analysis?

- **Causal analysis** is the determination of signal direction of the bonds.
- Energetic connection is interpreted as a **bi-directional signal flow**.
- The result is a **causal bond graph** which can be seen as a compact **block diagram**.
- The element ports can impose **constraints** on the connection bonds depending on its nature.

Acausal to Causal Bond Graphs (2) : Causality Constraints

Fixed Causality:

When the equations allow only one of the two variables to be the outgoing variable,

1. At Sources:

•Effort-out causality

•Flow-out causality

Another situation,

2. Non-linear Elements:

• There is no relation between port variables

• The equations are not invertible ('singular') Eg. Division by zero

This is possible at R, GY, TF, C and I elements

Acausal to Causal Bond Graphs (3) : Causality Constraints

Constrained Causality:

Relations exist between the different ports of the element.

TF:

One port has effort-out causality and the other has flow-out causality.

GY:

Both ports have either effort-out causality or flow-out causality.

0-junction:

All efforts are the same and hence just one bond brings in the effort.

1-junction:

All flows are equal hence just one bond brings in the flow.

Acausal to Causal Bond Graphs (4) : Causality Constraints

Preferred Causality:

Applicable at storage elements where we need to make a choice about whether to perform **numerical differentiation** or **numerical integration**.

Eg.

A voltage *u* is imposed on an electrical capacitor (a C-element), the current is the result of the constitutive equation of the capacitor.

Flow-out causality

$$i = C\frac{du}{dt}$$

Needs info about ***future time points*** hence physically not realizable. Also, function must be **differentiable**.

Effort-out causality

$$u = u_0 + \int i\,dt$$

Physically Intuitive!

Needs initial state data.

Implication: C-element has effort-out causality and I-element has flow-out causality

Acausal to Causal Bond Graphs (5) : Causality Constraints

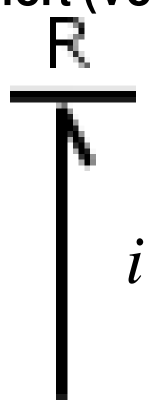
Indifferent Causality:

Indifferent causality is used when there are no causal constraints!

Eg.

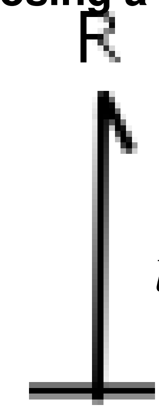
At a **linear R** it does not matter which of the port variables is the output.

Imposing an effort (Voltage)



$$i = \frac{u}{R}$$

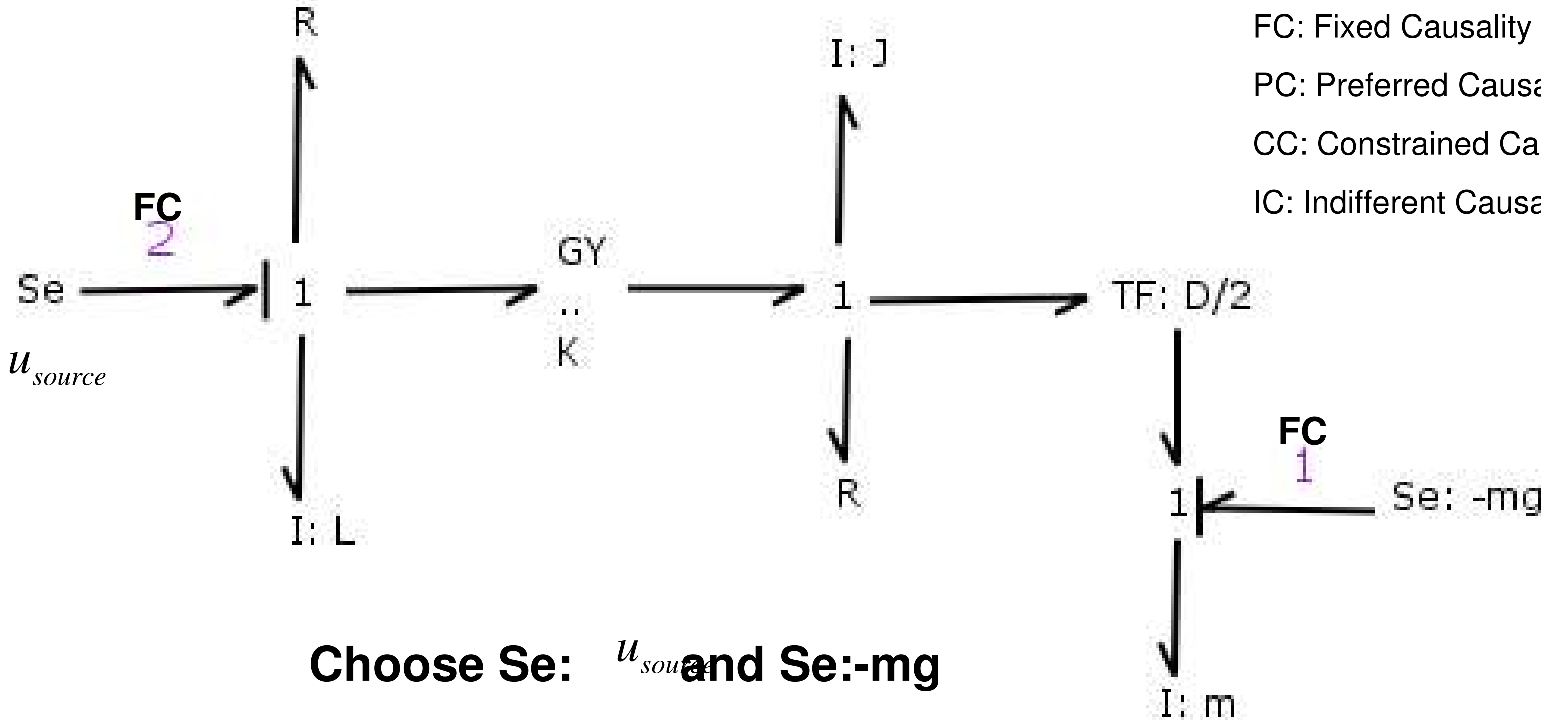
Imposing a flow (Current)



$$u = iR$$

Doesn't Matter!

Acausal to Causal Bond Graphs (6) : Causality Analysis Procedure



FC: Fixed Causality

PC: Preferred Causality

CC: Constrained Causality

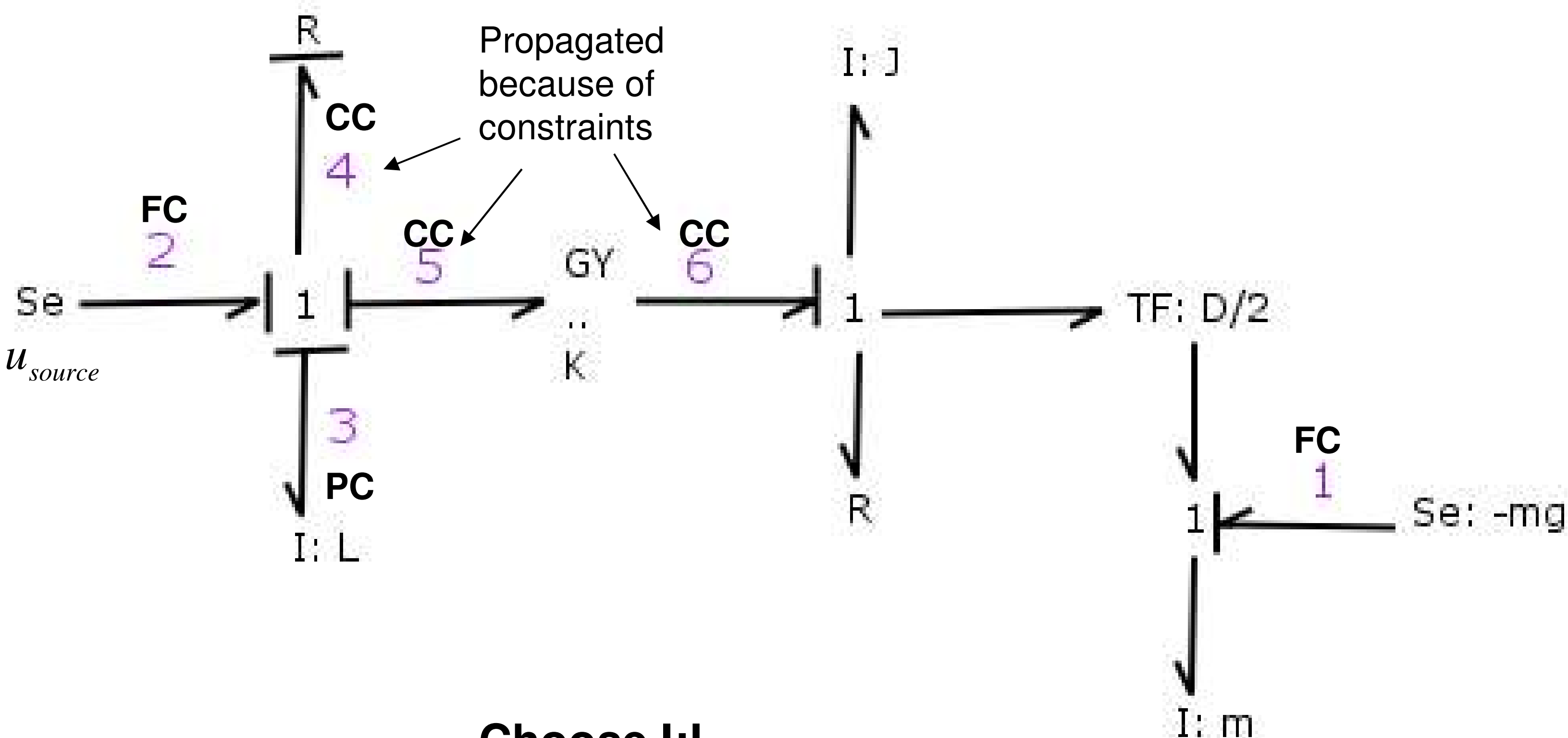
IC: Indifferent Causality

Choose Se: u_{source} **and Se:-mg**

1a. Choose a *fixed causality of a source element*, assign its causality, and propagate this assignment through the graph using causal constraints. Go on until all sources have their causality assigned.

1b. Choose a not yet causal port with *fixed causality (non-invertible equations),* assign its causality, and propagate this assignment through the graph using causal constraints. Go on until all ports with fixed causality have their causalities assigned. **(Not Applicable in this example)**

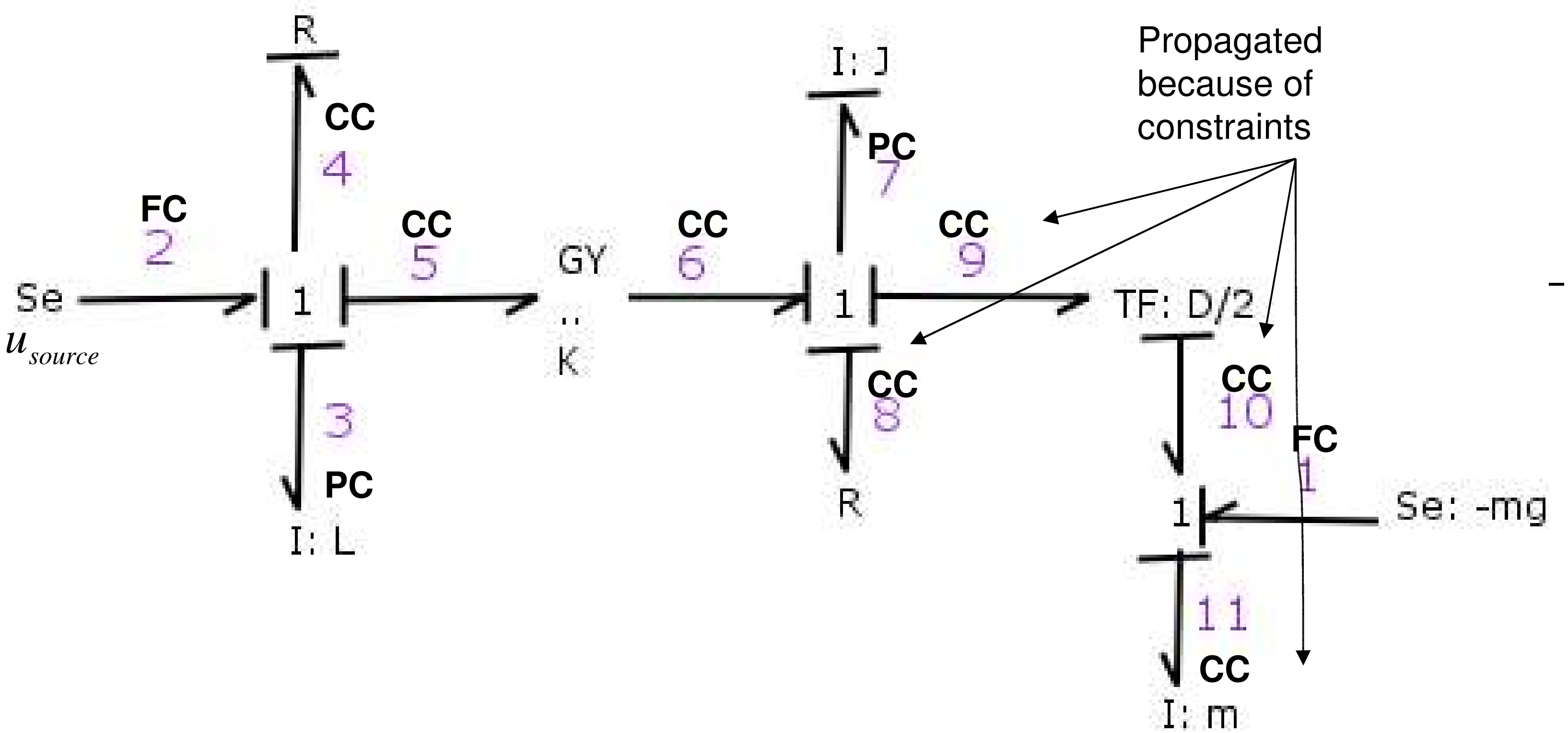
Acausal to Causal Bond Graphs (7) : Causality Analysis Procedure



Choose I:L

2. Choose a not yet causal port with *preferred* causality (storage elements), assign its causality, and propagate this assignment through the graph using the causal constraints. Go on until all ports with preferred causality have their causalities assigned.

Acausal to Causal Bond Graphs (8) : Causality Analysis Procedure



Continued… Choose I:J

Acausal to Causal Bond Graphs (9) : Causality Analysis Procedure

Not applicable in our example since all causalities have been already assigned!

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3. Choose a not yet causal port with *indifferent causality*, assign its causality, and propagate this assignment through the graph using the causal constraints. Go on until all ports with indifferent causality have their causality assigned.

Model Insight via Causal Analysis(1)

• When model is **completely causal after step 1a**. The model has **no dynamics**.

• If a **causal conflict** arises at **step 1a or 1b** then the problem is **ill-posed**. Eg. Two effort sources connected to a 0-Junction.

•At conflict in **step 1b** (non-invertible equations), we could perhaps **reduce the fixedness**. Eg. A valve/diode having zero current while blocking can be made invertible by allowing a small resistance.

• When a conflict arises at **step 2**, a storage element receives a **non-preferred causality**. This implies that this storage element doesn't represent a **state variable**. Such a storage element is often called a **dependent storage element**. This implies that a storage element was not taken into account while modeling. **Eg.** Elastic cable in the hoisting device.

•A causal conflict in **step 3** possibly means that there is an **algebraic loop.**

Model Insight via Causal Analysis(2)

Remedies:

• Add Elements

• Change bond graph such that the conflict disappears

• Dealing with algebraic loops by adding a one step delay or by using an **implicit integration scheme**.

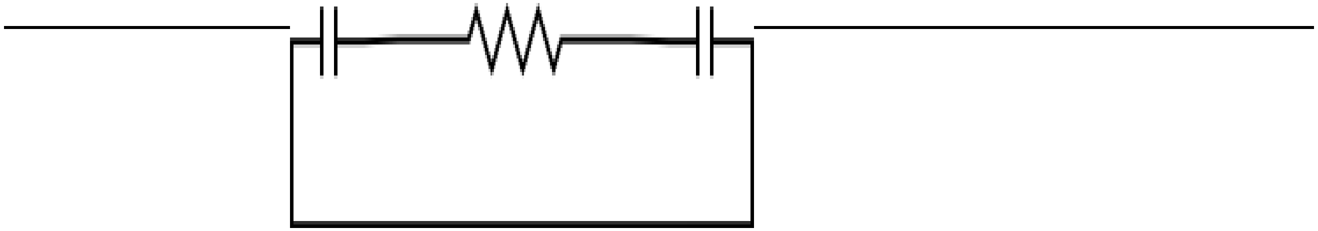
Other issues:

Algebraic loops and loops between a dependant and an independent storage element are called **zero-order causal paths (ZCP).** These occur in rigid body mechanical systems and result in complex equations.

Order of set of state equations

- Order of the system: Number of initial conditions
- Order of set of state equations <= Order of the system

Sometimes storage elements can depend on one another.



Recipe to check whether this kind of storage elements show up:

Perform **integral preference** and **differential preference** causality assignment and compared.

•**Dependent storage elements:** In both cases not their preferred causality

•**Semi-dependent storage elements:** In one case preferred and not-preferred in the other. **INDICATES** that a storage element was not taken into account.

Generation of Equations

1. We first write a set of mixed Differential Algebraic Equations (DAEs). This system comprises of **2n equations** of a bond graph have **n bonds**, **n equations compute an effort** and **n equations compute a flow or derivatives of them.**

2. We then eliminate the algebraic equations:
   - Eliminate identities coming from sources
   - We substitute the multiplications with a parameter.
   - At last we substitute summation equations of the junctions in the differential equations of the storage elements.

Beware! In case of dependent storage variables we need to take care that accompanying state variables do not get eliminated. These are called **semi-state variables.**

Mixed DAE to ODE by Example (1)

Mixed DAE system for hoisting device

$$e_2 = u_{source}$$
$$\frac{df_3}{dt} = \frac{1}{L} e_3$$
$$e_4 = R_{el} f_4$$
$$f_2 = f_3$$
$$f_4 = f_3$$
$$f_5 = f_3$$
$$e_3 = e_2 - e_4 - e_5$$
$$e_5 = K f_6$$
$$e_6 = K f_5$$
$$\frac{df_7}{dt} = \frac{1}{J} e_7$$
$$e_8 = R_{bearing} f_8$$
$$f_6 = f_7$$
$$f_8 = f_7$$
$$f_9 = f_7$$

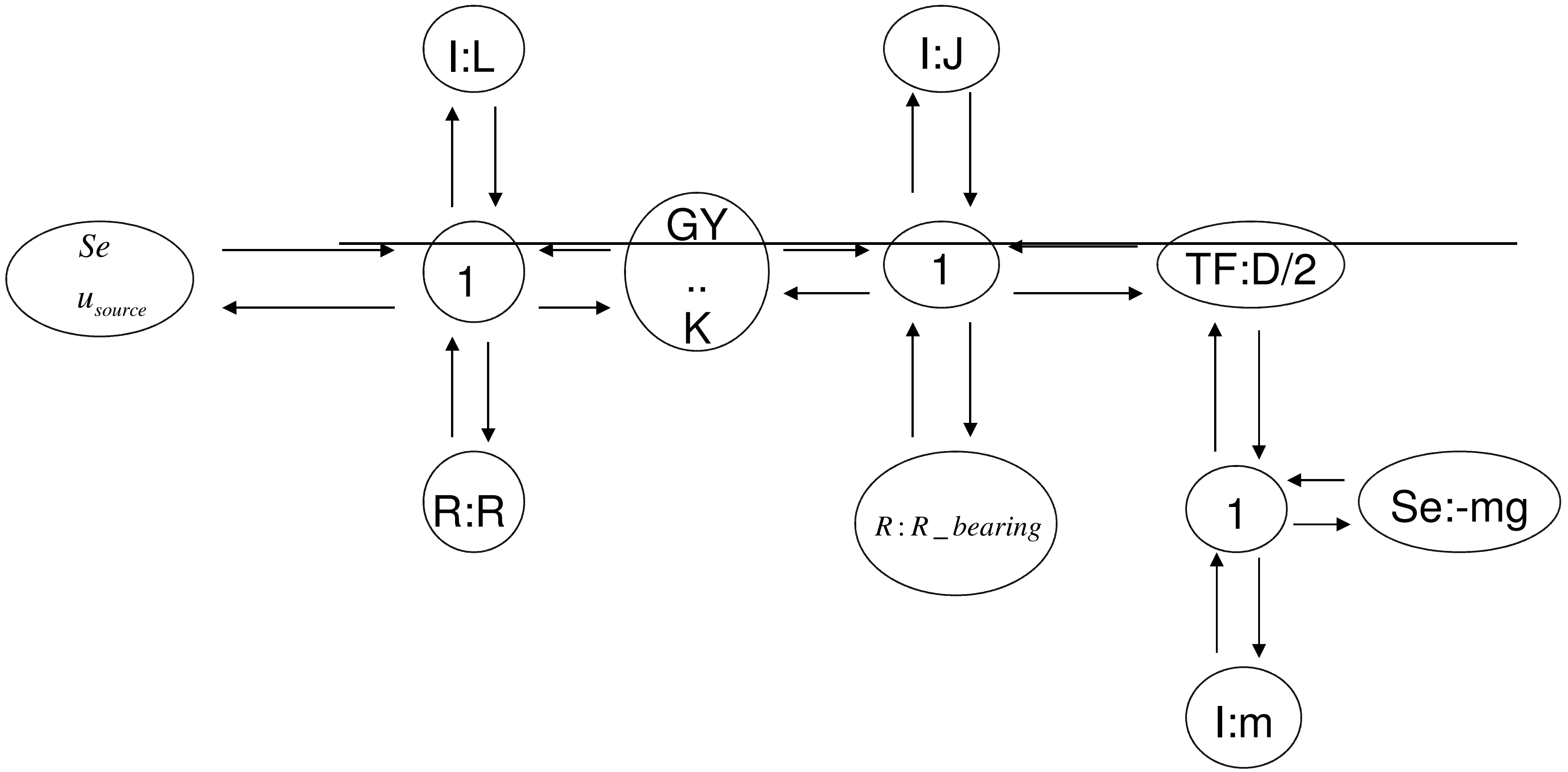
$$e_7 = e_6 - e_8 - e_9$$
$$e_9 = -\frac{D}{2} e_{10}$$
$$f_{10} = -\frac{D}{2} f_9$$
$$f_1 = f_{10}$$
$$f_{11} = f_{10}$$
$$e_{10} = e_{11} - e_1$$
$$e_1 = -mg$$
$$e_{11} = m \frac{df_{11}}{dt}$$

Mixed DAE to ODE by Example (2)

Resulting **linear system of ODEs**

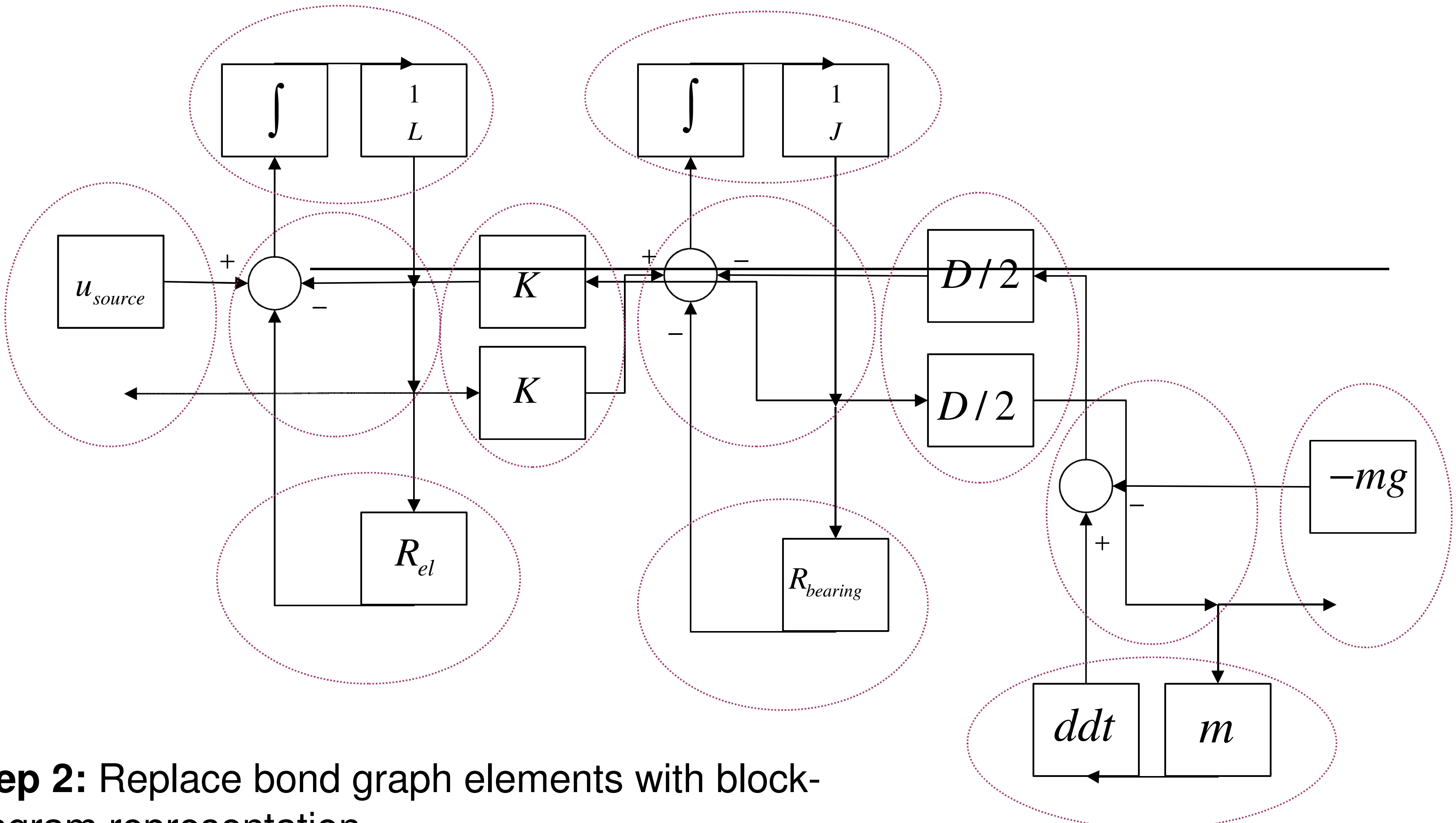
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{D}{2} \\ 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} f_3 \\ f_7 \\ f_{11} \end{bmatrix} = \begin{bmatrix} -\frac{R_{el}}{L} & -\frac{K}{L} & 0 \\ \frac{K}{J} & -\frac{R_{bearing}}{J} & 0 \\ 0 & \frac{D}{2} & 1 \end{bmatrix} \begin{bmatrix} f_3 \\ f_7 \\ f_{11} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{D}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{source} \\ mg \end{bmatrix}$$

Expansion to Block Diagrams (1)



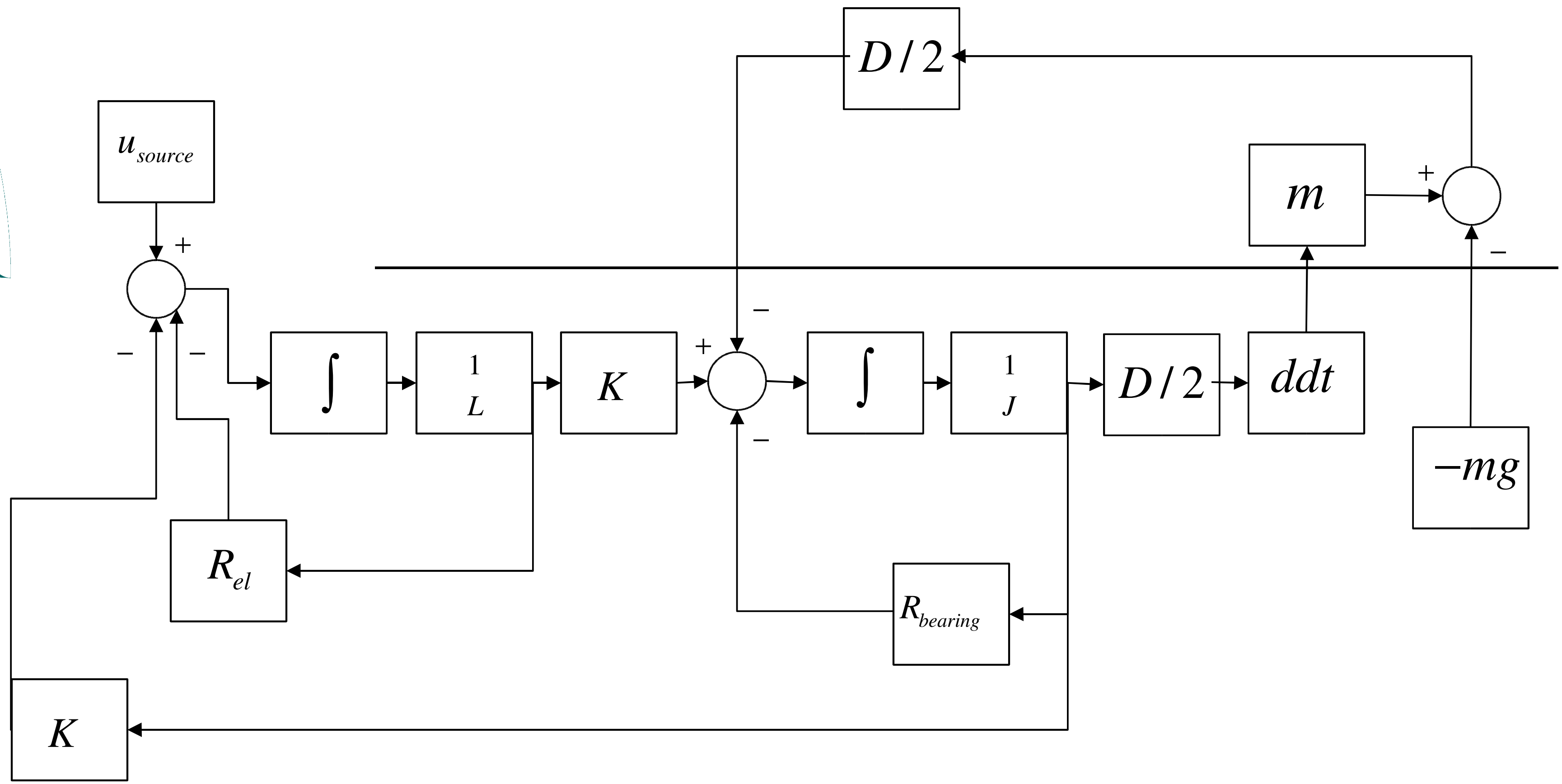
Step 1: Expand all bonds to **bilateral signal flows**

Expansion to Block Diagrams (2)



Step 2: Replace bond graph elements with block-diagram representation

Expansion to Block Diagrams (3)



Step 3: Redraw the block diagram in **standard form**. All integrators in an on going stream (from left to right), and all other operations as feedback loops

Simulation

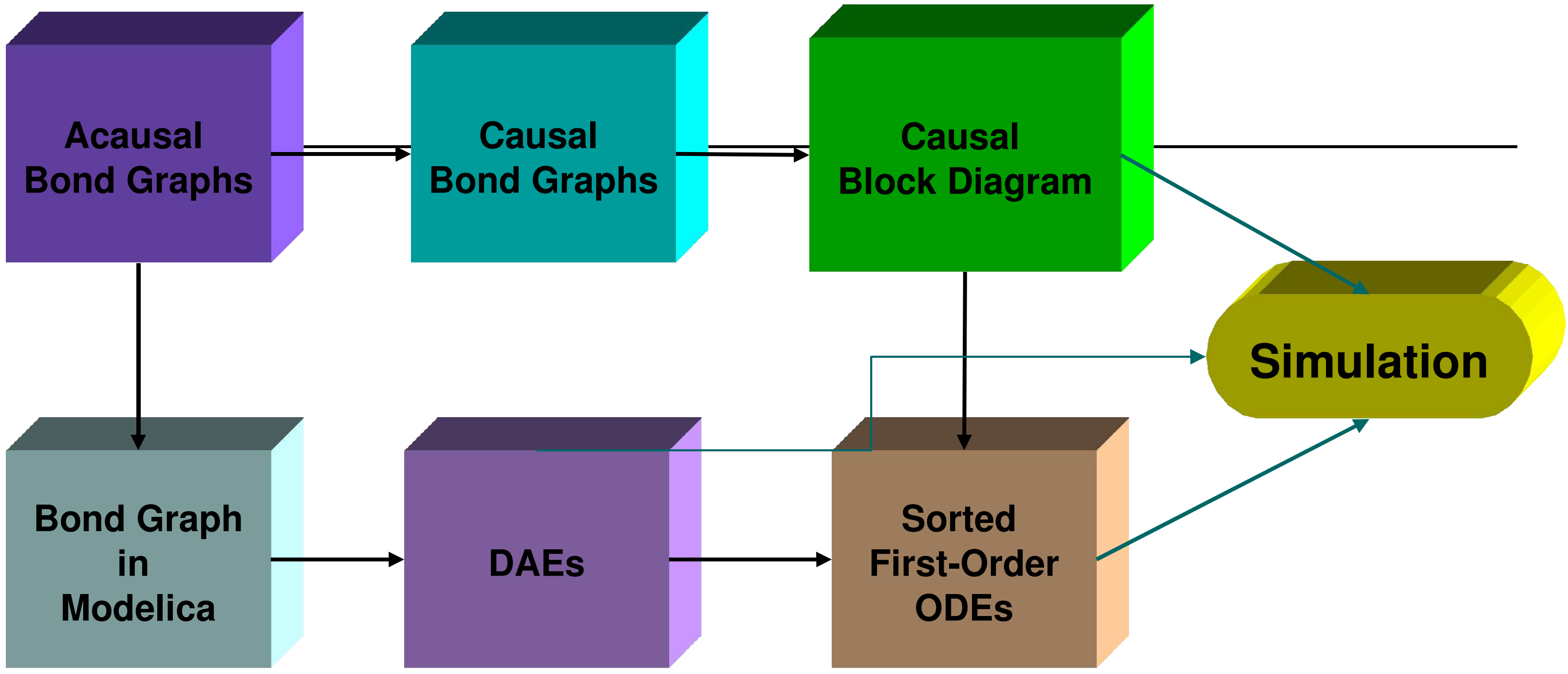
Equations coming from the bond-graph model is the **simulation model.** These are first-order ODEs or DAEs and are solved using numerical integration.

4 aspects that govern the selection of a numerical integrator:

- Presence of implicit equations
- Presence of discontinuities
- Numerical stiffness
- Oscillatory parts

The Big Picture

Model Transformation using Graph Grammars for the Bond Graph Formalism



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- Peter Gawthrop, Lorcan Smith, Metamodeling: Bond Graphs and Dynamics Systems, Prentice Hall 1996