Foundations of Modelling and Simulation

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Hierarchy of System Specification of Structure and Behaviour

- Basis of System Specification:
  sets theory, time base, segments and trajectories
- Hierarchy of System Specification (**causal**, **deterministic**)
  1. I/O Observation Frame
  2. I/O Observation Relation
  3. I/O Function Observation
  4. I/O System
- Multicomponent Specifications
- Non-causal models

ref: Wayne Waymore, Bernard Zeigler, George Klir, . . .

Set Theory

Properties:

$$\{1, 2, \ldots, 9\}$$

$$\{a, b, \ldots, z\}$$

$$\mathbb{N}, \mathbb{N}^+, \mathbb{N}^+_\infty$$

$$\mathbb{R}, \mathbb{R}^+, \mathbb{R}^+_\infty$$

$$EV = \{ARRIVAL, DEPARTURE\}$$

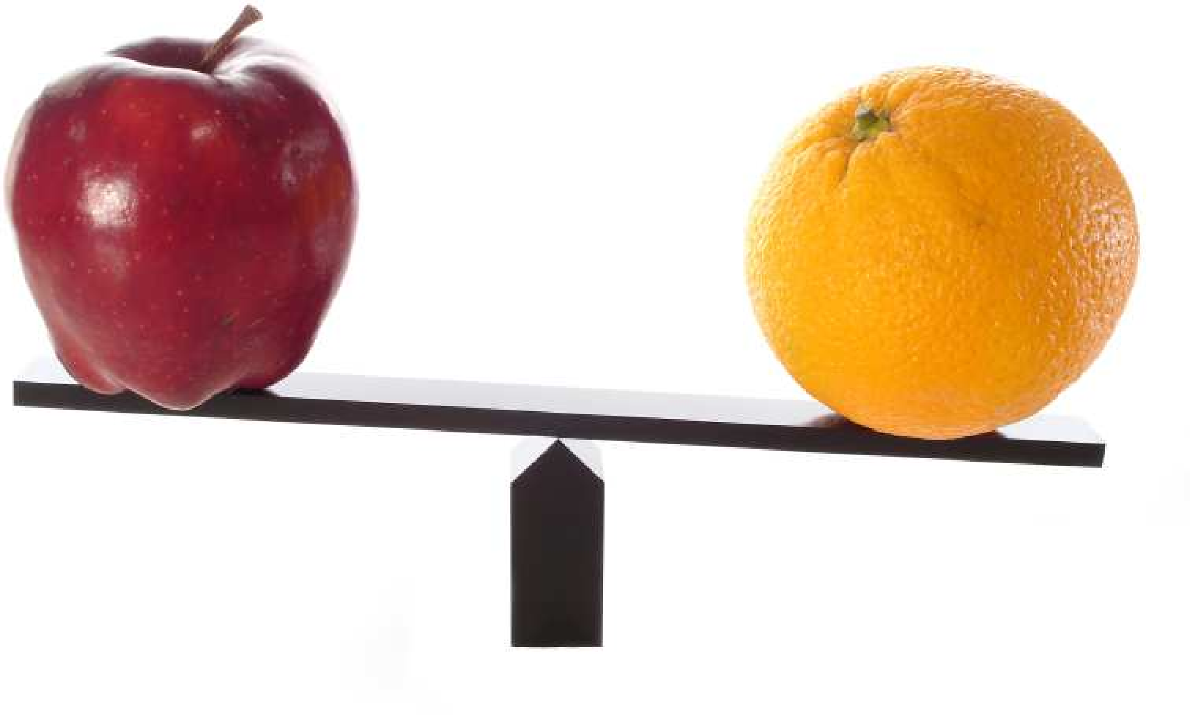
$$EV^\phi = EV \cup \{\phi\}$$

Structuring:

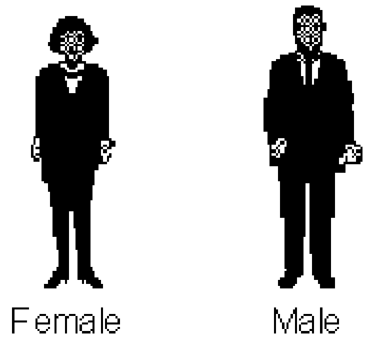
$$A \times B = \{(a, b) | a \in A, b \in B\}$$

$$G = (E, V), V \subseteq E \times E$$

Comparing things



Nominal Scale: e.g., gender

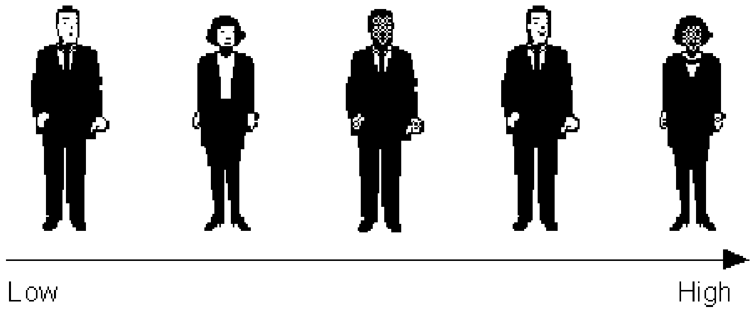


A scale that assigns a *category label* to an individual.
Establishes no explicit ordering on the category labels.

Only a notion of *equivalence* "$=$" is defined with properties:

1. Reflexivity: $x = x \vee x \neq x$.
2. Symmetry of equivalence: $x = y \Leftrightarrow y = x$.
3. Transitivity: $x = y \wedge y = z \rightarrow x = z$.

Ordinal Scale: e.g., degree of happiness



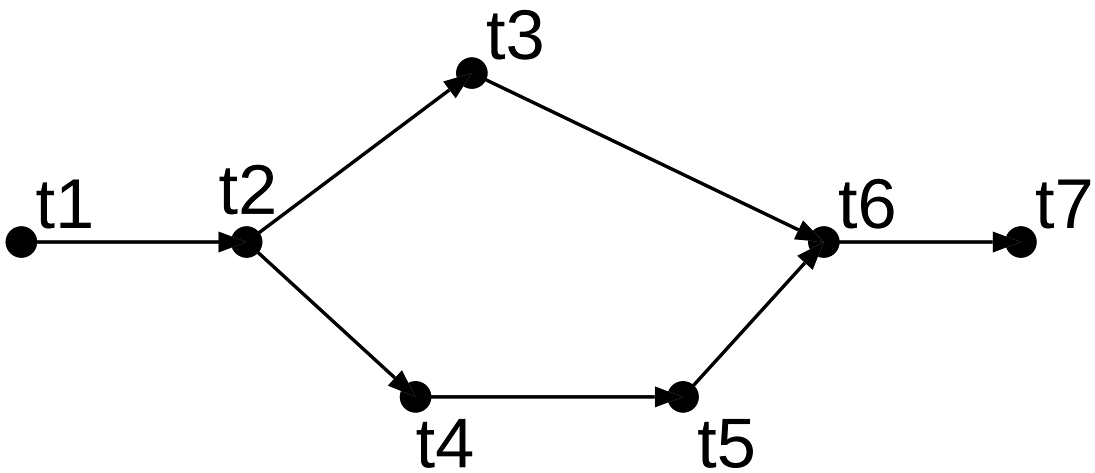
A scale in which data can be *ranked*, but in which no arithmetic transformations are meaningful. It is meaningless to talk about difference (distance).

In addition to equivalence, a notion of *order* $<$ is defined with properties:

1. Symmetry of equivalence: $x = y \Leftrightarrow y = x$.
2. Asymmetry of order: $x < y \rightarrow y \not< x$.
3. Irreflexivity: $x \not< x$.
4. Transitivity: $x < y \wedge y < z \rightarrow x < z$.

Partial ordering

The ordering may be *partial* (some data items cannot be compared).



The ordering may be *total* (all data items can be compared).

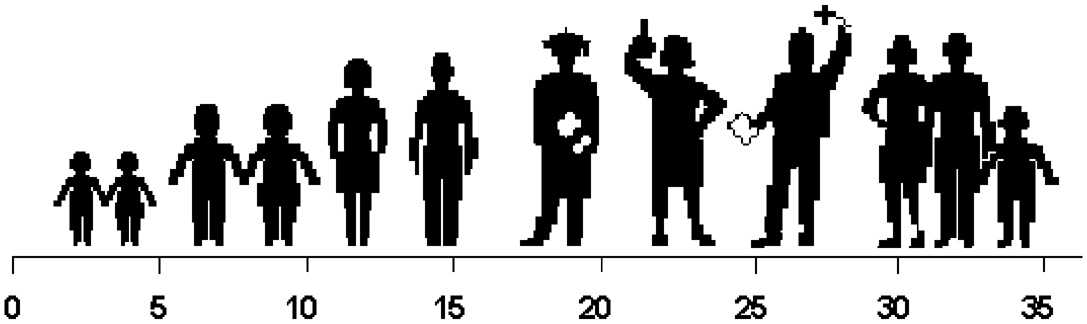
$$\forall x, y \in X : x < y \vee y < x \vee x = y$$

Interval Scale: e.g., Shoe Size

A scale where *distances* between data are meaningful. On interval measurement scales, one unit on the scale represents the *same magnitude* of the characteristic being measured across the whole range of the scale. Interval scales do not have a “true” zero point, however, and therefore it is not possible to make statements about how many times higher one value is than another.

In addition to equivalence and order, a notion of *interval* is defined. The choice of a zero point is arbitrary.

Ratio Scale: e.g., age



Both *intervals* between values and *ratios* of values are meaningful. A meaningful *zero* point is known. "A is twice as old as B".

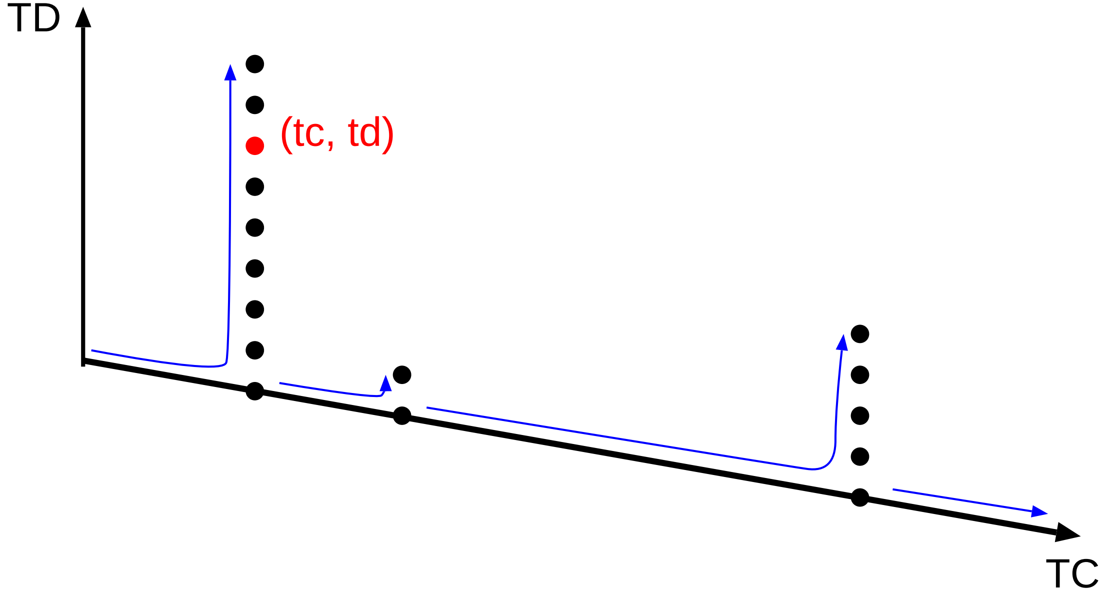
Time Base

- Simulation of **Dynamic** Systems: irreversible passage of *time*.

- Time Base T:
  - $\{NOW\}$ (instantaneous)
  - $\mathbb{R}$: *continuous-time*
  - $\mathbb{N}$ or isomorphic: *discrete-time*
- Ordering:
  - Ordinal Scale (possibly partial ordering, for concurrency)
  - Interval Scale
  - Ratio Scale

Time Bases for hybrid system models

Time Bases for hybrid system models

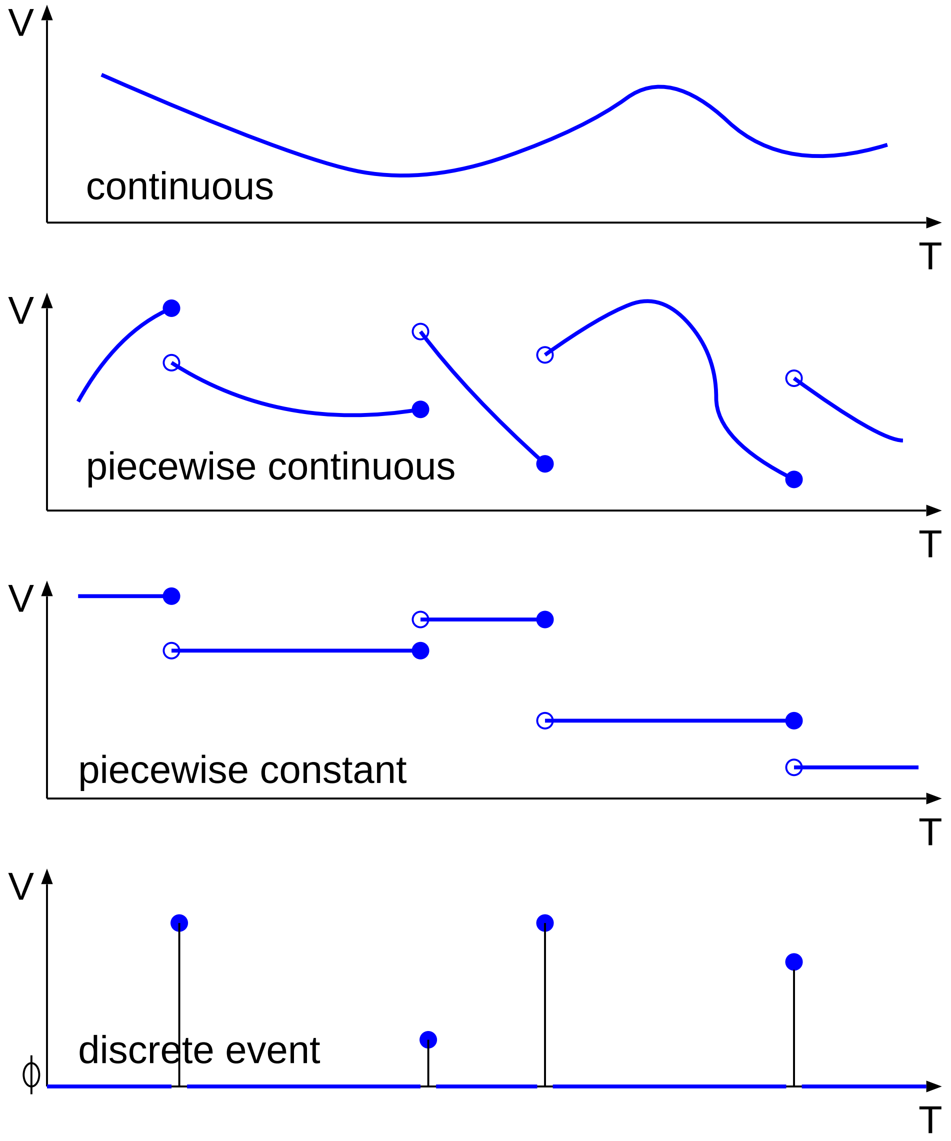


"nested time" for nested experiments.

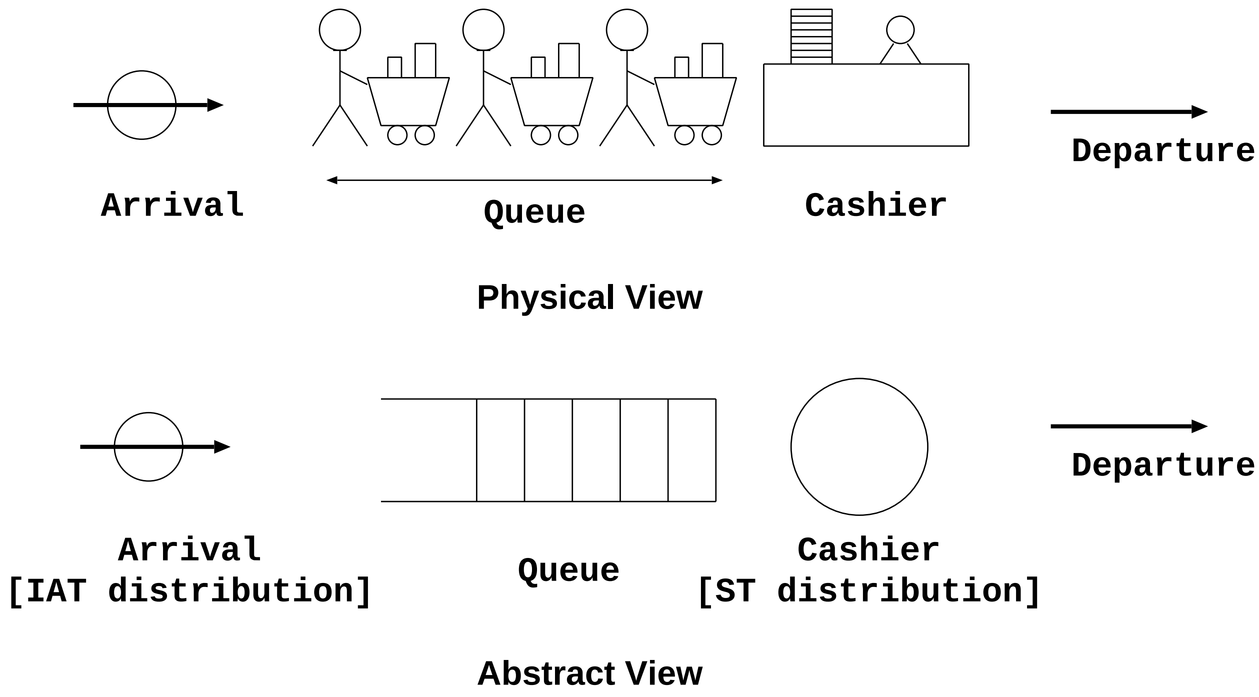
Behaviour $\equiv$ Evolution over Time

- With time base, describe *evolution over time*
- Time function, **trajectory**, signal: $f : T \rightarrow V$
- Restriction to $T' \subseteq T$
  $f|T' : T' \rightarrow V$, $\forall t \in T' : f|T'(t) = f(t)$
  - Past of f: $f|T_{t\rangle}$
  - Future of f: $f|T_{\langle t}$
- Restriction to an interval: **segment**
  $\omega : \langle t_1, t_2 \rangle \rightarrow V$

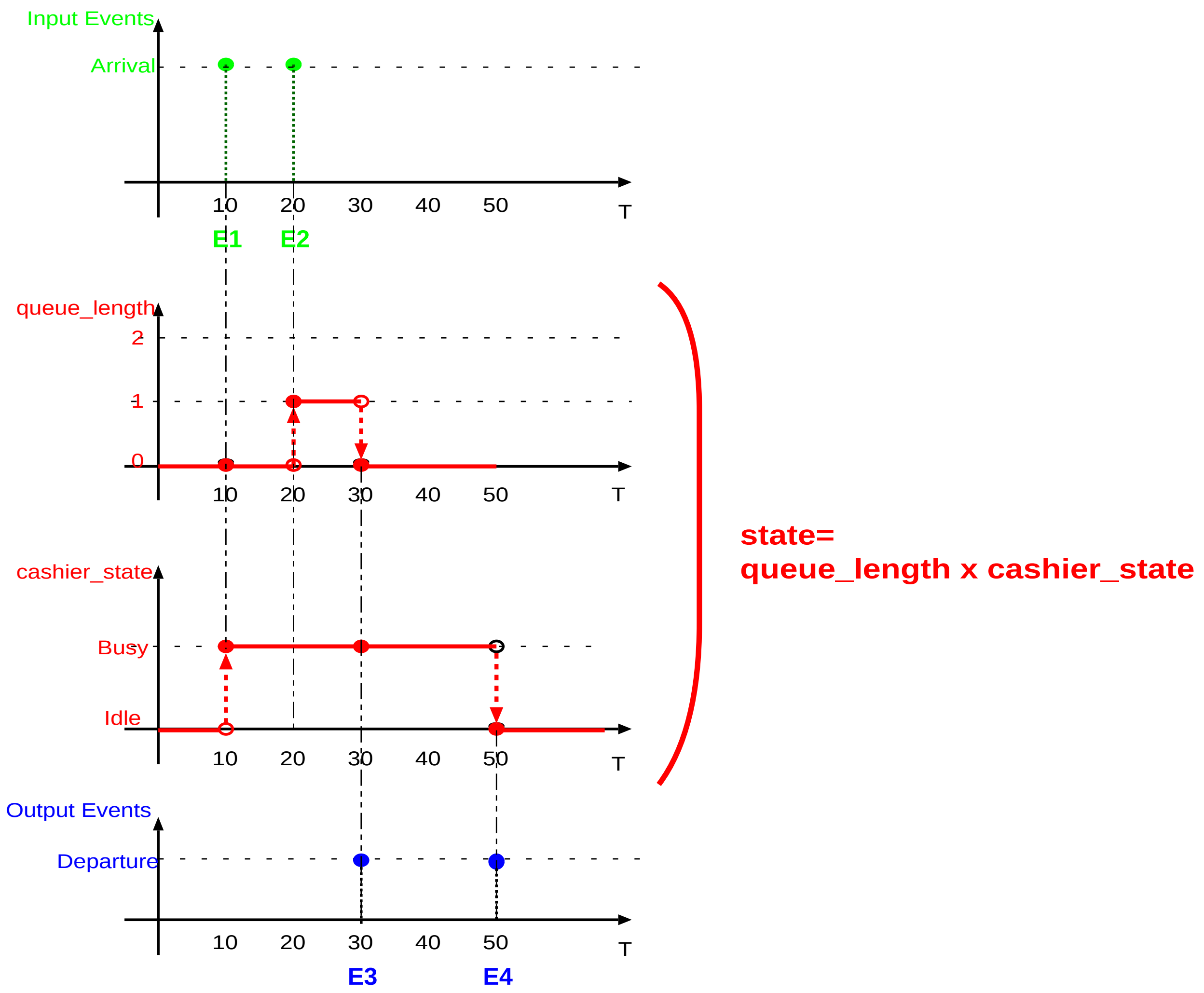
Types of Segments



Cashier-Queue System



Trajectories



I/O Observation Frame (causal)

$$O = \langle T, X, Y \rangle$$

- T is *time-base*: $\mathbb{N}$ (discrete-time), $\mathbb{R}$ (continuous-time)
- X input value set: $\mathbb{R}^n, EV^{\phi}$
- Y output value set: system response

I/O Relation Observation

$$IORO = \langle T, X, \Omega, Y, R \rangle$$

- $\langle T, X, Y \rangle$ is Observation Frame
- Ω is the set of all possible input segments
- R is the *I/O relation*
  $\Omega \subseteq (X, T)$, $R \subseteq \Omega \times (Y, T)$
  $(\omega, \rho) \in R \Rightarrow dom(\omega) = dom(\rho)$
- $\omega : \langle t_i, t_f \rangle \rightarrow X$: input *segment*
- $\rho : \langle t_i, t_f \rangle \rightarrow Y$: output *segment*
- note: not really necessary to observe over same time domain

I/O Function Observation

$$IOFO = \langle T, X, \Omega, Y, F \rangle$$

- $\langle T, X, \Omega, Y, R \rangle$ is a Relation Observation
- Ω is the set of all possible input segments
- F is the *set of I/O functions*
  $f \in F \Rightarrow f \subset \Omega \times (Y, T)$, where
  f is a **function** such that $dom(f(\omega)) = dom(\omega)$
- *f = initial state*: **unique** response to ω
- $R = \bigcup_{f \in F} f$

I/O System

- From **Descriptive Variables** (properties) to **State**.
- **State** summarizes the past behaviour of the system.
- Future is uniquely determined by
  - **current state**
  - **future input**

$$SYS = \langle T, X, \Omega, Q, \delta, Y, \lambda \rangle$$

T	time base
X	input set
$\omega : T \rightarrow X$	input segment
Q	state set
$\delta : \Omega \times Q \rightarrow Q$	transition function
Y	output set
$\lambda : Q \rightarrow Y$ (or $Q \times X \rightarrow Y$)	output function

$$\forall t_x \in [t_i, t_f] : \delta(\omega_{[t_i,t_f]}, q_i) = \delta(\omega_{[t_x,t_f]}, \delta(\omega_{[t_i,t_x]}, q_i))$$

From I/O System specification to I/O Function Observation

For a given initial condition q and a given input segment ω, we can define a *state trajectory* $STRAJ_{q,\omega}$ from SYS

$$STRAJ_{q,\omega} : dom(\omega) \rightarrow Q,$$

with

$$STRAJ_{q,\omega}(t) = \delta(\omega_{t\rangle}, \forall t \in dom(\omega).$$

From this state trajectory, an *output trajectory* $OTRAJ_{q,\omega}$ may be constructed

$$OTRAJ_{q,\omega} : dom(\omega) \rightarrow Y,$$

with

$$OTRAJ_{q,\omega}(t) = \lambda(STRAJ_{q,\omega}(t), \omega(t)), \forall t \in dom(\omega).$$

Thus, for every q (initial state), it is possible to construct

$$\mathcal{T}_q : \Omega \rightarrow (Y, T),$$

where

$$\mathcal{T}_q(\omega) = OTRAJ_{q,\omega}, \forall \omega \in \Omega.$$

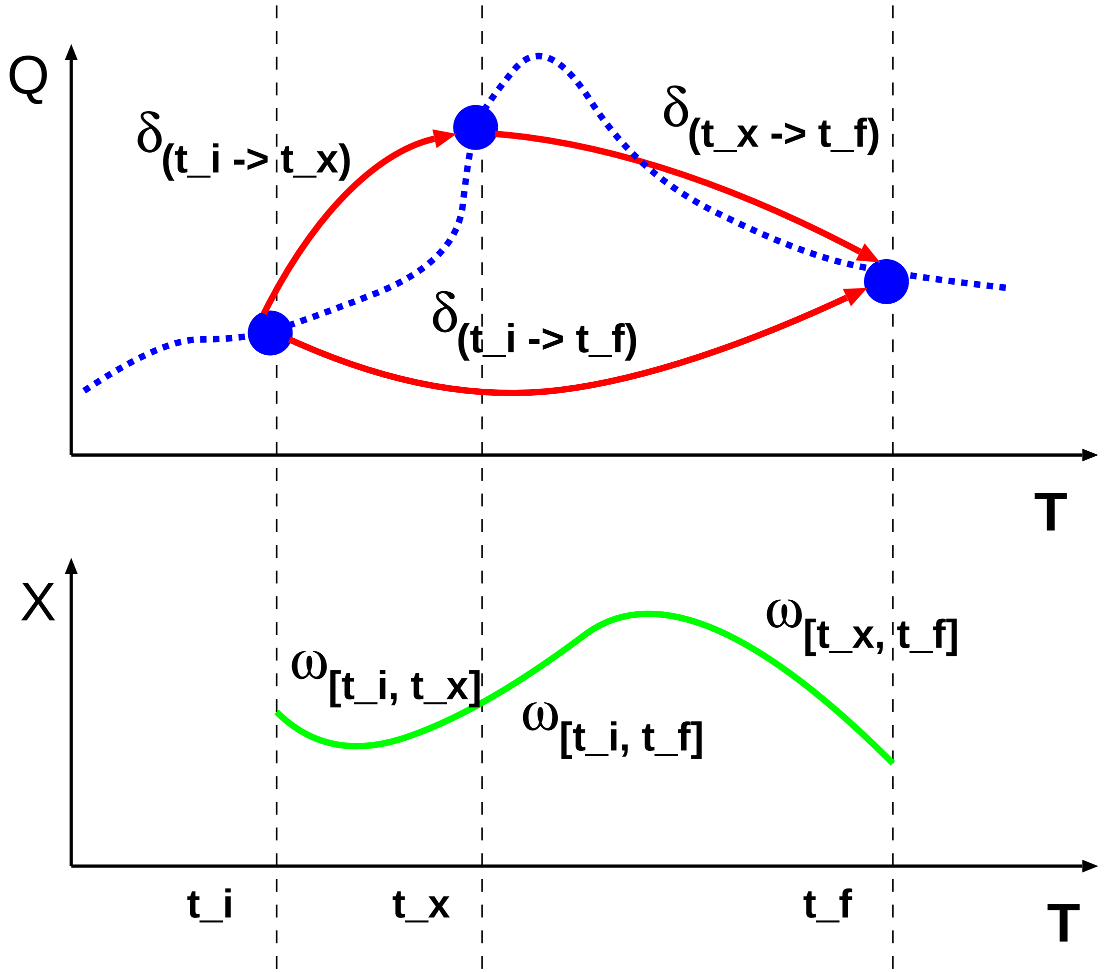
The I/O Function Observation associated with SYS is then

$$IOFO = \langle T, X, \Omega, Y, \{\mathcal{T}_q(\omega) | q \in Q\} \rangle.$$

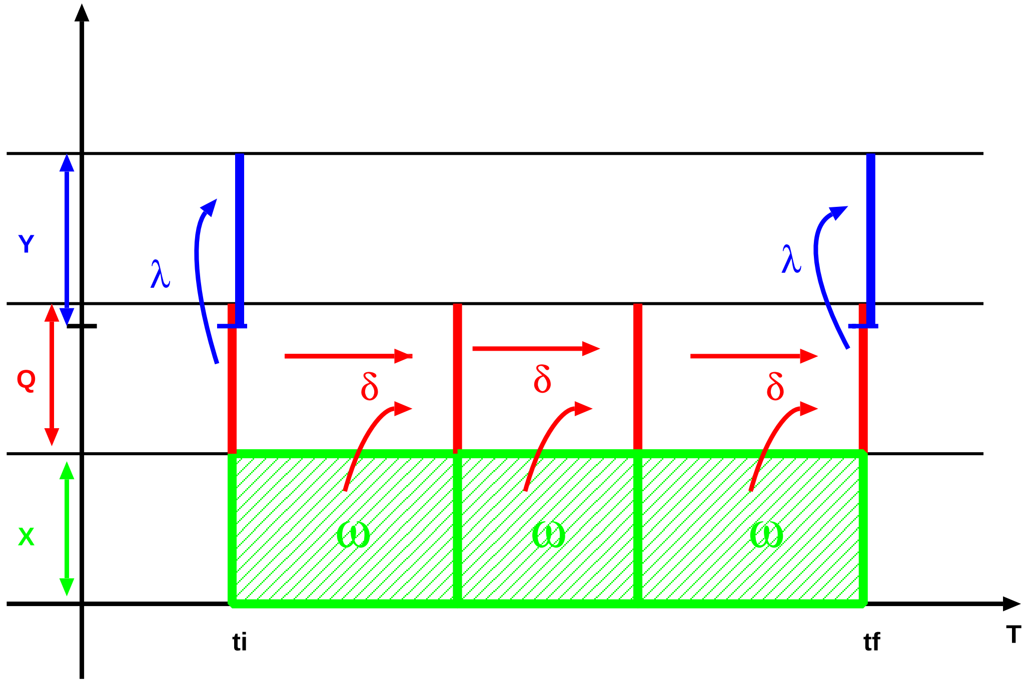
Subsequently, we may derive the I/O Relation Observation by constructing the relation R as the union of all I/O functions:

$$R = \{(\omega, \rho) | \omega \in \Omega, \rho = OTRAJ_{q,\omega}, q \in Q\}.$$

Composition Property



Simulator: step through time



Formalism classification based on general system model

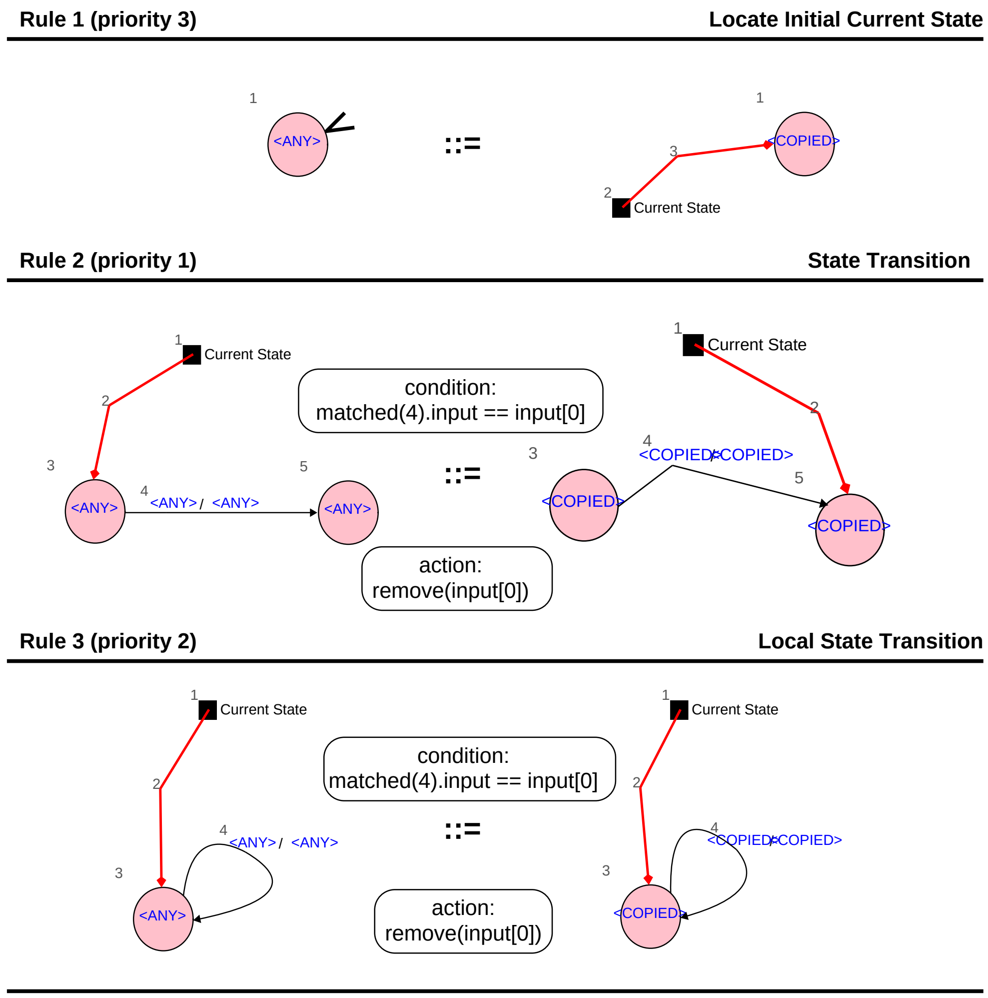
	T: **Continuous**	T: **Discrete**	T: $\{$**NOW**$\}$
Q: **Continuous**	ODE, DEVS	Difference Eqns. (DTSS)	Algebraic Eqns.
Q: **Discrete**	Discrete-event	Finite State Automata	Integer Eqns.

Basis for **general, standard software architecture of simulators**

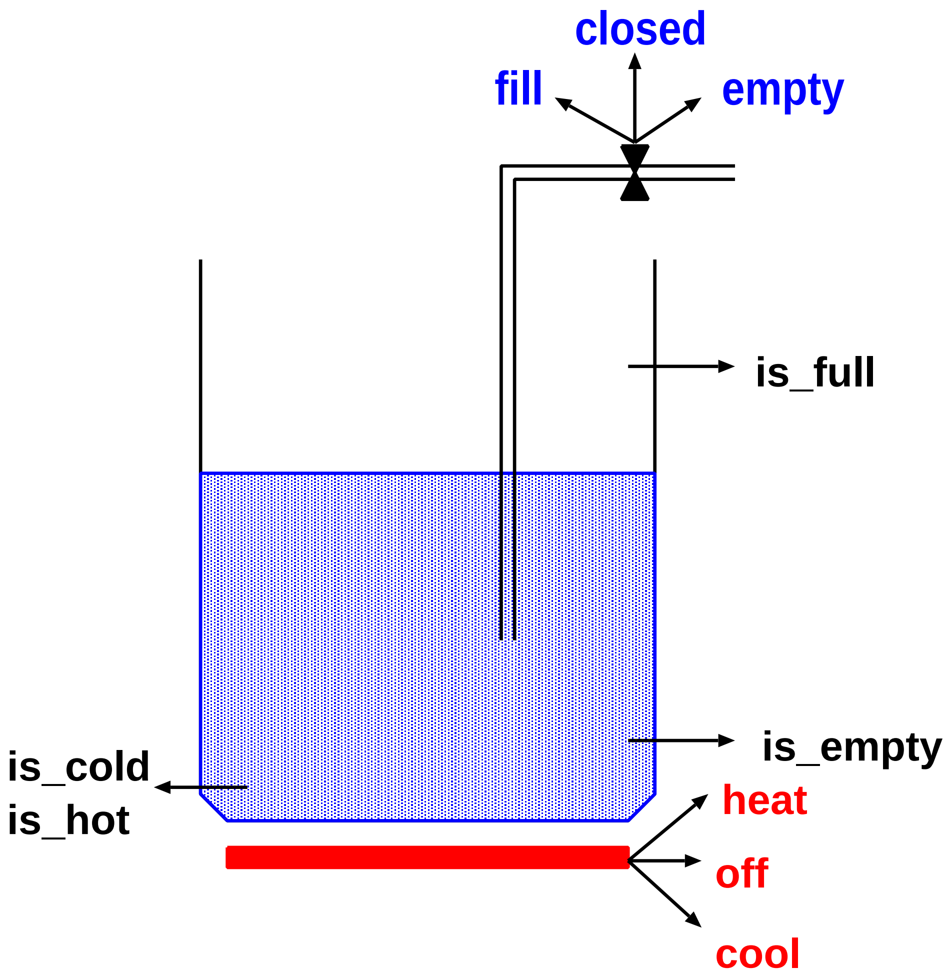
Further classifications based on **structure of formalisms**

(in particular of δ)

Rule-based specification of δ



System under study: T, h controlled liquid



Detailed (continuous) view, ALG + ODE

Inputs (discontinuous $\rightarrow$ hybrid model):

- Emptying, filling flow rate ϕ
- Rate of adding/removing heat W

Parameters:

- Temperature of influent T_{in}
- Cross-section surface of vessel A
- Specific heat of liquid c
- Density of liquid ρ

State variables:

- Temperature T
- Level of liquid l

Outputs (sensors):

- $is\_low, is\_high, is\_cold, is\_hot$

$$
\left\{
\begin{array}{rcl}
\frac{dT}{dt} & = & \frac{1}{l}\left[\frac{W}{c\rho A} - \phi(T - T_{in})\right] \\
\frac{dl}{dt} & = & \phi \\
is\_low & = & (l < l_{low}) \\
is\_high & = & (l > l_{high}) \\
is\_cold & = & (T < T_{cold}) \\
is\_hot & = & (T > T_{hot})
\end{array}
\right.
$$

$$SYS^{ODE}_{VESSEL} = \langle \mathcal{T}, X, \Omega, Q, \delta, Y, \lambda \rangle$$

$$\mathcal{T} = \mathbb{R}$$

$$X = \mathbb{R} \times \mathbb{R} = \{(W, \phi)\}$$

$$\omega : \mathcal{T} \to X$$

$$Q = \mathbb{R}^+ \times \mathbb{R}^+ = \{(T, l)\}$$

$$\delta : \Omega \times Q \to Q$$

$$\delta(\omega_{[t_i, t_f]}, (T(t_i), l(t_i))) =$$

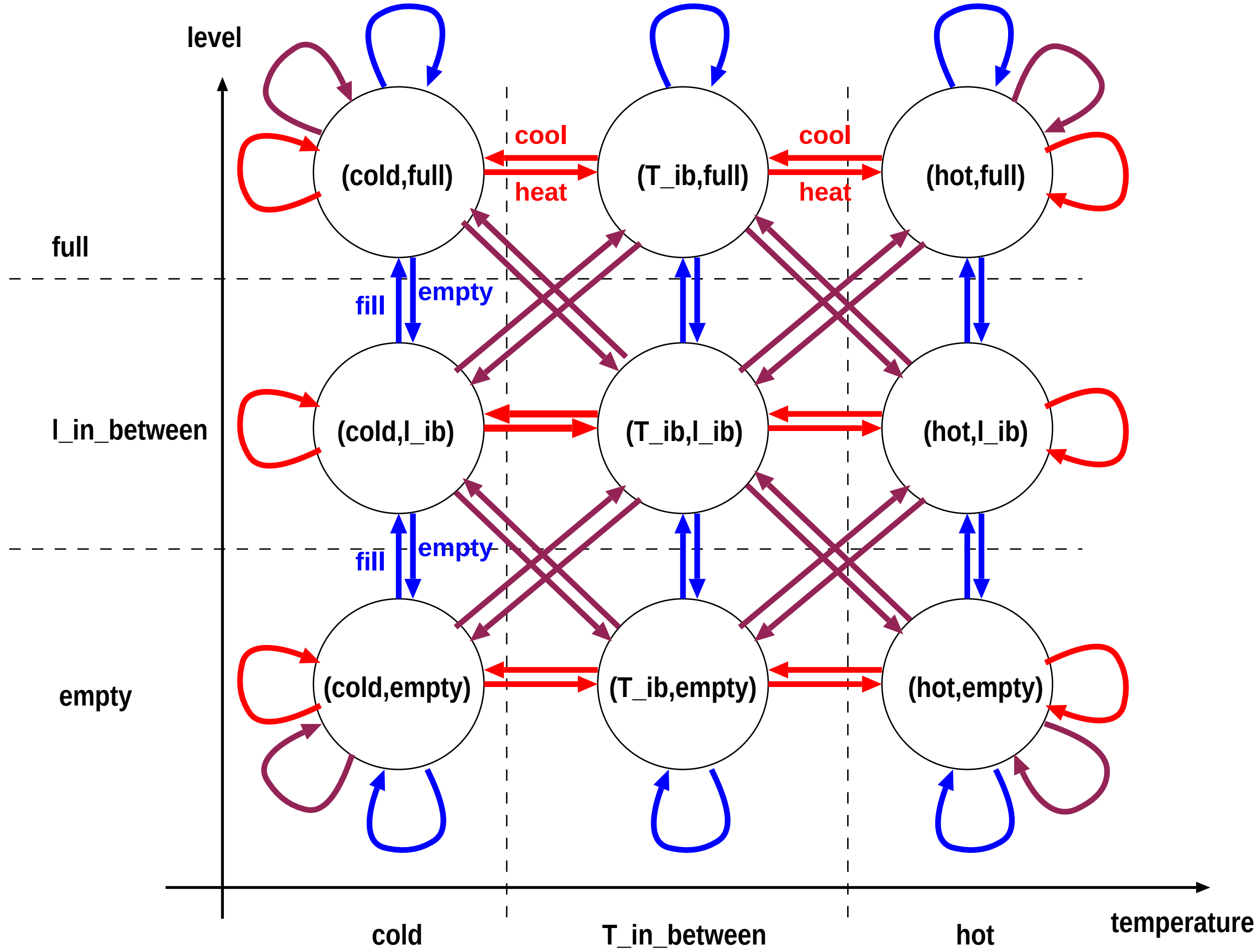
$$(T(t_i) + \int_{t_i}^{t_f} \frac{1}{l(\alpha)} [\frac{W(\alpha)}{c\rho A} - \phi(\alpha) T(\alpha)] d\alpha,\ l(t_i) + \int_{t_i}^{t_f} \phi(\alpha) d\alpha)$$

$$Y = \mathbb{B} \times \mathbb{B} \times \mathbb{B} \times \mathbb{B} = \{(is\_low, is\_high, is\_cold, is\_hot)\}$$

$$\lambda : Q \to Y$$

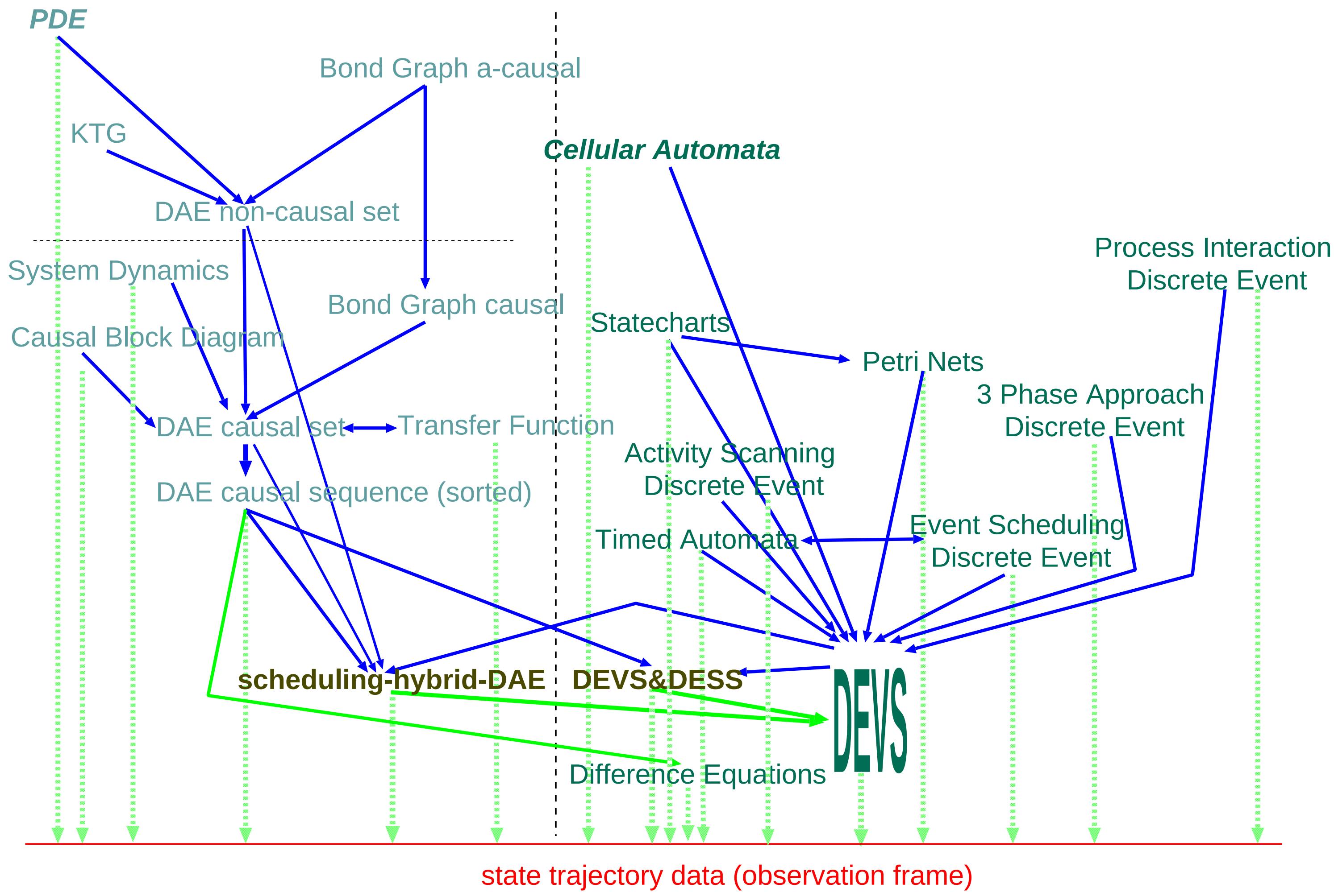
$$\lambda(T, l) = ((l < l_{low}), (l > l_{high}), (T < T_{cold}), (T > T_{hot}))$$

High-abstraction-level (discrete) view: FSA

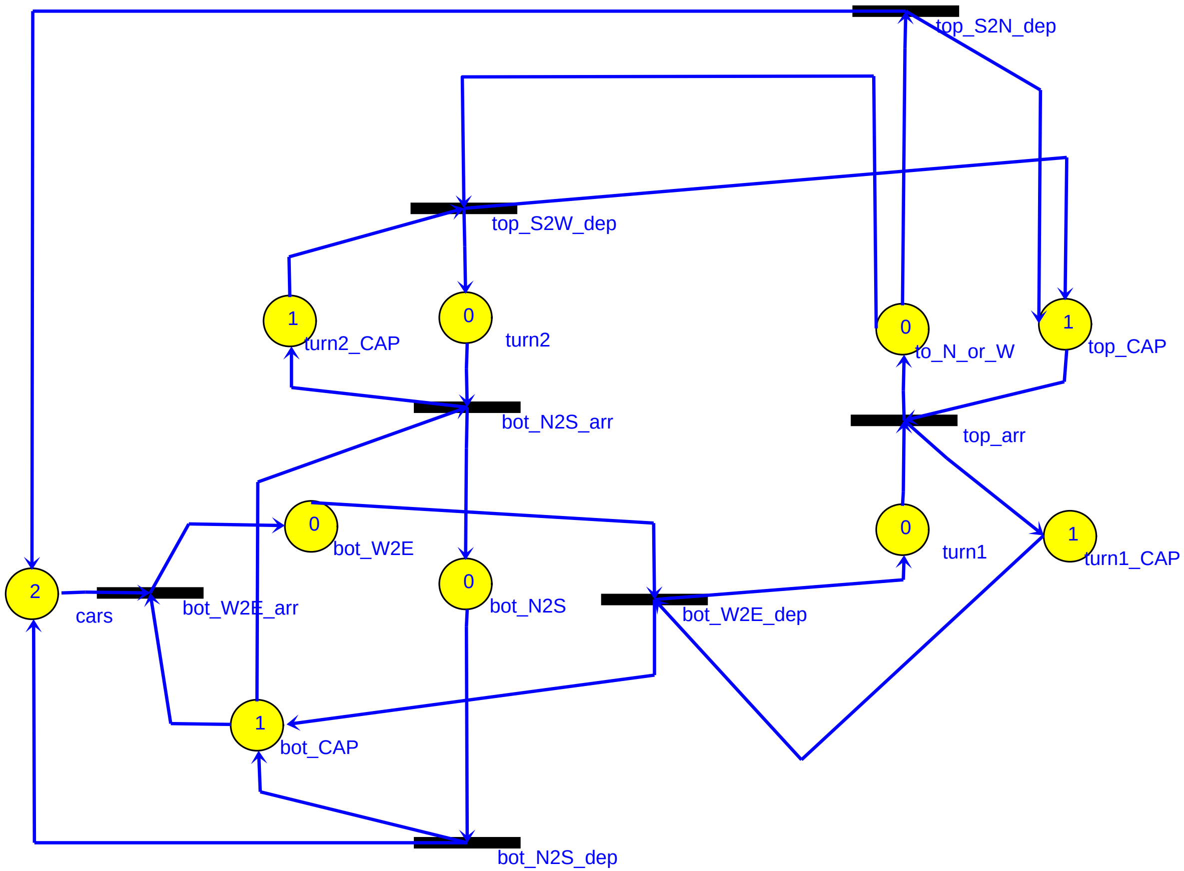


at this level: verification of properties possible

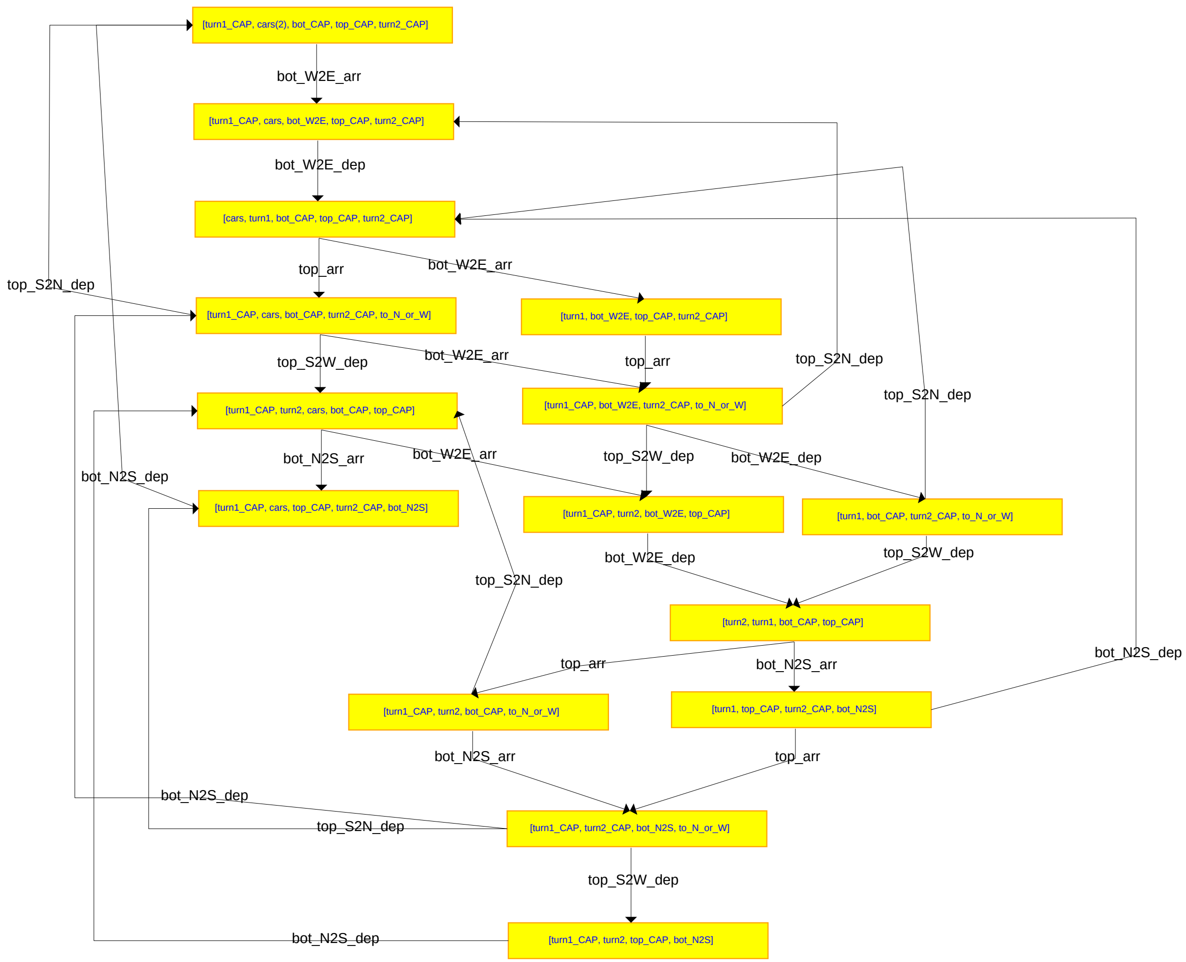
don't build simulator (Operational Semantics) but Transform (Transformational Semantics)



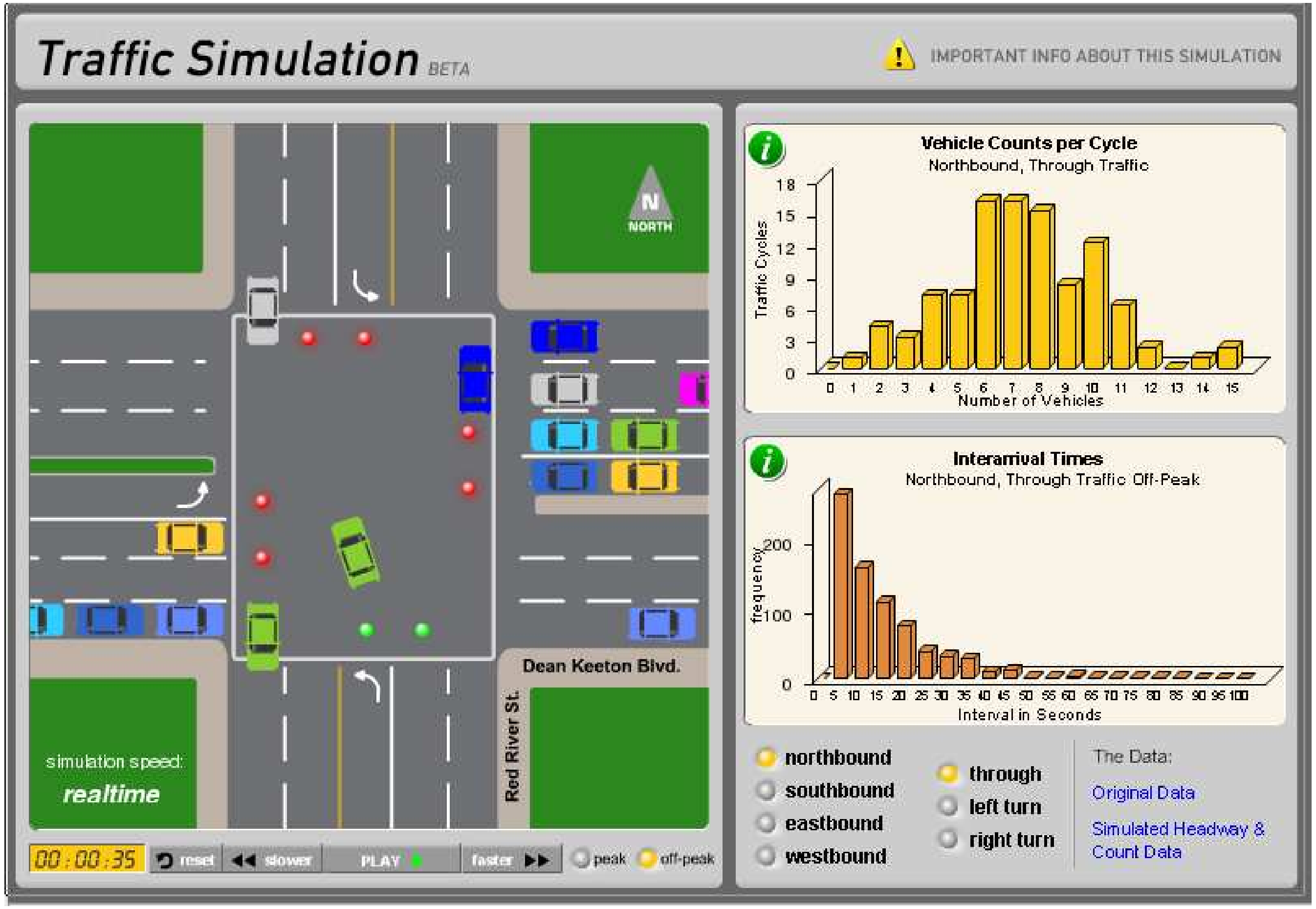
Non-determinism: Traffic network Petri Net



All traces → Reachability Graph

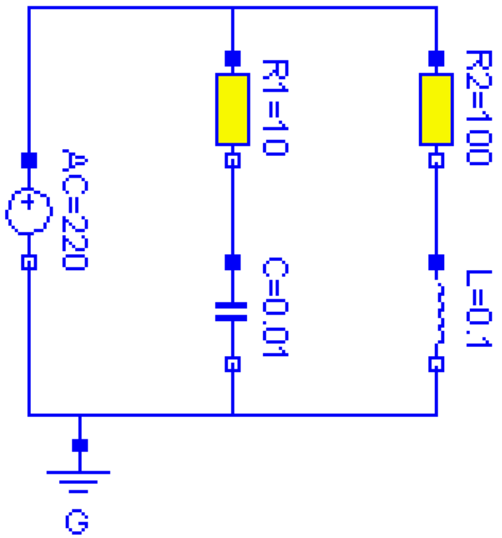


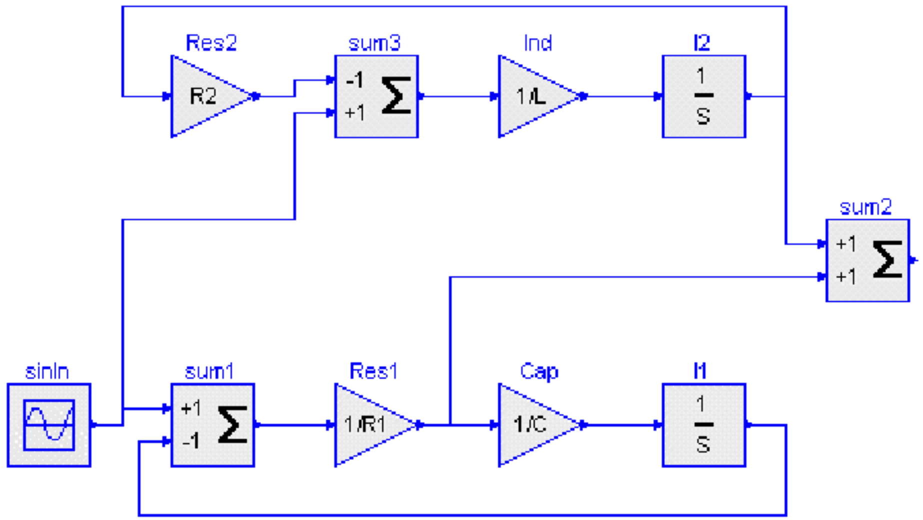
Probabilistic → Monte-Carlo Simulation



www.engr.utexas.edu/trafficSims/

Causality: Modelica vs. Matlab/Simulink

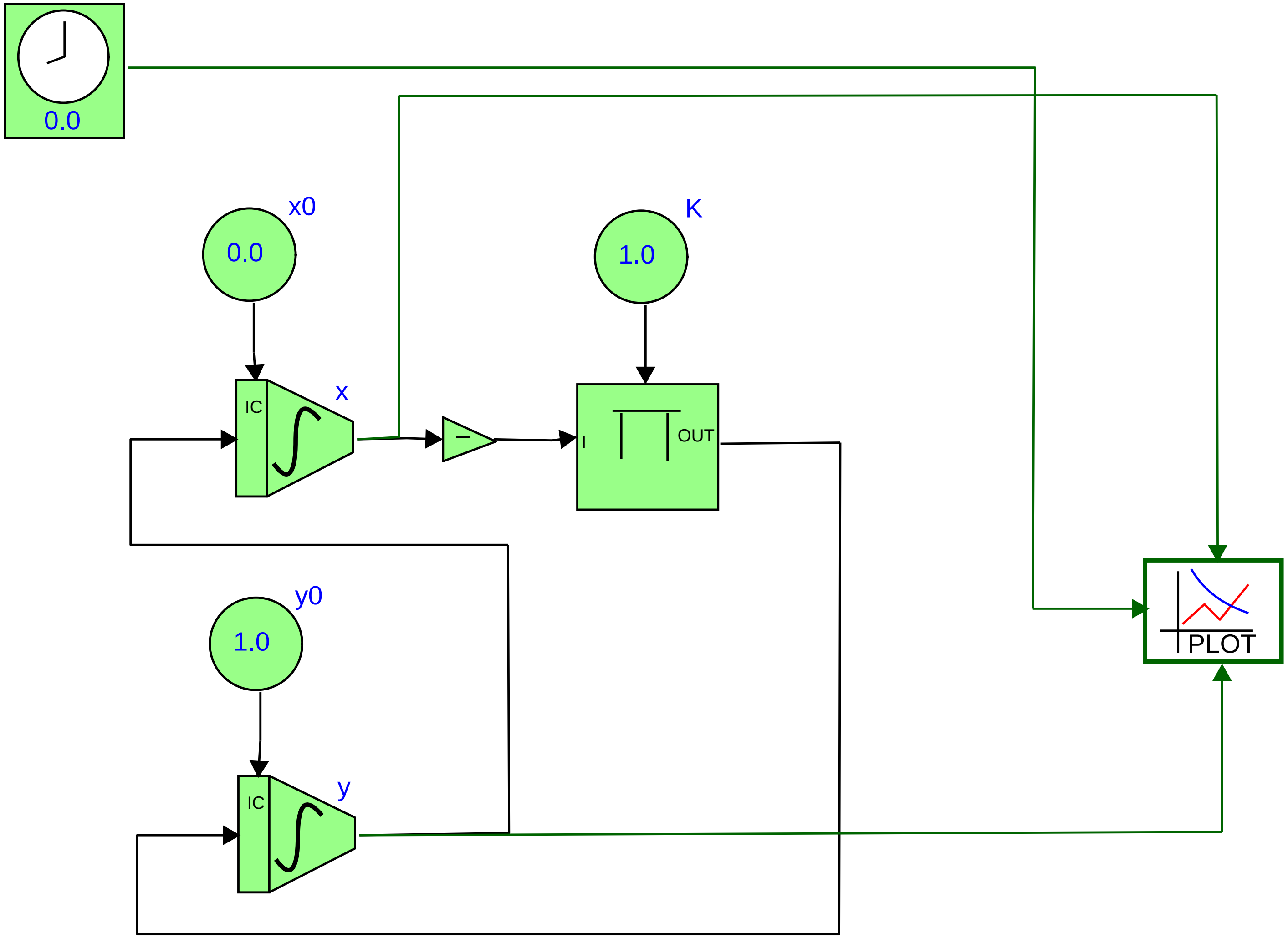




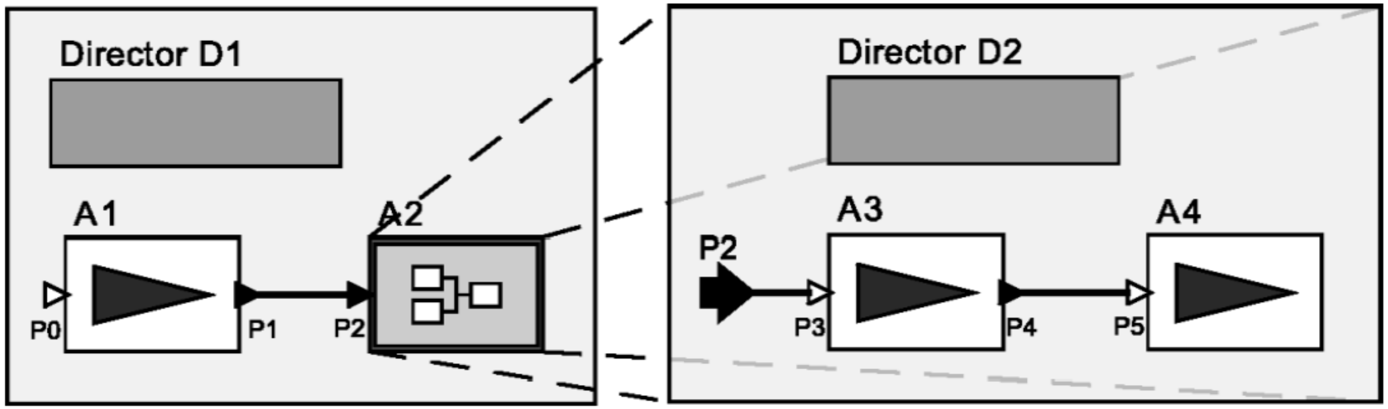
Multicomponent Specification

- Collections of *interacting* components
- *Compositional* modelling
  - *Modular* (interaction through ports only).
    Encapsulated. Allows for *hierarchical (de-)composition*.
  - *non-modular* (direct interaction between components).
    Not encapsulated. "global" variable access. Direct interaction through transition function

Causal Block Diagram



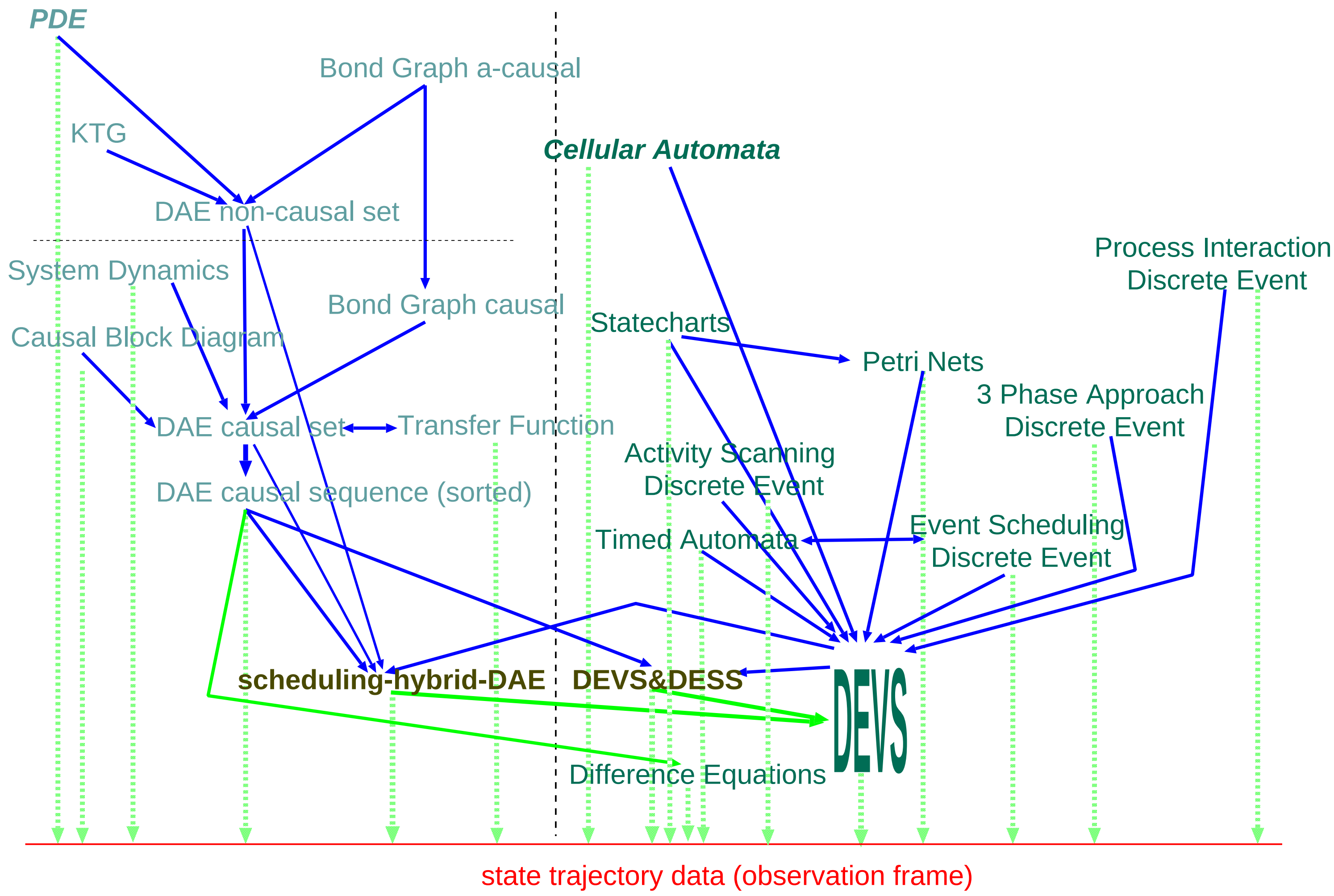
Multi-formalism / Heterogeneous MoC (Ptolemy)



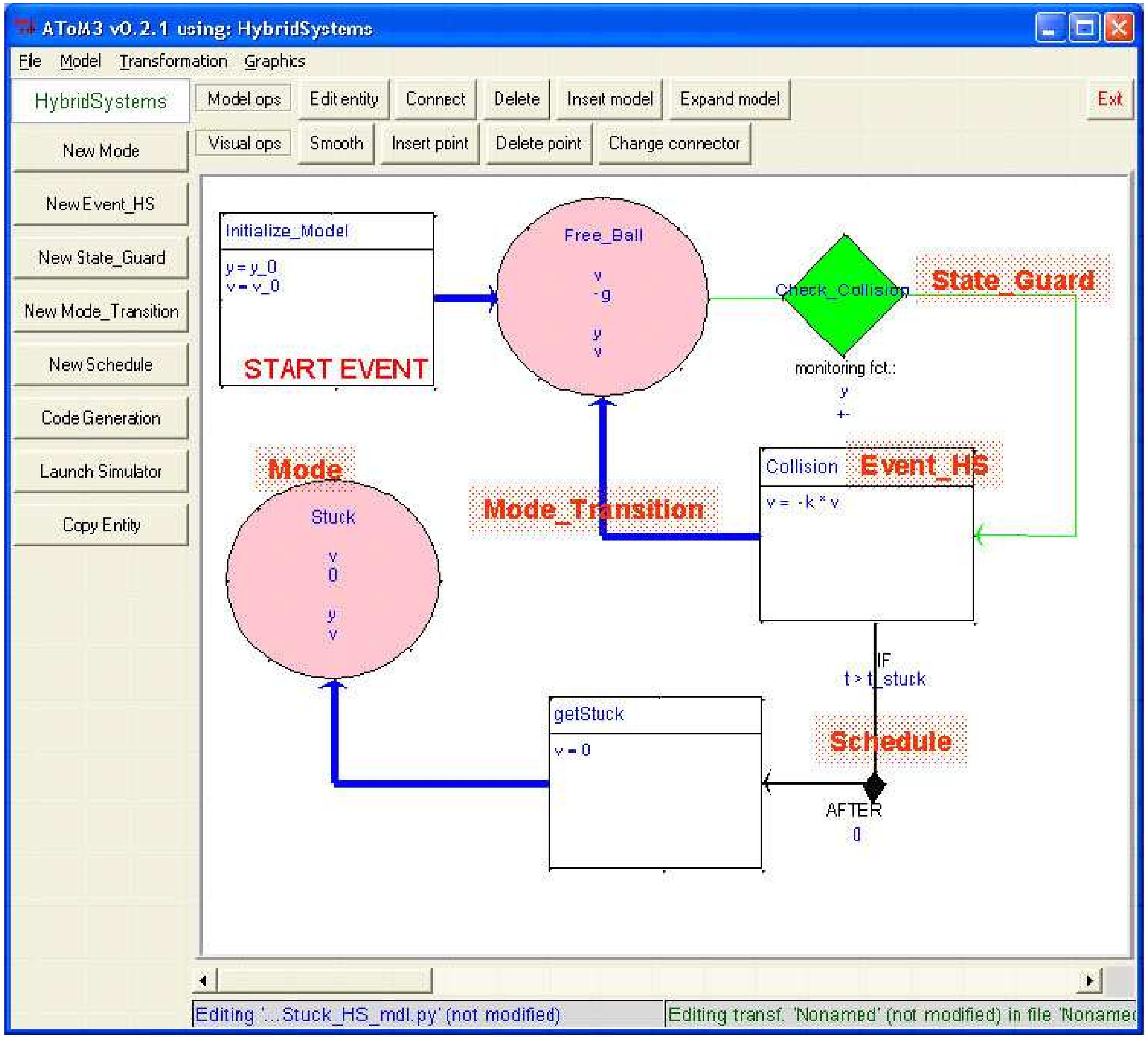
solution:

- co-simulation
- formalism transformation (using graph transformation)

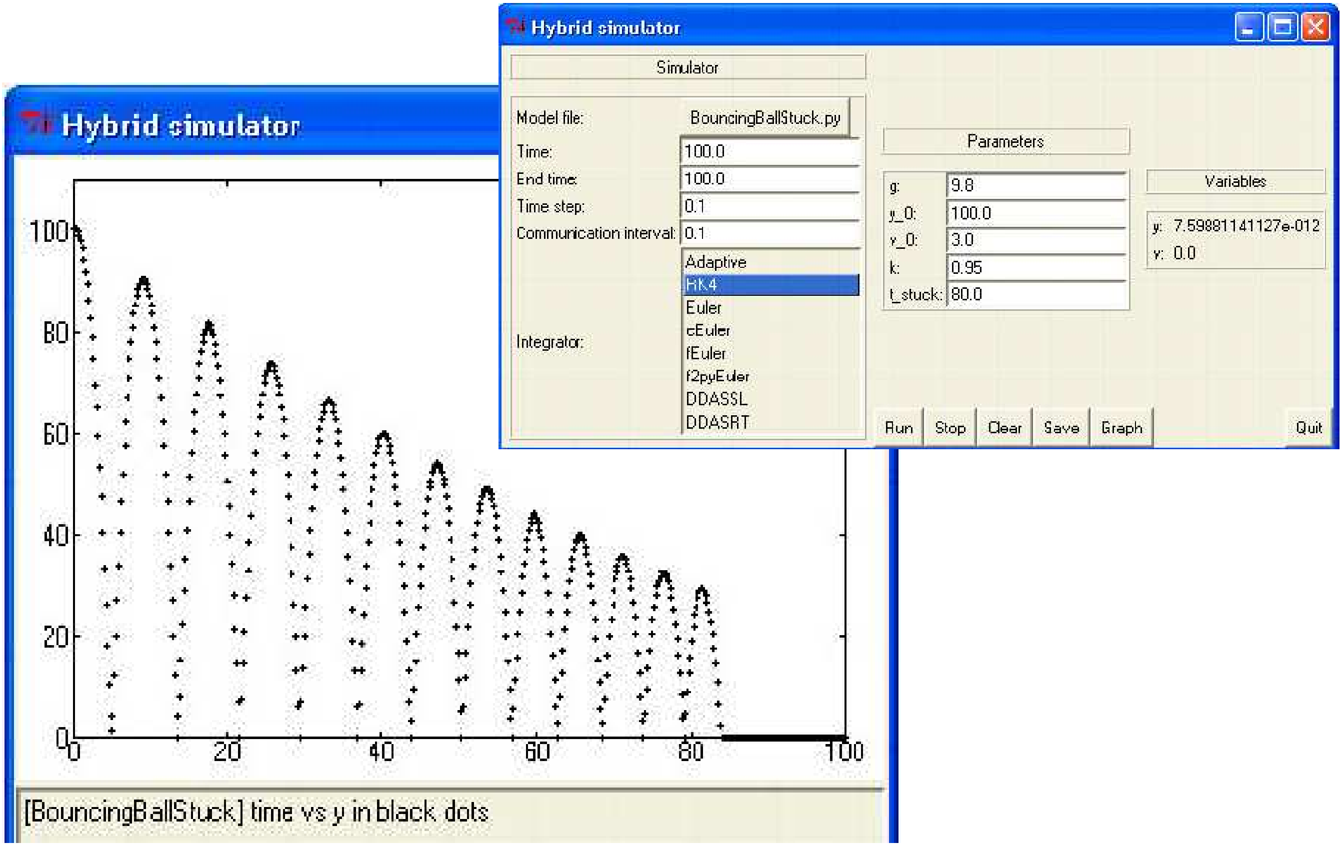
Transform to common Formalism



Hybrid Simulation



Simulation Trace



A Zoo of Formalisms