EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

24 Nov. 2015 - 24 Nov. 2018

IC1404 - Multi-Paradigm Modelling for Cyber-Physical Systems (MPM4CPS)

Training School: Causal Block Diagrams

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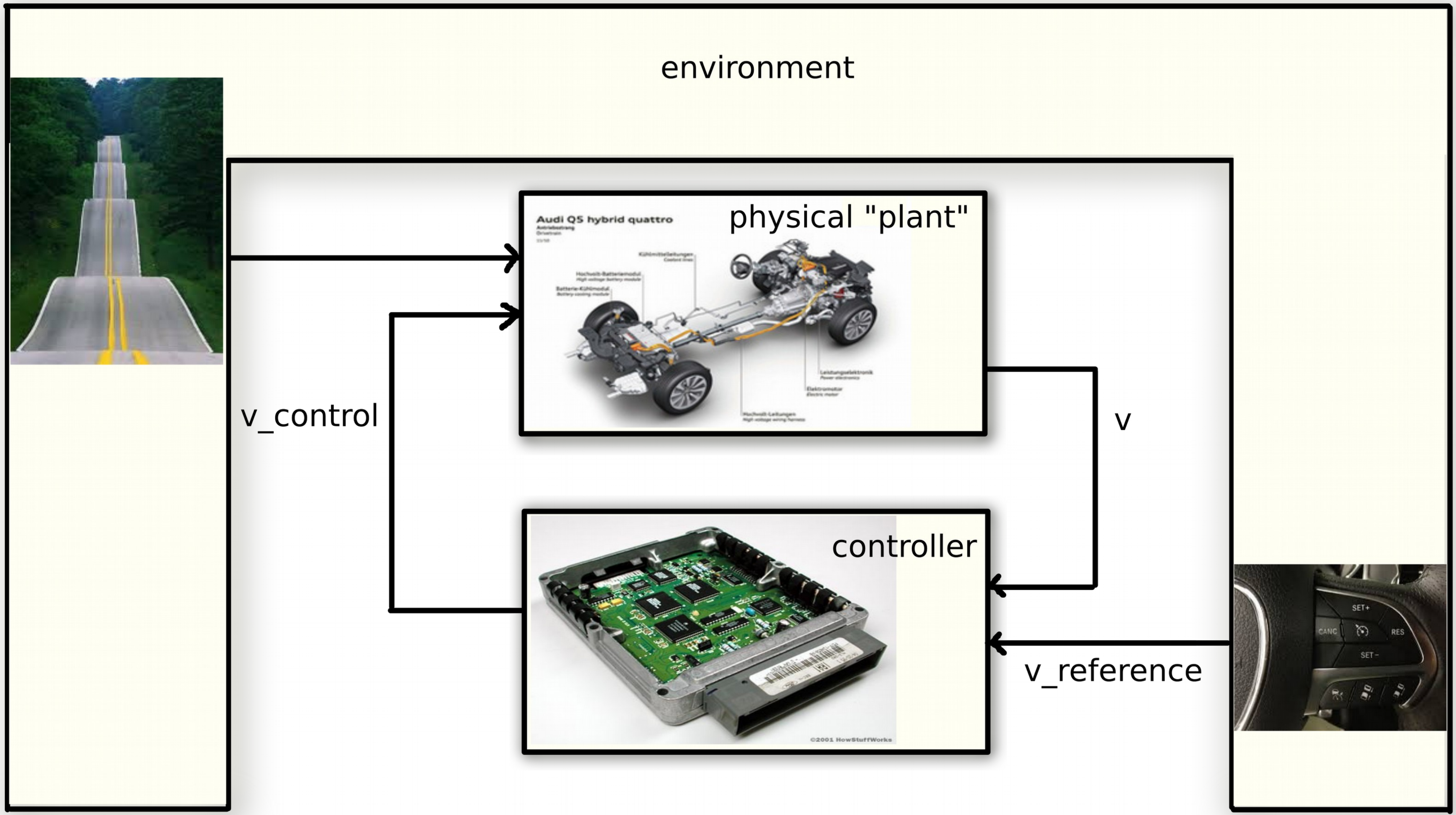
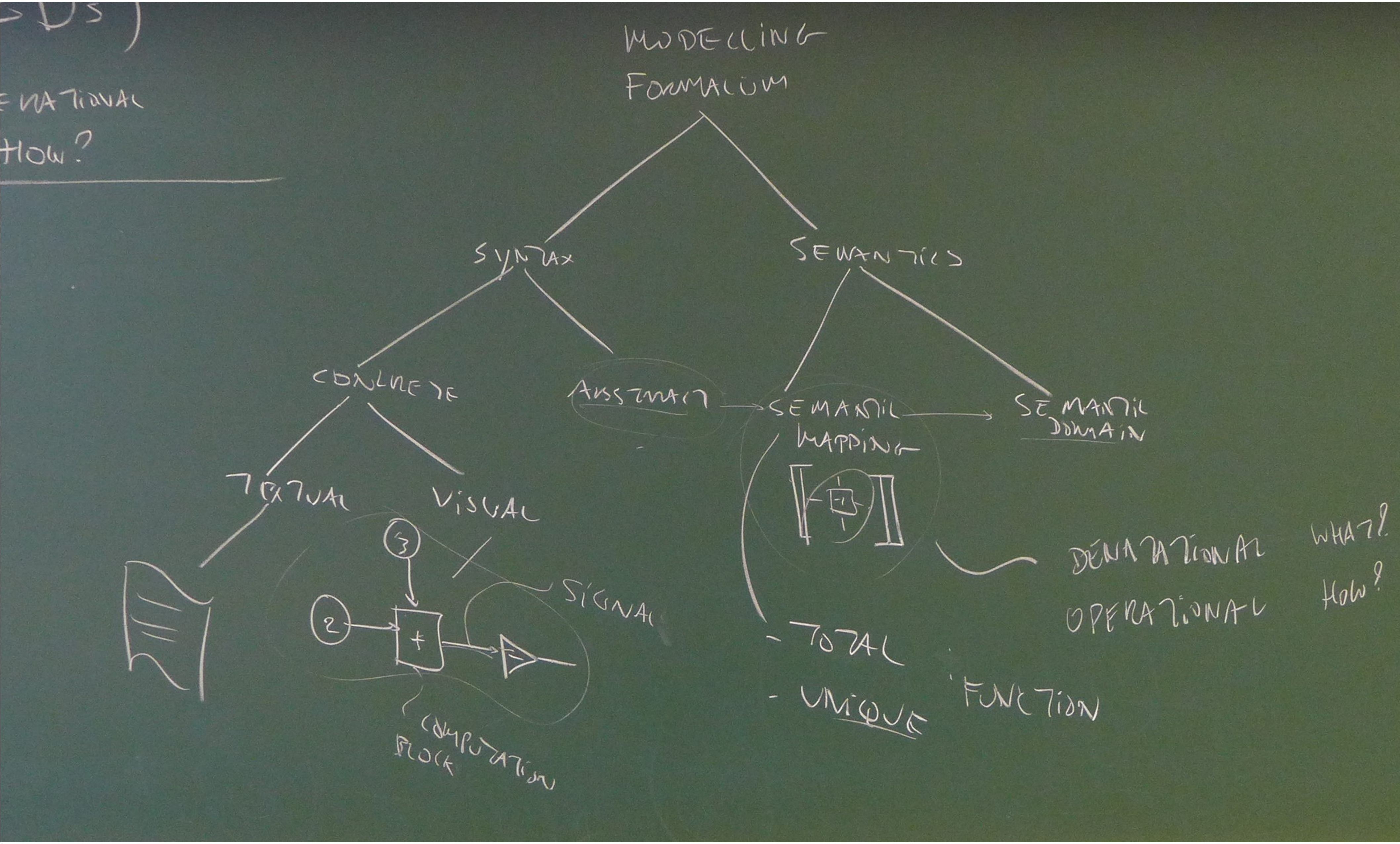
19 November 2018

Universiteit Antwerpen
Ansymo Antwerp Systems & Software Modelling University of Antwerp
Nexor Cyber-Physical Systems University of Antwerp
FLANDERS MAKE MANUFACTURING INNOVATION NETWORK
McGill
MPM4CPS

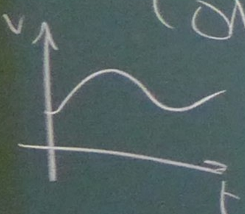


environment
physical "plant"
Audi Q5 hybrid quattro
v_control
v
controller
v_reference
SET+
CANC
RES
SET−
©2001 HowStuffWorks

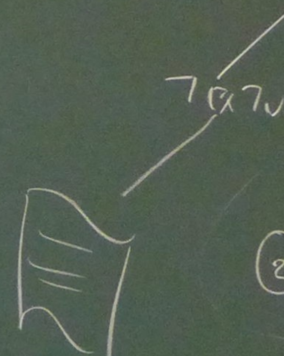

HOW?
MODELLING FORMALISM
SYNTAX
SEMANTICS
CONCRETE
ABSTRACT
SEMANTIC MAPPING
SEMANTIC DOMAIN
TEXTUAL
VISUAL
3
2
+
SIGNAL
COMPUTATION BLOCK
DENOTATIONAL WHAT?
OPERATIONAL HOW?
- TOTAL
- UNIQUE
FUNCTION

CAUSAL BLOCK DIAGRAMS (CBDs)

CBD TYPE	TIME BASE	SEMANTICS: DENOTATIONAL $\boxed{A}$ WHAT?	$\cong$ OPERATIONAL $\boxed{B}$ HOW?
ALGEBRAIC (I)	$\{now\}$		
DISCRETE-TIME (II)	$\mathbb{N}$		
CONTINUOUS-TIME (III)	$\mathbb{R}$		

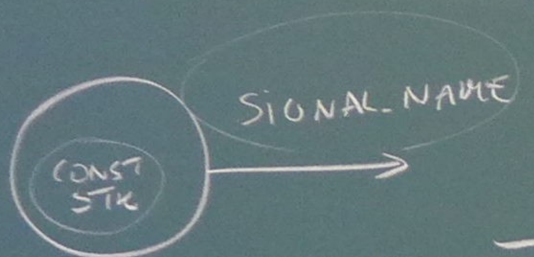


TEXTU

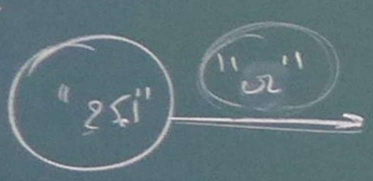


AIG CBD DENOTATIONAL SEMANTICS (I) $\boxed{A}$

$[\![\cdot]\!]$: String × String $\longrightarrow$ $\mathbb{R}$

$[\![$ 

$]\!]$ = VAR(SIGNAL_NAME) = VAL(CONST_STR) $\in \mathbb{R}$

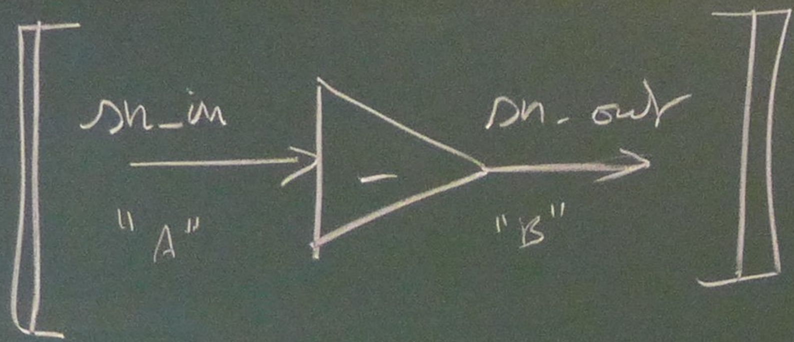
$[\![$ 

$]\!]$ = $a \in \mathbb{R}$

$a = [\![\text{"25"}]\!] = 25.1 \in \mathbb{R}$ ~ float

DICTIONARY SIGNAL_VALUES =
{ "a" : 25.1 }

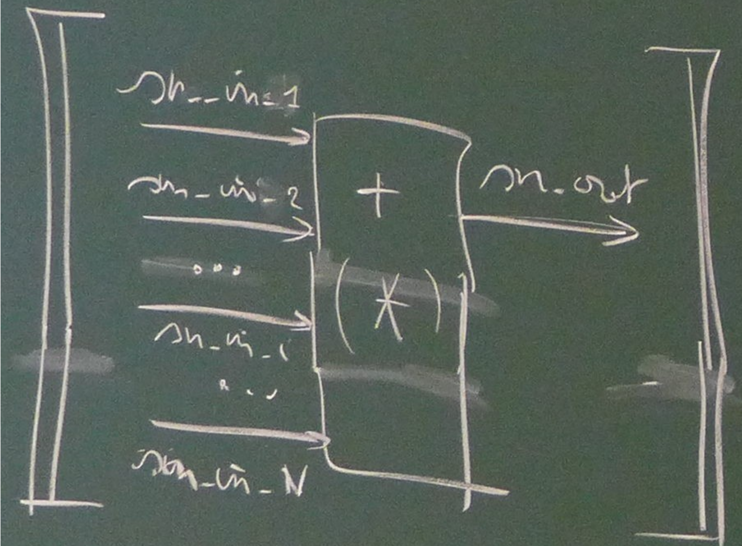
SIGNAL_VALUES["a"]



$$= \quad VAR(sn\_out) = -VAR(sn\_in)$$

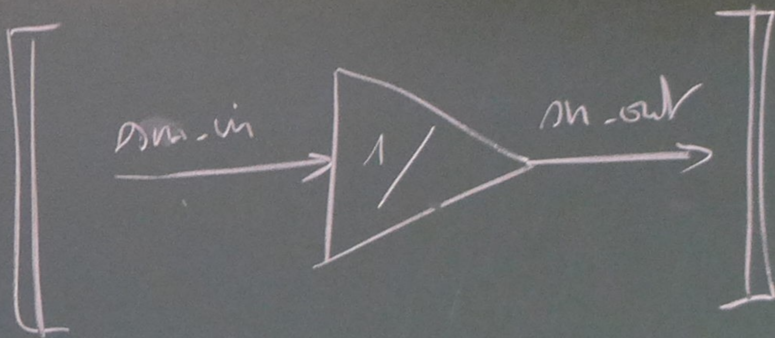
$$B = -A \quad , \quad A, B \in \mathbb{R}$$

{ "A" : 25 , "B" : -25 }



$$= \quad VAR(sn\_out) = \sum_{i=1}^{N} VAR(sn\_in\_i)$$

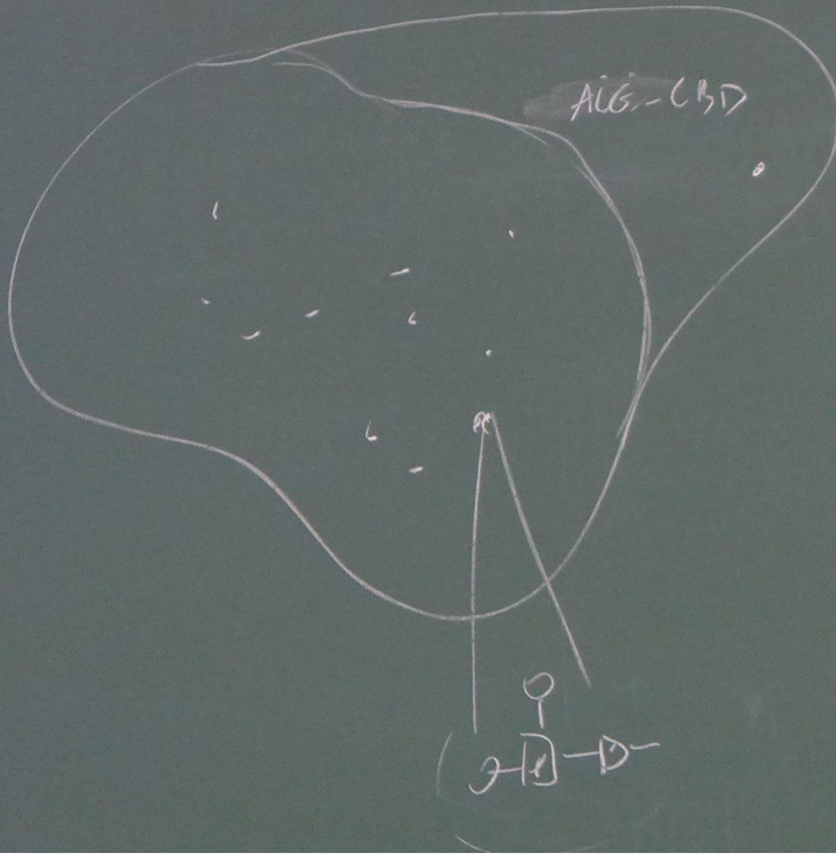
$$\left(\prod_{i=1}^{N} \right)$$



$$= \quad VAR(sn\_out) = \frac{1}{VAR(sn\_in)}$$

if $VAR(sn\_in) \neq \phi$

ELSE ——— $= \phi$



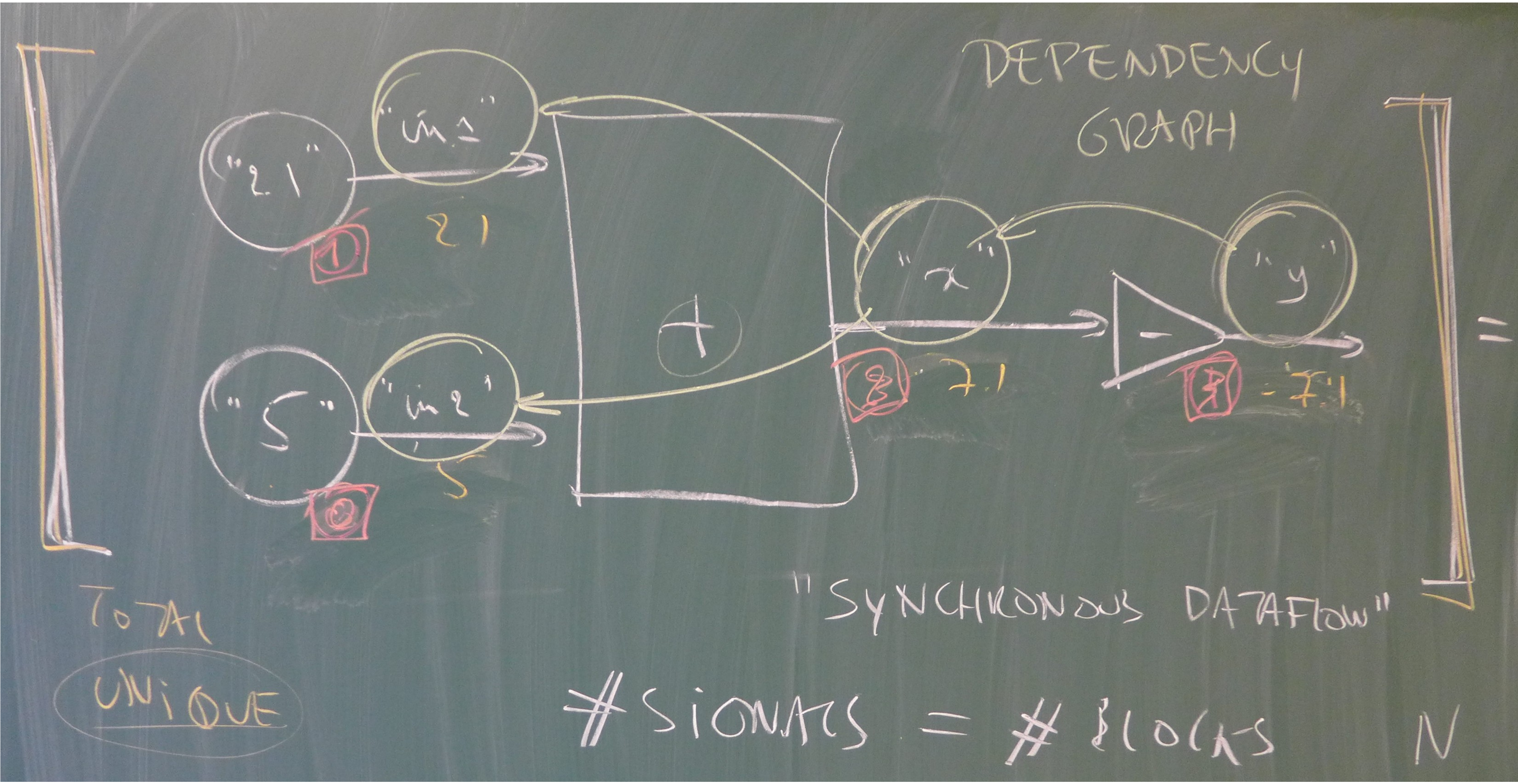
i)

1)) MODEL $\notin$ CBD_LANG

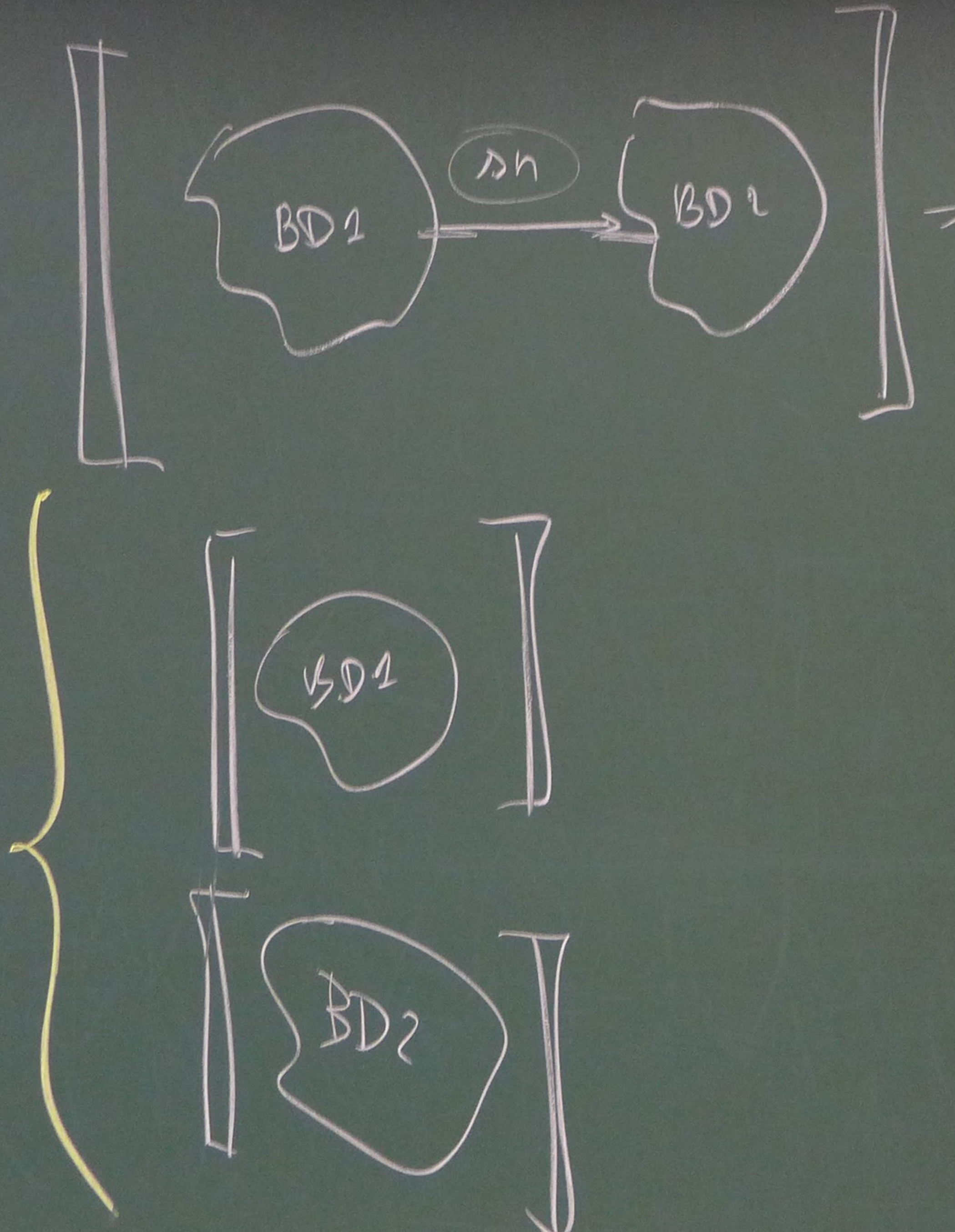
CHECK ~~STATIC~~ ?

DYNAMIC RAISE EXCEPTION

2))


DEPENDENCY GRAPH
"21"
"in1"
21
"5"
"in2"
5
+
"x"
7.1
"y"
-
-7.1
=
TOTAL
UNIQUE
"SYNCHRONOUS DATAFLOW"
#SIGNALS = #BLOCKS
N

COMPOSITION



$$
\left\{
\begin{array}{l}
in1 = 2.1 \\
in2 = 5 \\
x = in1 + in2 \\
y = -x
\end{array}
\right)
$$

TOTAL
UNIQUE

$in1, in2, x, y \in \mathbb{R}$

$$\begin{cases} 1 \times in1 + 0 \times in2 + 0 \times x + 0 \times y = 2.1 \\ 0 \times in1 + 1\, in2 + 0\, x + 0\, y = 5 \\ 1 \times in1 + 1 \times in2 - 1\, x + 0\, y = 0 \\ 0 \times in1 + 0 \times in2 + 1\, x + 1\, y = 0 \end{cases}$$

$$N \; \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}}_{M} \begin{bmatrix} in1 \\ in2 \\ x \\ y \end{bmatrix} = \begin{bmatrix} 2.1 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

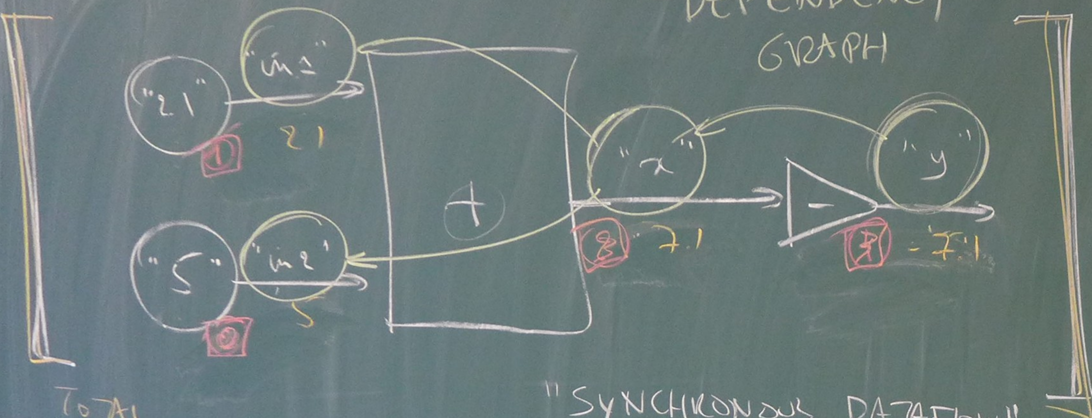
$DET(M) \neq 0$

LU DECOMPOSITION

GAUSS

$O(N^2)$

DEPENDENCY GRAPH



=

$$\begin{cases} in1 = 2.1 \\ in2 = 5 \\ x = in1 + in2 \\ y = -x \end{cases}$$

$in1, in2, x, y \in \mathbb{R}$

TOTAL UNIQUE

TOTAL UNIQUE

"SYNCHRONOUS DATAFLOW"

\# SIGNALS = \# BLOCKS N

I [B] ALG-CBD OPERATIONAL SEMANTICS

$\Theta(N)$

[A] NAIEVE : $N \times N$

$\Theta(N^2)$

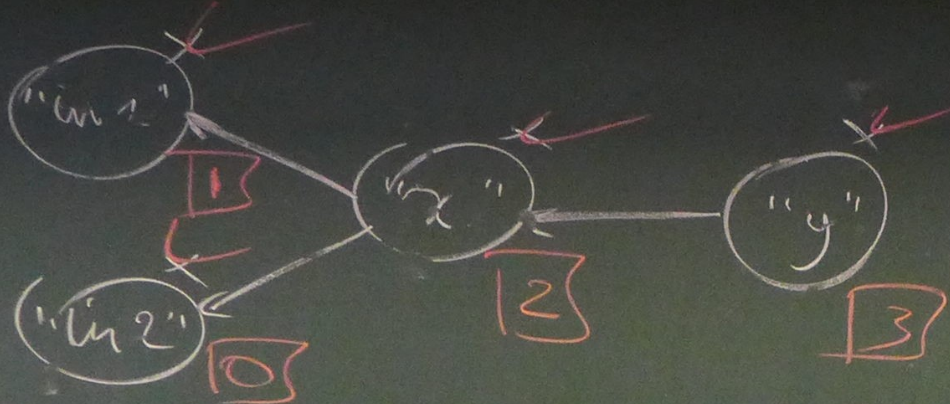
$\Theta(n)$ DEPENDENCY_GRAPH(MO…

[B] SCHEDULE = SORT(MODEL)

FOR BLOCK IN SCHEDULE

$\Theta(n)$ COMPUTE(BLOCK)

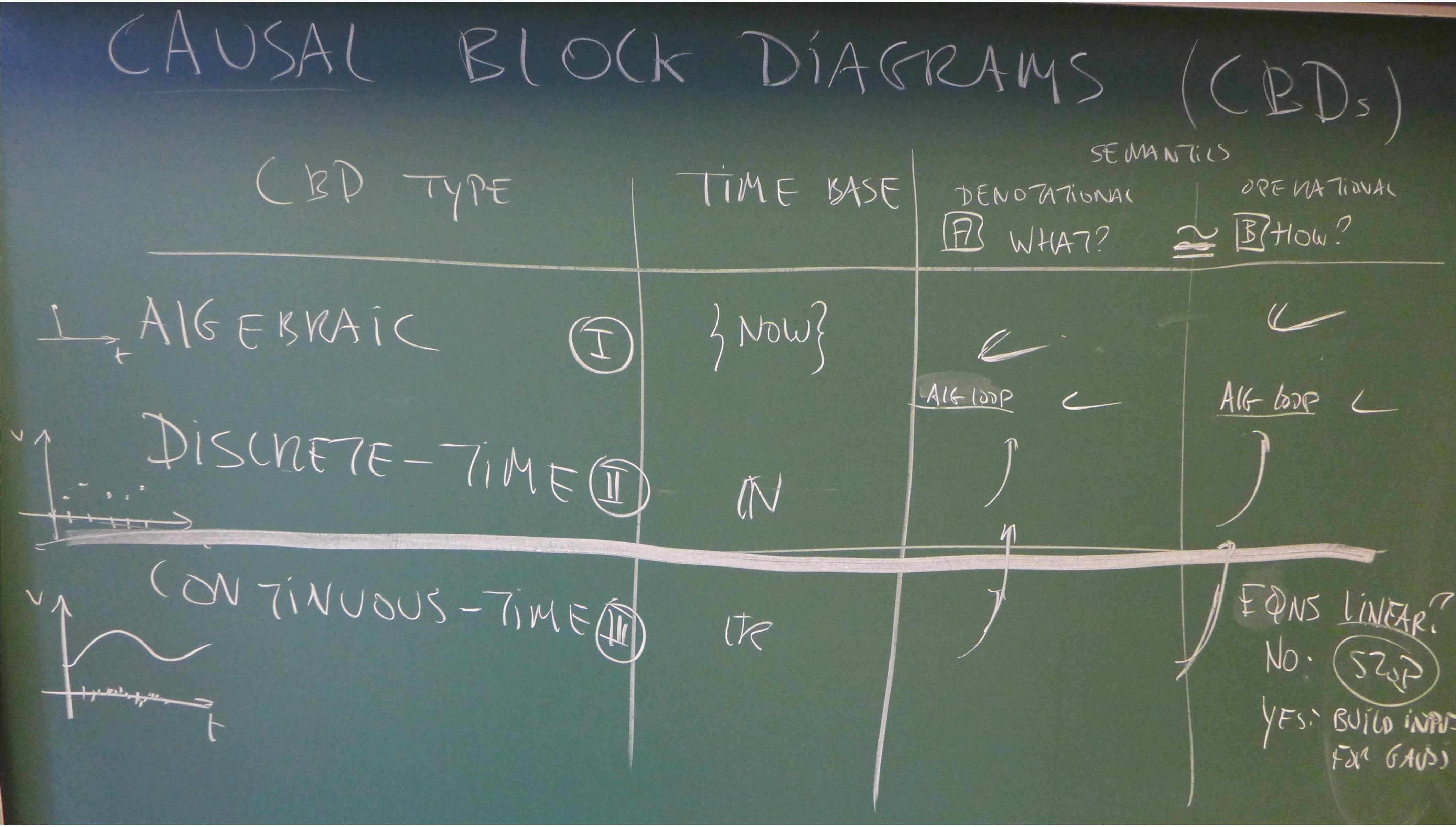
DEPGRAPH

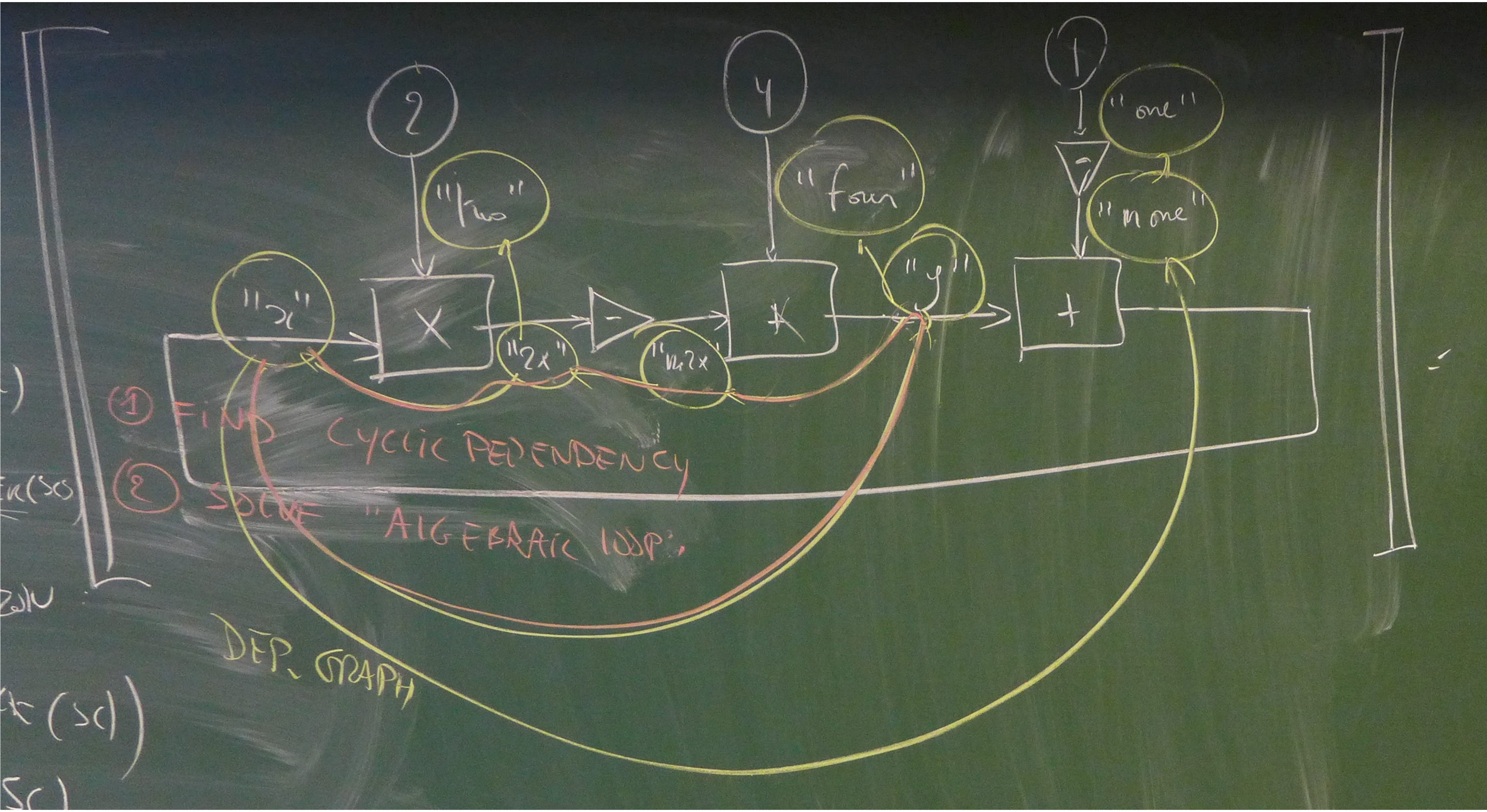


DFS_LABEL 4

```
DEPGRAPH = DEPENDENCIES(MODEL)
MARK ALL NODES NOT VISITED IN DEPGRAPH
DFS_LABEL = 0
WHILE NOT VISITED NODES REMAINING:
    NODE = PICK-NODE(DEPGRAPH)
    DFS_LABELLING(NODE)

DFS_LABELLING(NODE):
    IF NOT_VISITED(NODE)
        MARK NODE VISITED
        FOR CH IN NODE.CHILDREN
            DFS_LABELLING(CH)
        LABEL NODE WITH DFS_LABEL
        DFS_LABEL ++
```


CAUSAL BLOCK DIAGRAMS (CBDs)
SEMANTICS
CBD TYPE
TIME BASE
DENOTATIONAL
OPERATIONAL
F WHAT?
≅ B HOW?
ALGEBRAIC
I
{NOW}
ALG LOOP
ALG LOOP
DISCRETE-TIME
II
ℕ
CONTINUOUS-TIME
III
ℝ
EQNS LINEAR?
NO: STOP
YES: BUILD INPU
FOR GAUSS



2
"two"
"x"
×
"2x"
−
"m2x"
4
"four"
×
"y"
1
"one"
−
"m one"
+
① FIND CYCLIC DEPENDENCY
② SOLVE "ALGEBRAIC LOOP"
DEP. GRAPH

DENOT. SEMANTICS

$$\begin{cases} 2x + y = 4 \\ x - y = -1 \end{cases}$$

$$\underbrace{\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} 4 \\ -1 \end{bmatrix}}_{B}$$

GAUSS $\Theta(N^2)$

CRAMER

$$x = \frac{\begin{vmatrix} 4 & 1 \\ -1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-4+1}{-3} = 1$$

$$\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} \neq \emptyset \quad \text{UNIQUE}$$

$$y = \frac{\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-2-4}{-3} = 2$$

$$\begin{cases} y = 4 - 2x \\ x = y - 1 \end{cases} \qquad \begin{matrix} y = 4 - 2x \\ 3x = 3 \end{matrix} \qquad \begin{matrix} y = 2 \\ x = 1 \end{matrix}$$

$x, y \in \mathbb{R}$

$$\begin{cases} 2x^2 + xy = 16 & = \varepsilon_1' \\ xy + \sin(e^y) + 1 & = \varepsilon_2' \end{cases}$$

(x_1, y_1)

$\downarrow$

(x_2, y_2)

SCS = STRONG_COMPS(MODEL)

loop

FOR SC IN SCHEDULEORDER(SCS)

if SC IS SINGLETON :

COMPUTE (BLOCK (SC))

ELSE

SOLVE(SC)

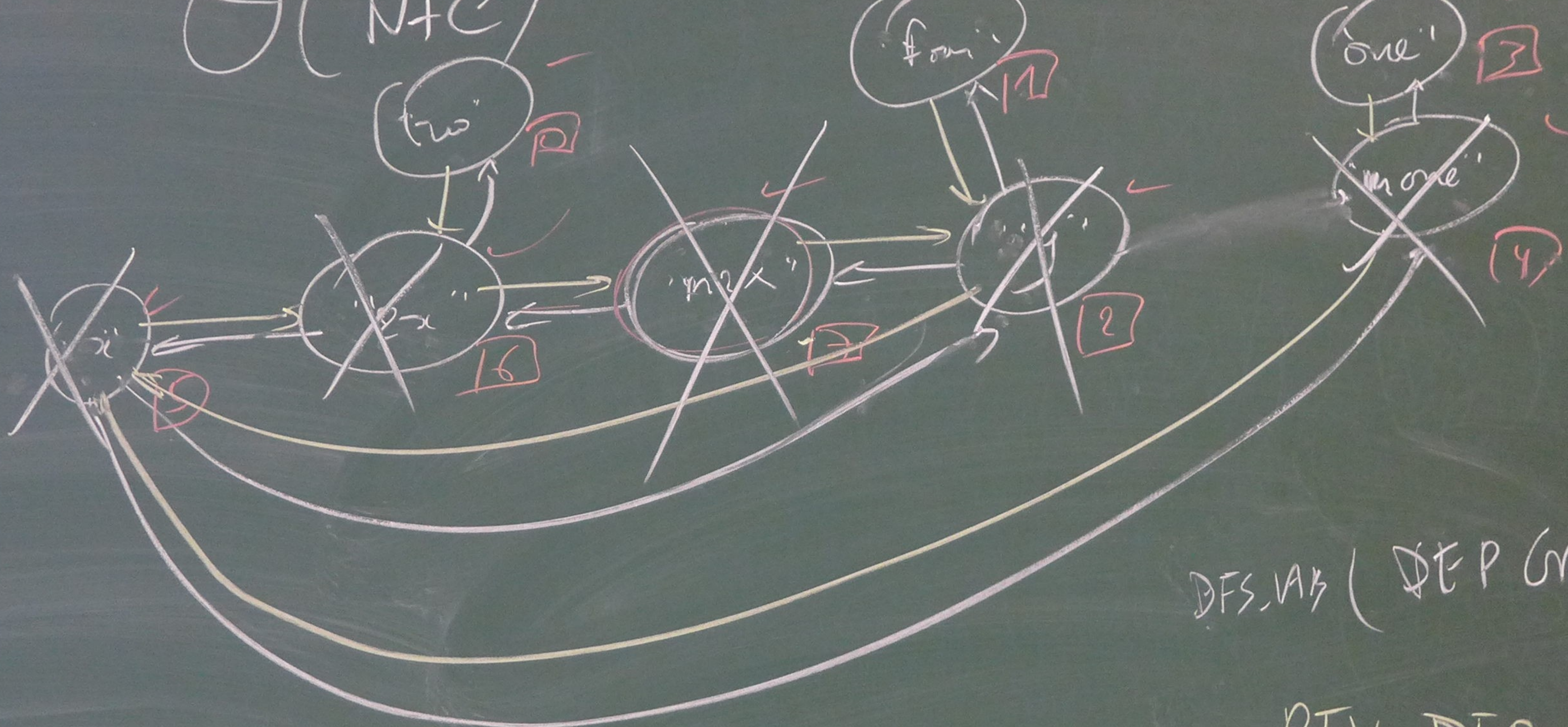
EQNS LINEAR?

NO: STOP

YES: BUILD INPUT FOR GAUSS & CALL

ROBERT TARJAN - 1974 "STRONGLY CONNECTED COMPONENT"

$\Theta(N+E)$

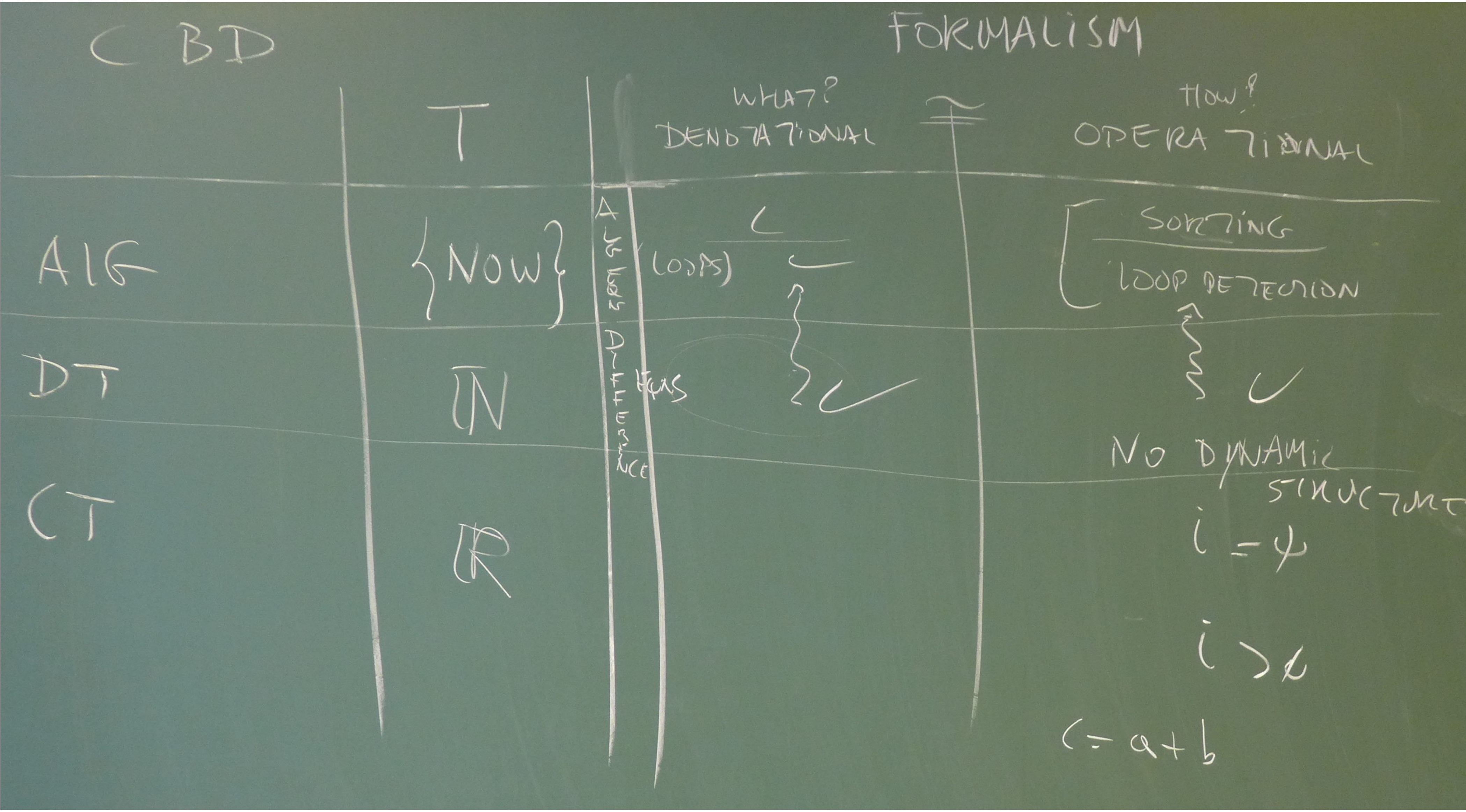


DFS_LAB(DEP GRAPH)

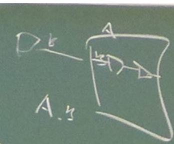
REV_DEP_GRAPH

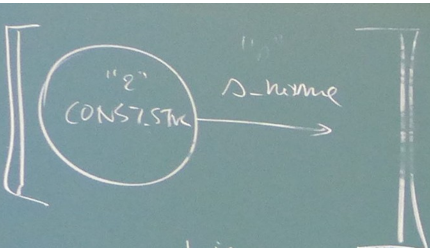
7 4 3 1 φ

[{"m2x", "y", "x", "2x"}, {"none"}, {"one"}, {"fou"}, {two}]


CBD
FORMALISM
T
WHAT?
DENOTATIONAL
HOW?
OPERATIONAL
ALG
{NOW}
(LOOPS)
SORTING
LOOP DETECTION
DT
ℕ
CT
ℝ
NO DYNAMIC STRUCTURE
i = 0
i > 0
c = a + b

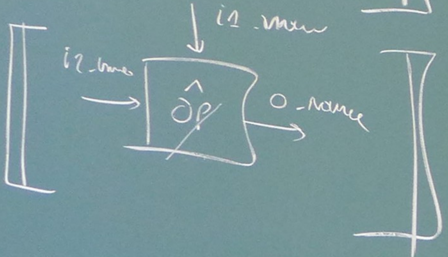
3/2





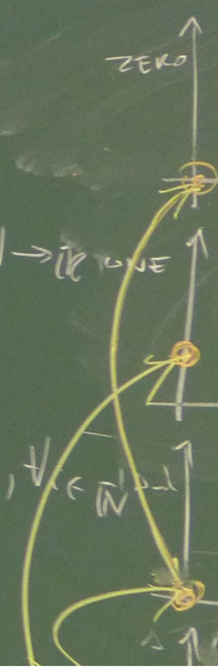
$$= \underbrace{VAR(s\_name)}_{s}(i) = VAL(CONST\_STR), \quad \forall i \in \mathbb{N}$$

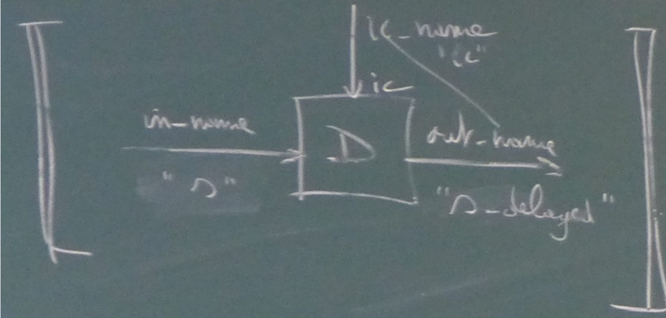
$$VAR(s\_name) \in (\mathbb{N} \to \mathbb{R})$$



$$= VAR(o\_name)(i) = VAR(i1\_name)(i) \; [\![\hat{op}]\!] \; VAR(i2\_name)(i), \quad \forall i \in \mathbb{N}$$

$$\in (\mathbb{N} \to \mathbb{R})$$





=

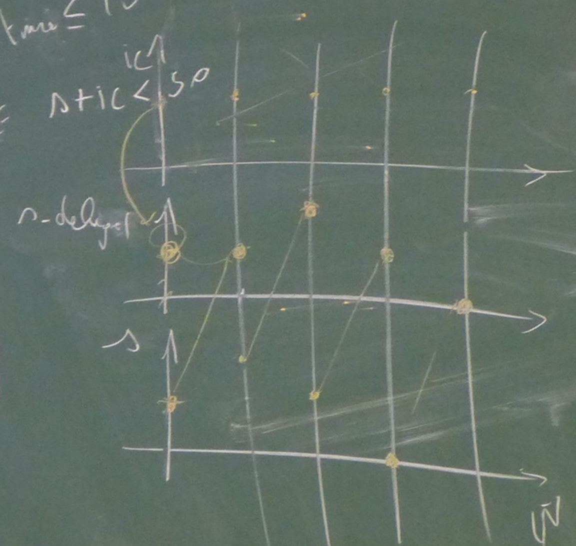
$$\mathrm{VAR}(\overset{s\_delay}{out\_name})(i) = \begin{cases} \mathrm{VAR}(\overset{s}{in\_name})(i-1), & \forall i \in \mathbb{N}_0 \\ \mathrm{VAR}(ic\_name)(0), & i = 0 \end{cases}$$

$$s\_delay_i = \begin{cases} s_{i-1}, & \forall i \in \mathbb{N}_0 \\ ic_0, & i = 0 \end{cases}$$

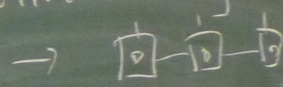
```
time = 0
while not term_condition
    SCHEDULE = TOPOLOGICAL_SORT(DEP_GRAPH(model))
    FOR BLOCKSET IN ORDER(SCHEDULE)
        COMPUTE/SOLVE(BLOCKSET)
    time += 1
```

TIME time ≤ 10

STATE s+ic ≤ 50



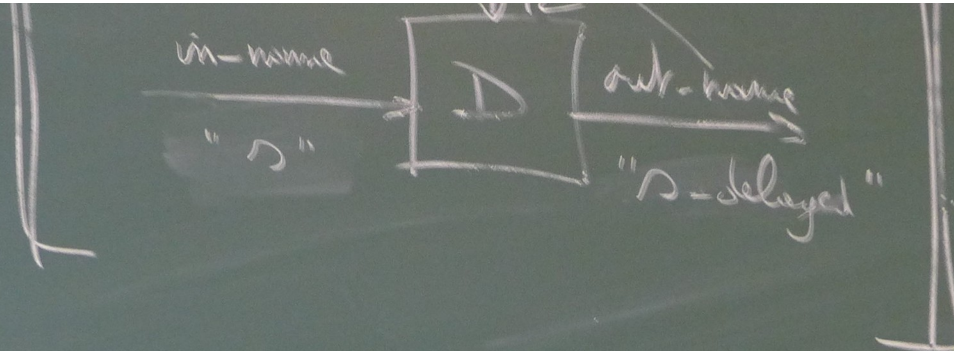
MULTIPLE DELAY



OPERATIONAL

SORTING

LOOP DETECTION



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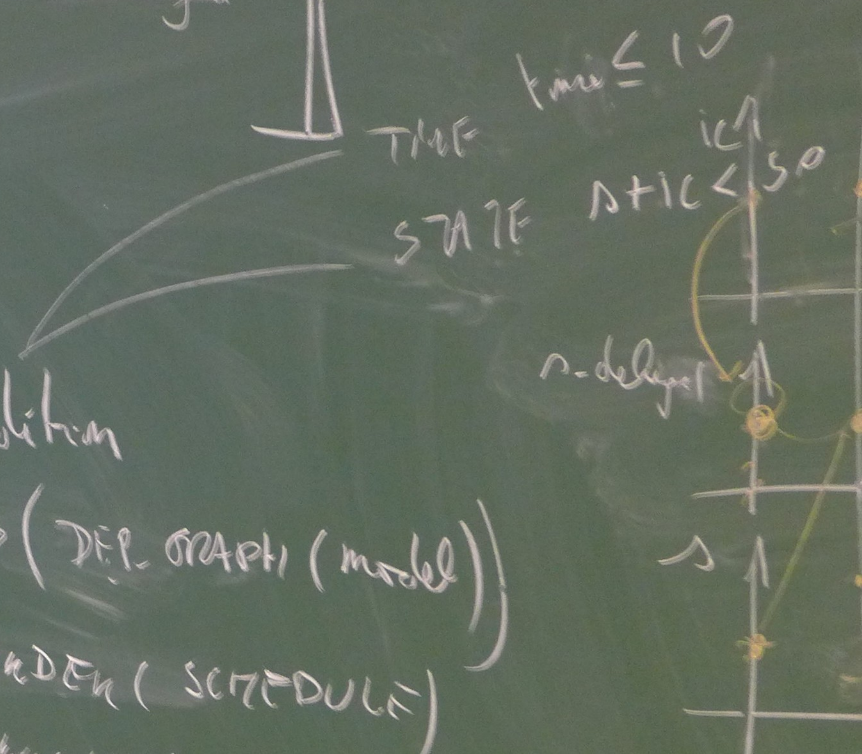


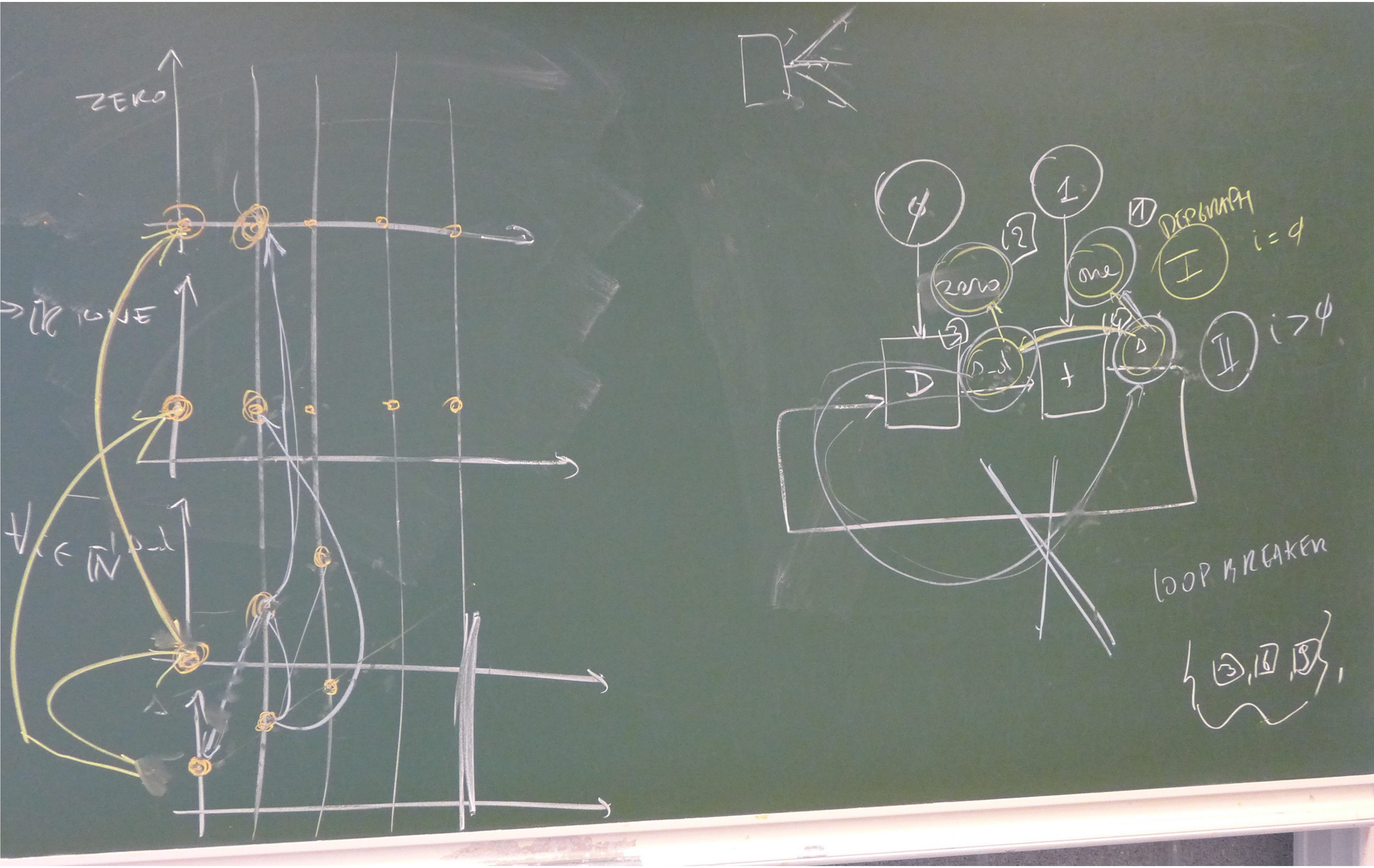
NO DYNAMIC STRUCTURE

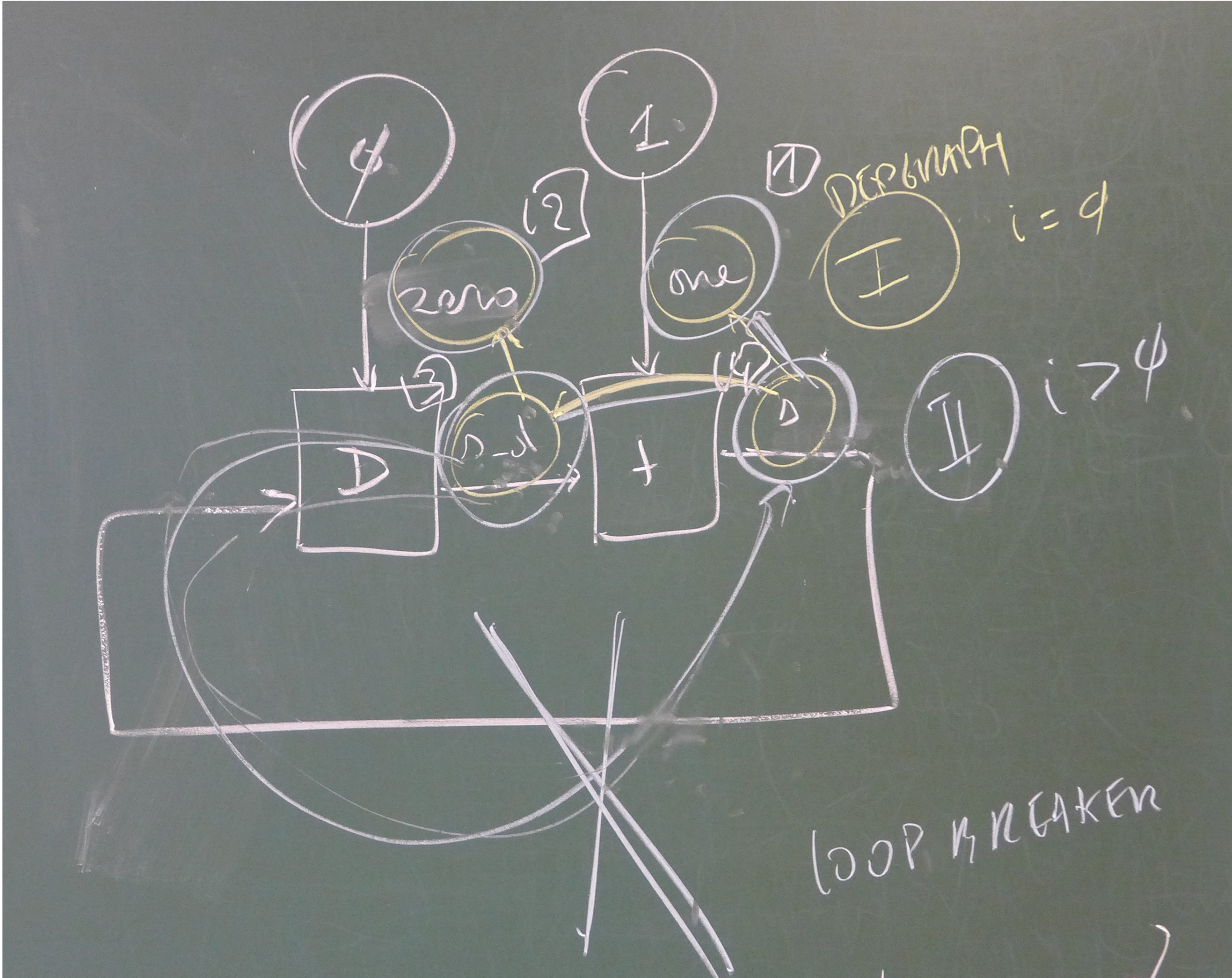
$i = \phi$

$i > 0$

$c = a + b$

```
time = ϕ
while not term_condition
    SCHEDULE = LOOPDET_SORT(DEP_GRAPH(model))
    FOR BLOCKSET IN ORDER(SCHEDULE)
        COMPUTESOLVE(BLOCKSET)
    time += 1
```


ZERO
ONE
zero
one
D
+
DEPGRAPH
I
i = 0
II
i > 0
LOOP BREAKER


0
1
zero
one
12
11
DEPGRAPH
i = 0
I
D_1
+
D
II
i > 0
LOOP BREAKER

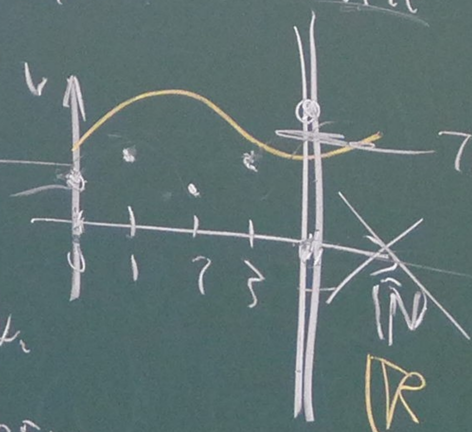
CAUSAL BLOCK DIAGRAMS (CBD)

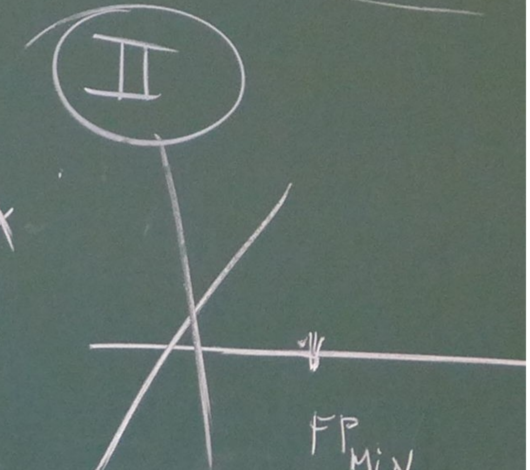
SEMANTICS

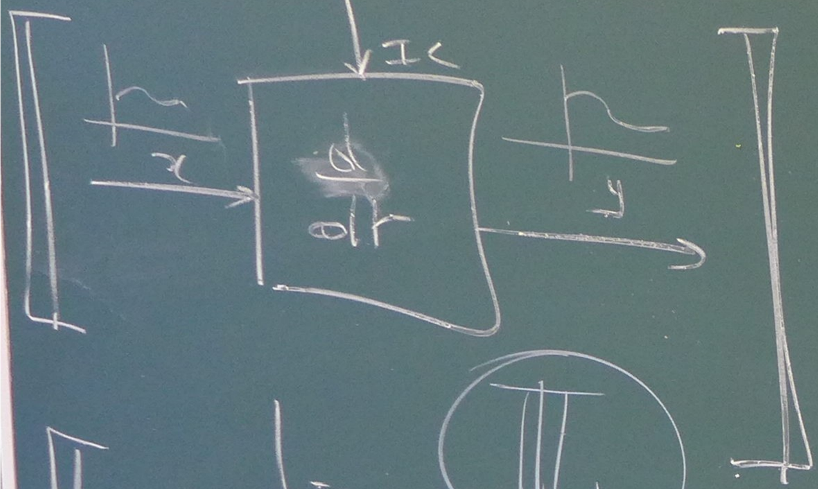
	T	DENOTATIONAL (SEM. DOM.)		OPERATIONAL
ALG	{NOW}	ALG EQNS	✓ INCL LOOPS	✓ INCL LOOPS
DT	$\mathbb{N}$	DIFFERENCE EQNS	✓	✓
CT	$\mathbb{R}$	DIFFERENTIAL INTEGRAL EQNS	TERM. COND × TIME × STATE	~ APPROX

RUN FOREVER:

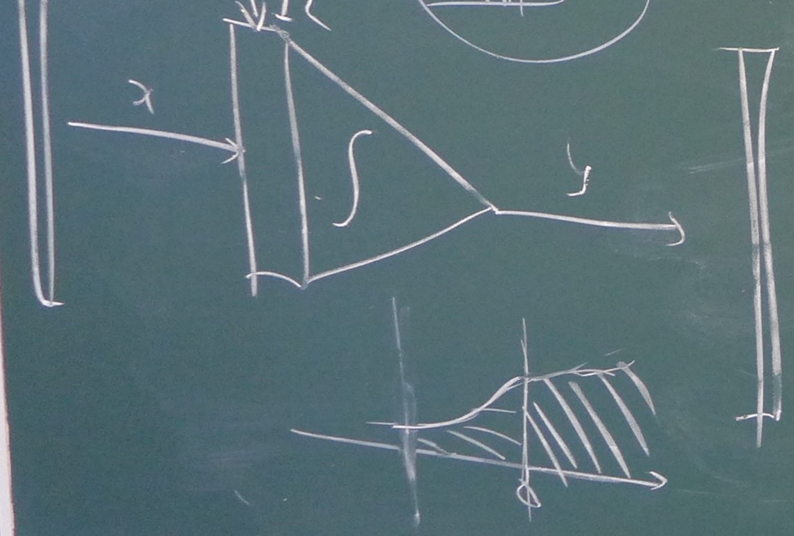
- ONLY RECORD DELAY
  → NO MEM OVFLOW
- TIME-TRANSLATION INV.
  → NO NUMERICAL OVERFLOW



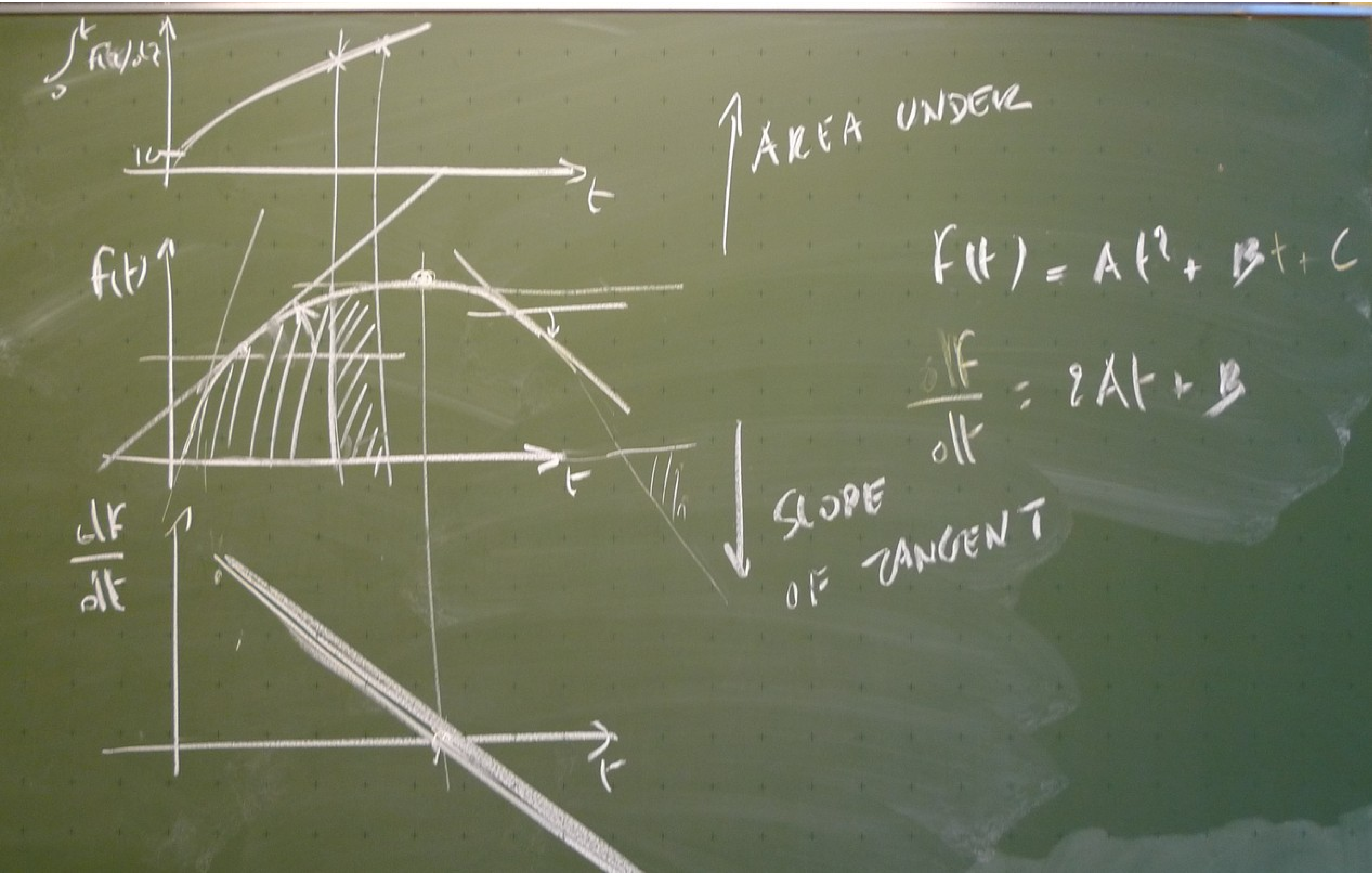


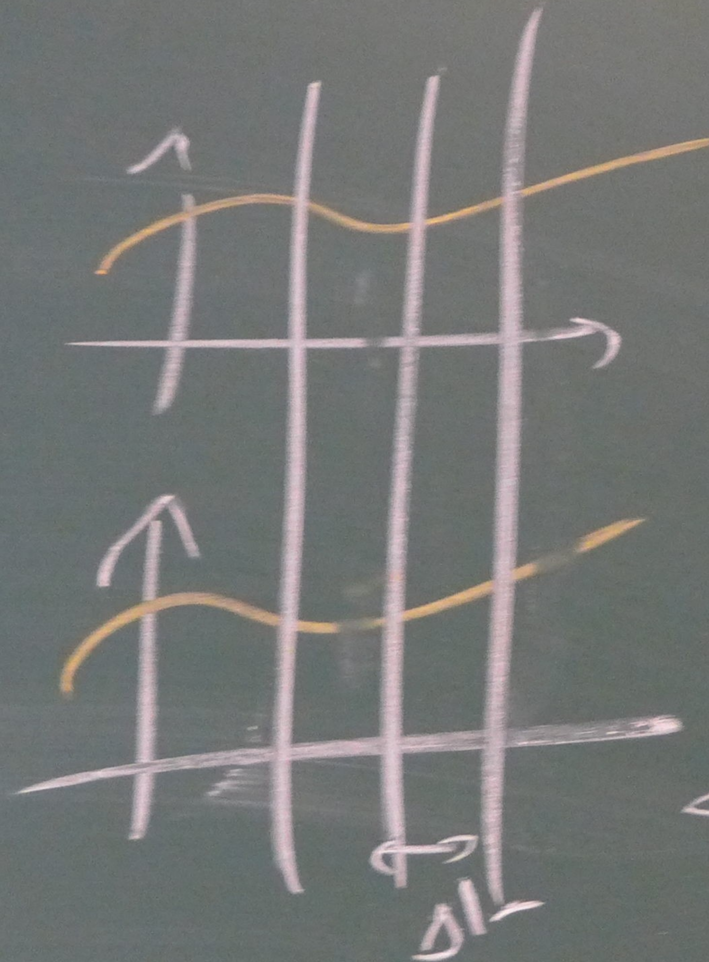


$$= \begin{cases} y(\tau) = \dfrac{dx}{dt}(\tau), & \forall \tau \in \mathbb{R} \setminus \{0\} \\ y(0) = IC \end{cases}$$



$$= \quad y(t) = IC + \int_0^t x(\tau)\, d\tau$$


∫₀ᵗ f(τ)dτ
10
t
f(t)
t
df/dt
t
AREA UNDER
SLOPE OF TANGENT
f(t) = At² + Bt + C
df/dt = 2At + B



SYNCHRONOUS

X

$\forall \tau \in \mathbb{R} \setminus \{0\}$

CT-CBD

DT-CBD

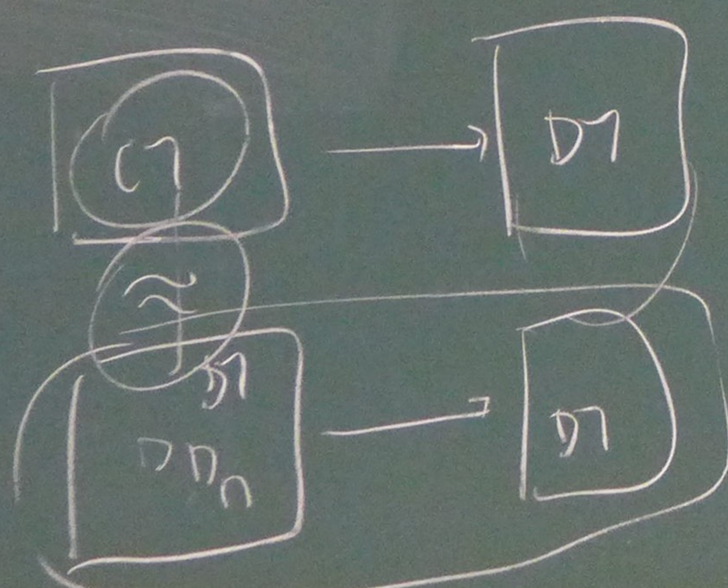
DIFFERENTIAL EQN.

DIFFERENCE EQNS

$\equiv$

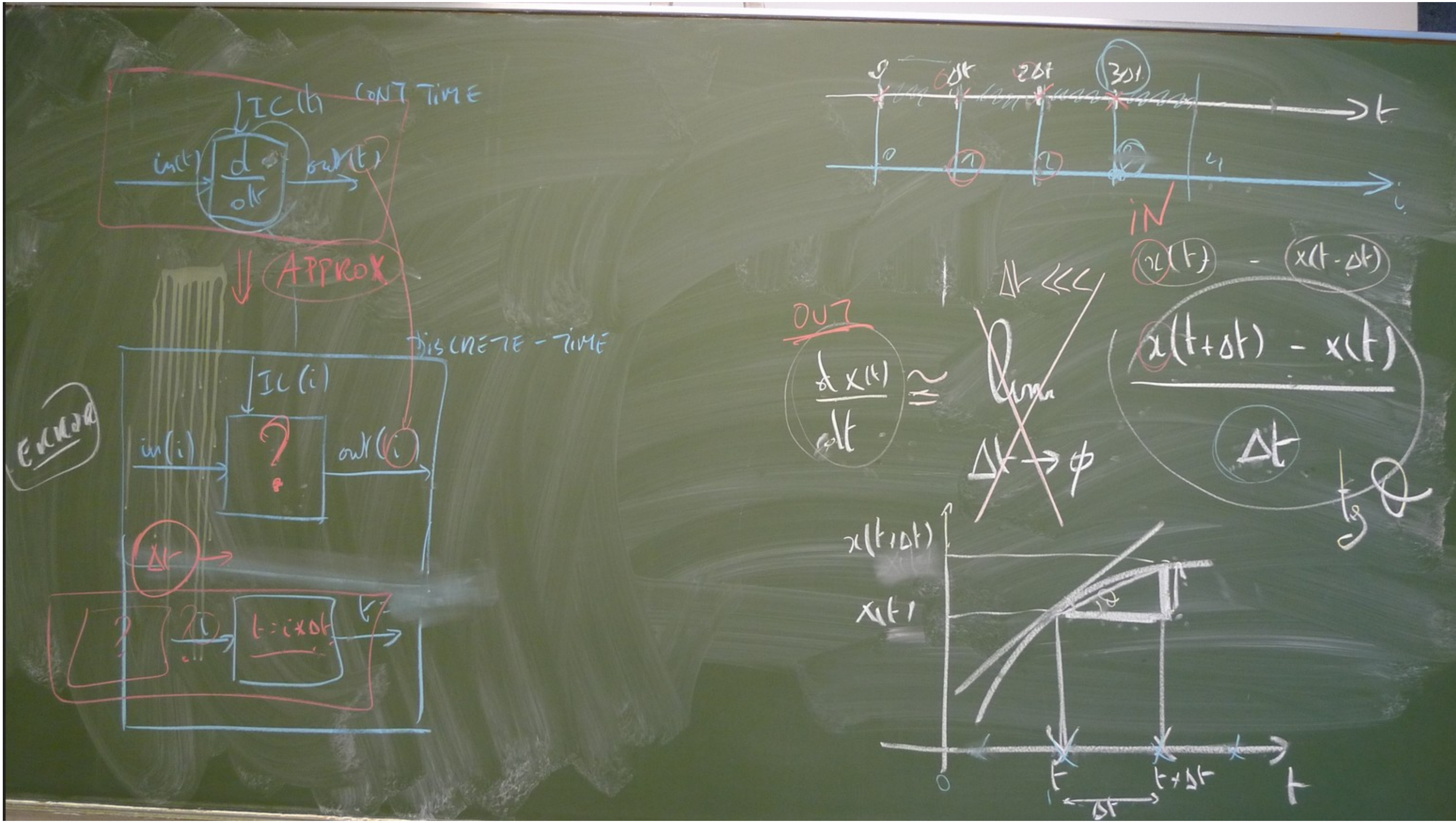
APPROX

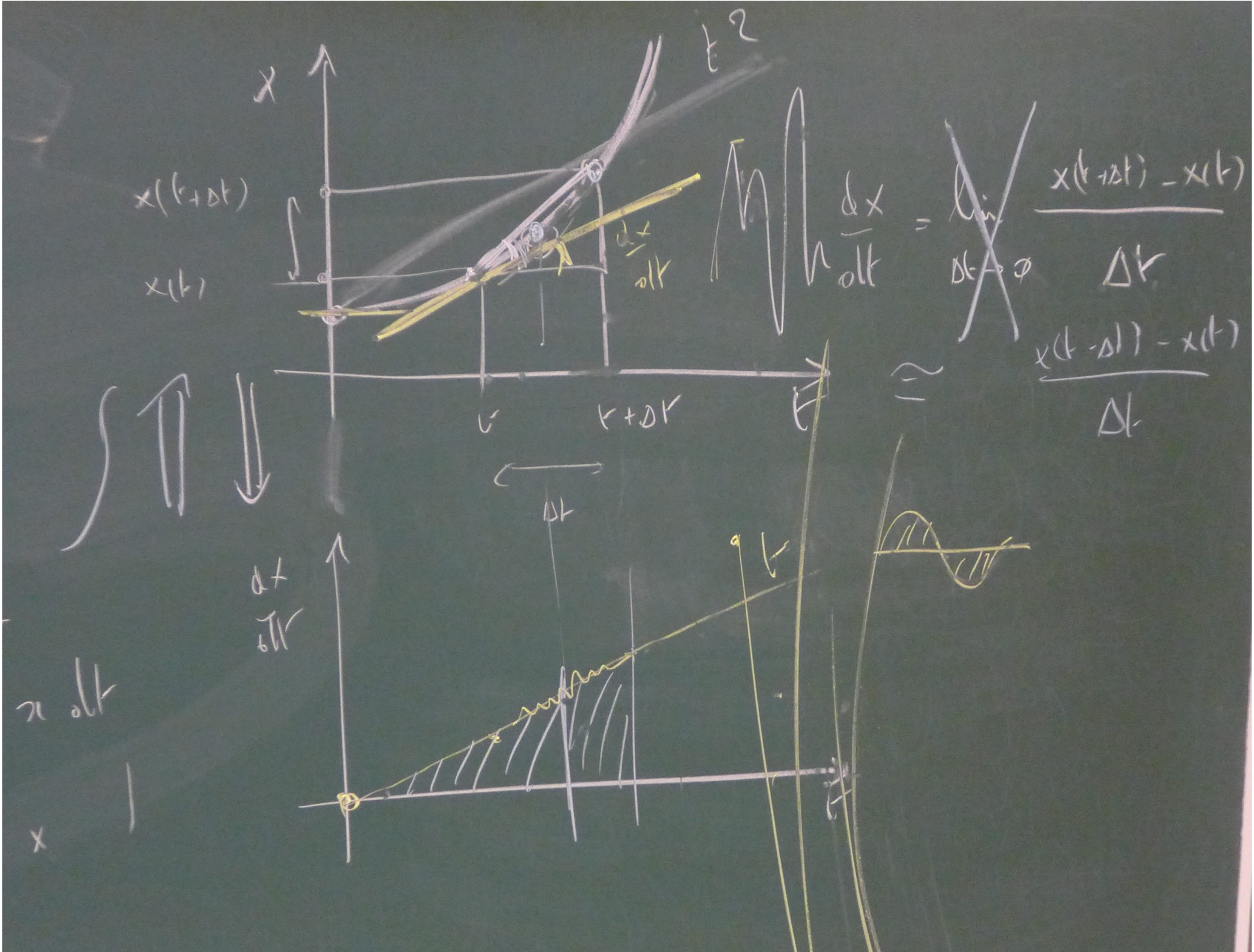
DT-CBD

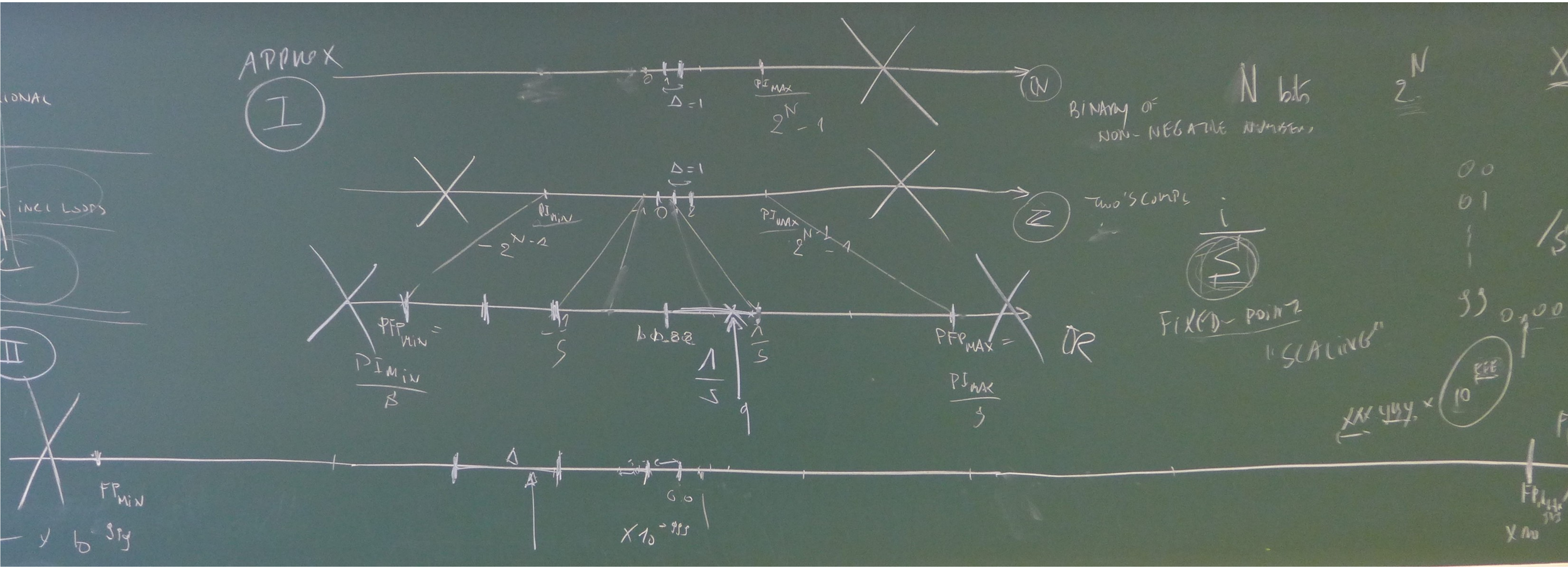


$$y = \int x \, dt$$

$$\frac{dy}{dt} = x$$


CONT TIME
IC(t)
in(t)
d/dt
out(t)
APPROX
DISCRETE - TIME
IC(i)
in(i)
?
out(i)
ERROR
Δt
t = i × Δt
0
Δt
2Δt
3Δt
t
IN
x(t) - x(t-Δt)
OUT
dx(t)/dt ≃
Δt <<<
Δt → φ
x(t+Δt) - x(t)
Δt
tg θ
x(t+Δt)
x(t)
t+Δt


x
t²
x(t+Δt)
x(t)
dx/dt
t
t+Δt
t
Δt
dx/dt = lim Δt→0 (x(t+Δt) − x(t))/Δt
≃ (x(t−Δt) − x(t))/Δt
dx/dt
x dt
x
t



APPROX
I
II
Δ=1
PI MAX
2^N - 1
N bits
2^N
BINARY OF
NON-NEGATIVE NUMBERS
Δ=1
PI MIN
-2^(N-1)
PI MAX
2^(N-1) - 1
TWO'S COMPL
Z
PFP MIN =
PI MIN / S
PFP MAX =
PI MAX / S
1/S
S
FIXED-POINT
"SCALING"
FP MIN
X 10^-999

N bits

2^N

NEGATIVE NUMBERS

COMPL

$\frac{i}{S}$

FIXED-POINT

"SCALING"

$X = \alpha y$

SCALE FACTOR

$$\begin{array}{ccc} 00 & & 0.00 \\ 01 & & 0.01 \\ \vdots & /S & \vdots \\ 99 & & 0.99 \end{array}$$

0.000......99

FIXED-POINT

$xxx.yyy \times 10^{EEE}$

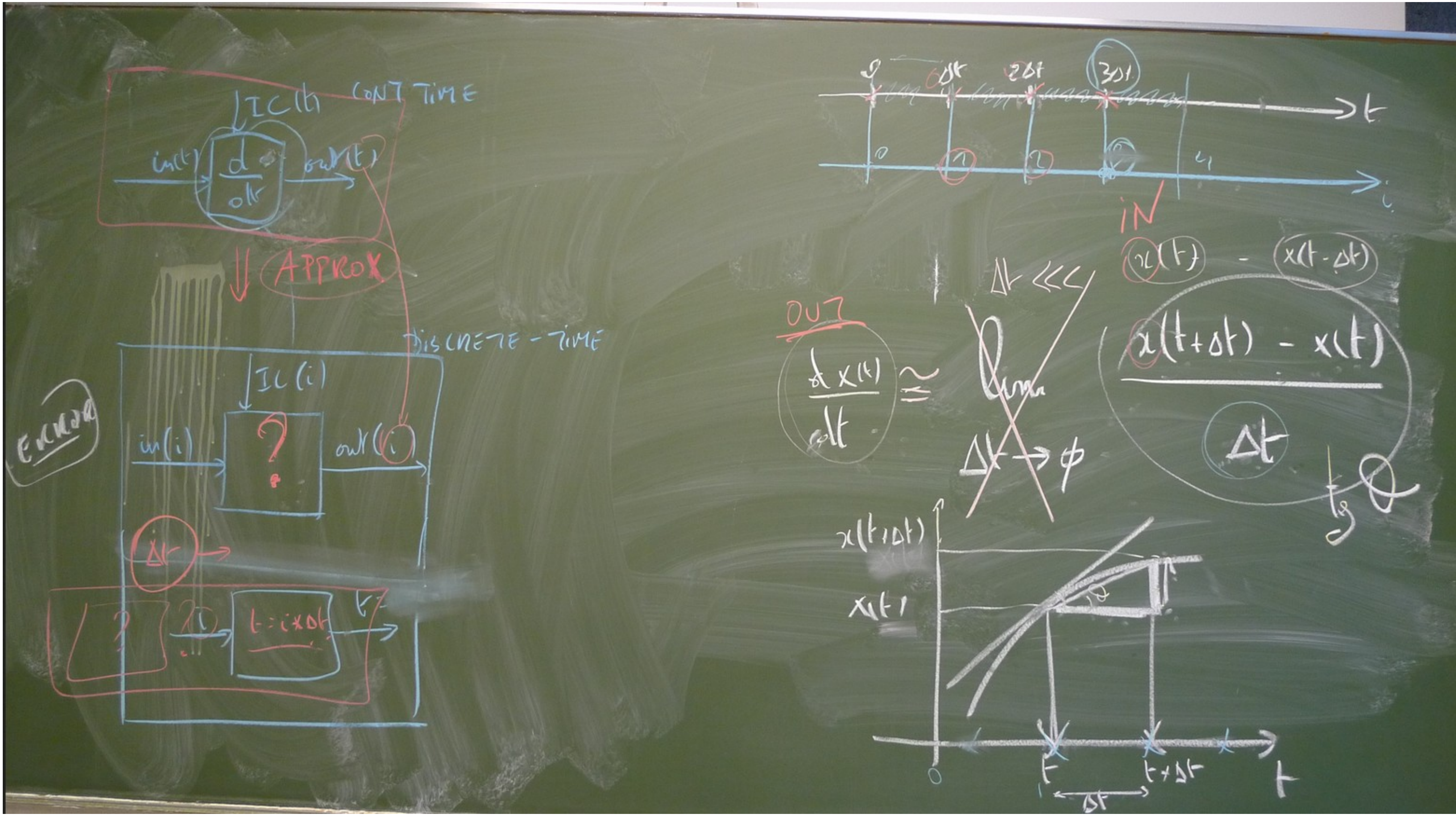
FLOATING-POINT

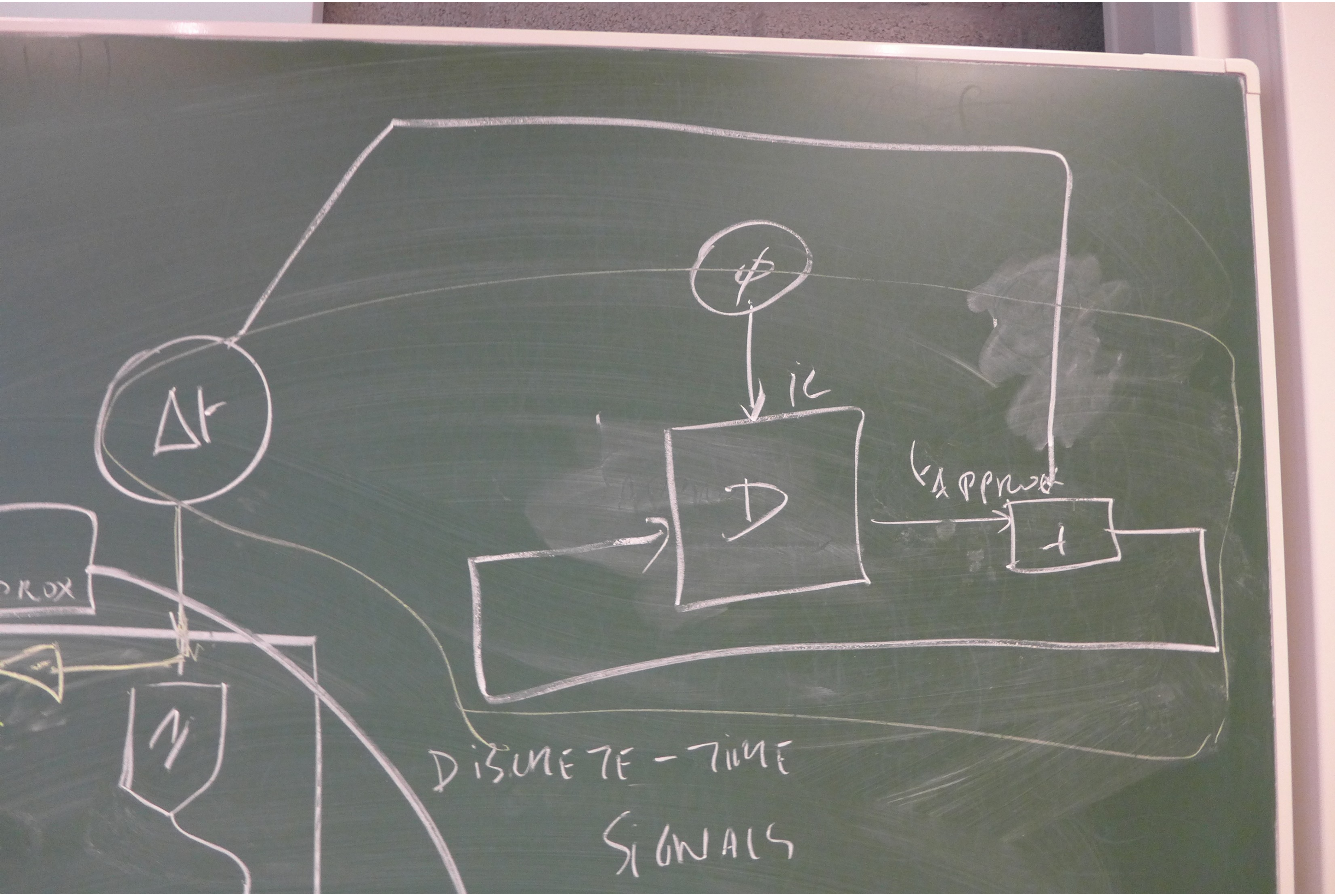
FP, MAX

yyy

X no

LP


CONT TIME
IC(t)
in(t)
d/dt
out(t)
APPROX
DISCRETE - TIME
IC(i)
in(i)
?
out(i)
ERROR
Δt
t = i × Δt
0
Δt
2Δt
3Δt
t
IN
x(t) - x(t-Δt)
OUT
dx(t)/dt ≃
Δt <<<
Δt → φ
x(t+Δt) - x(t)
Δt
tg θ
x(t+Δt)
x(t)
t+Δt


Δt
D
DISCRETE-TIME
SIGNALS

DISCRETIZED

$$y(t_{APPROX})_{APPROX} = \frac{x(t_{APPROX} + \Delta t)_{APPROX} - x(t_{APPROX})_{APPROX}}{\Delta t} \quad \text{FWD}$$

$i+1 \qquad i$

$$y(t_{APPROX})_{APPROX} = \frac{x(t_{APPROX})_{APPROX} - x(t_{APPROX} - \Delta t)_{APPROX}}{\Delta t} \quad \text{BWD}$$

CENTRAL

$$t_{APPROX}(i) = i\,\Delta t$$

ABSOLUTE

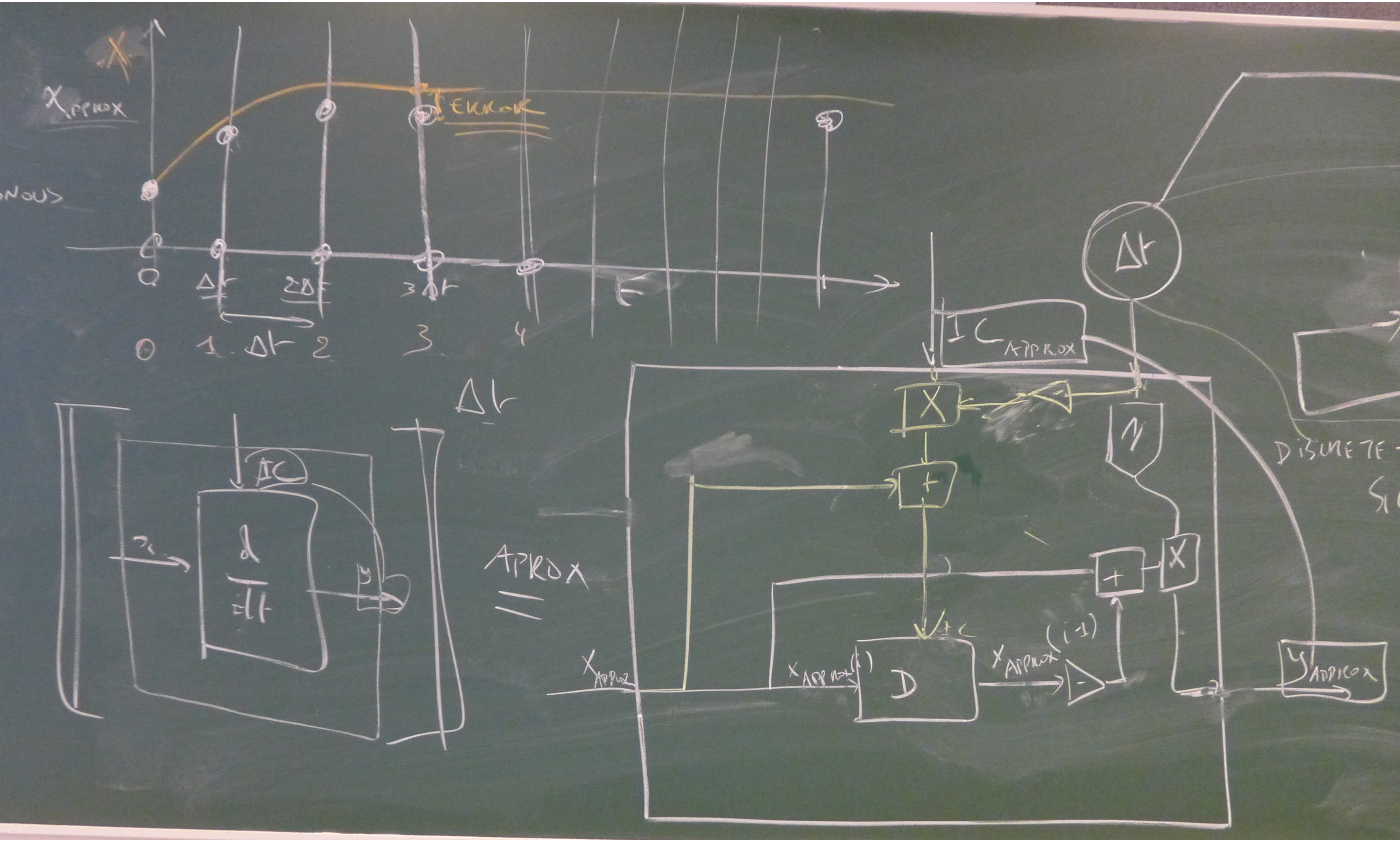
$$t_{APPROX}(i+1) = t_{APPROX}(i) + \Delta t$$

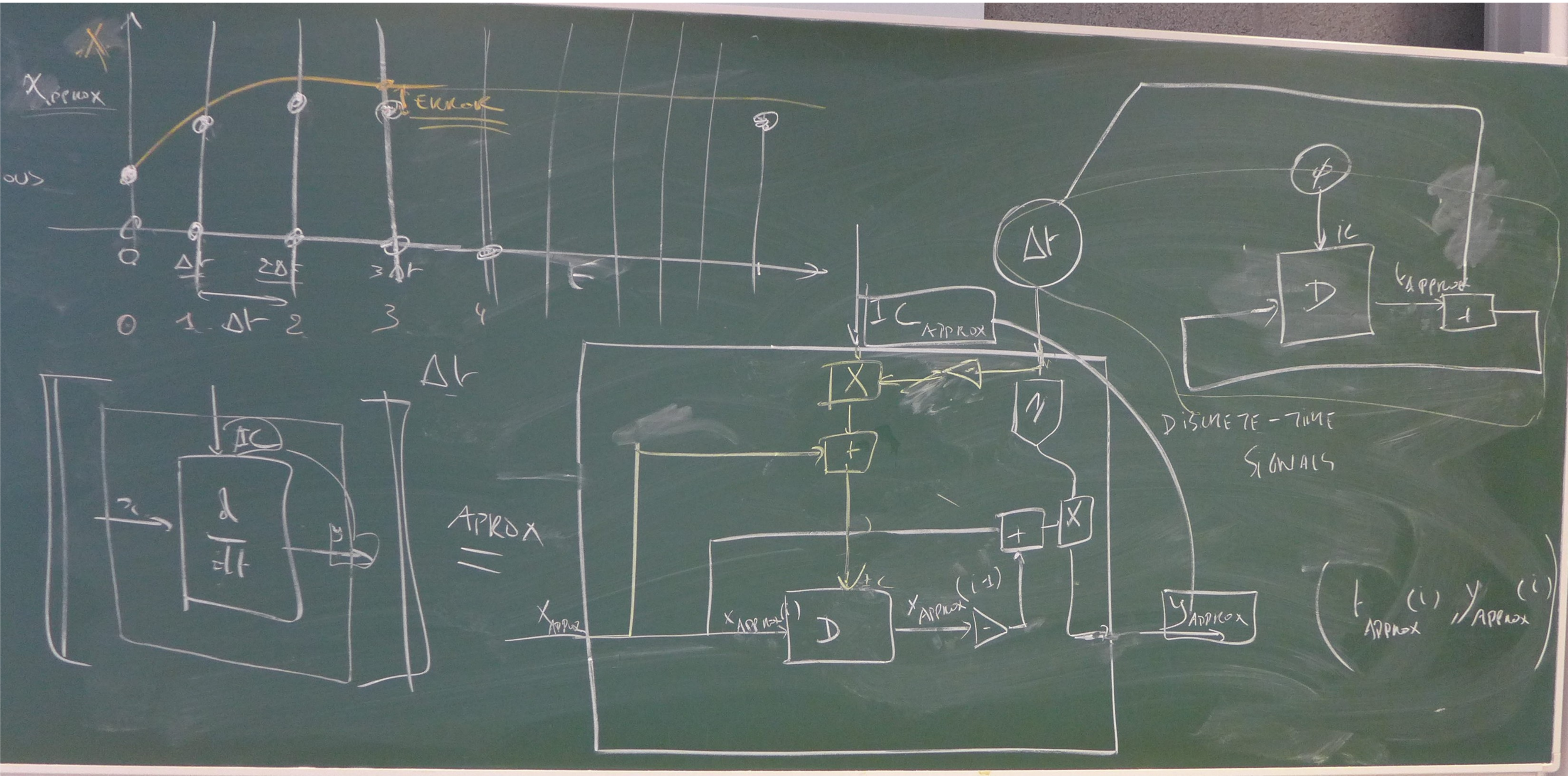
$$t_{APPROX}(\phi) = \phi$$

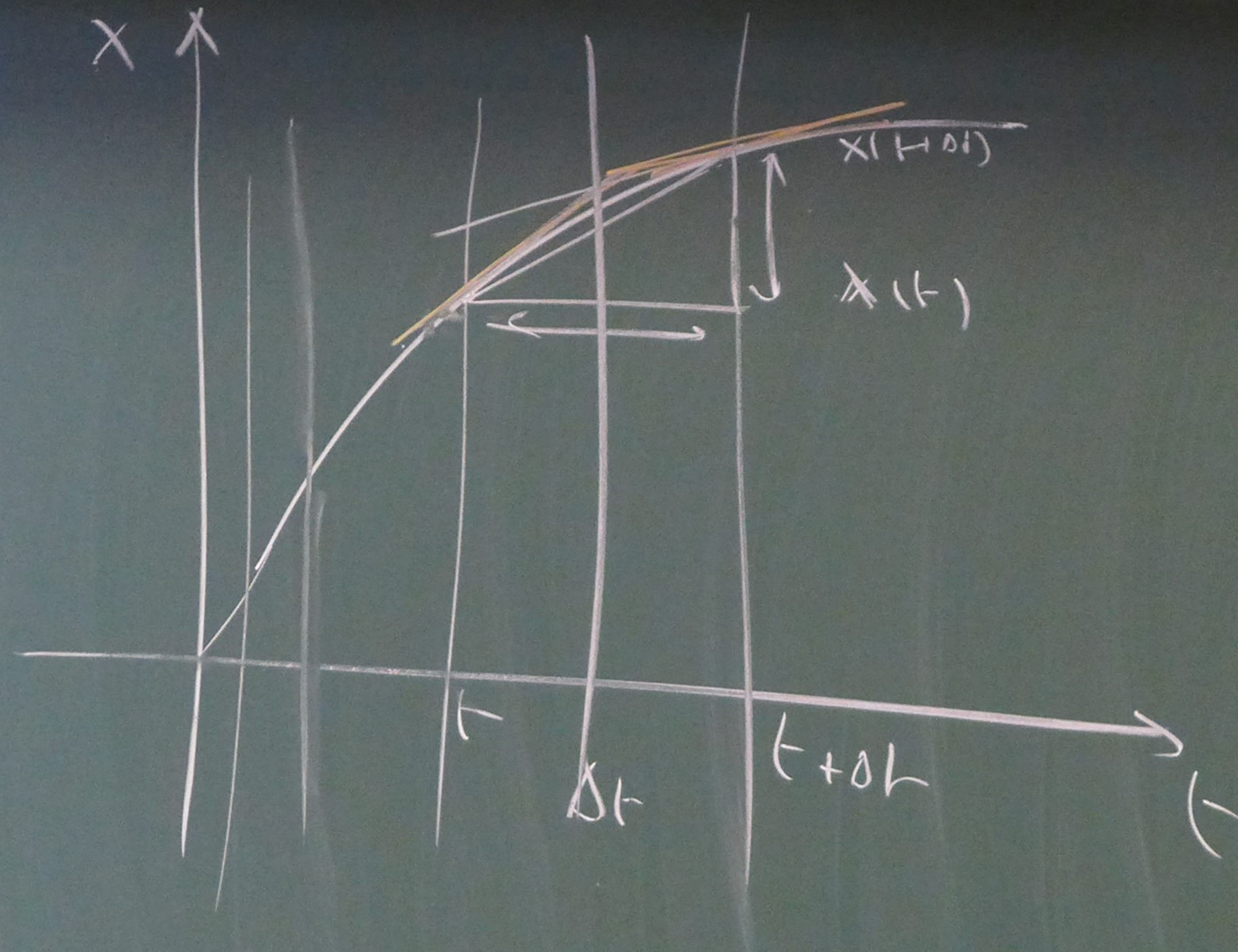
EXPLICIT MODEL t_{APPROX}

$$\Delta t \; IC_{APPROX} = X(0)_{APPROX} - DELAY_{IC}$$

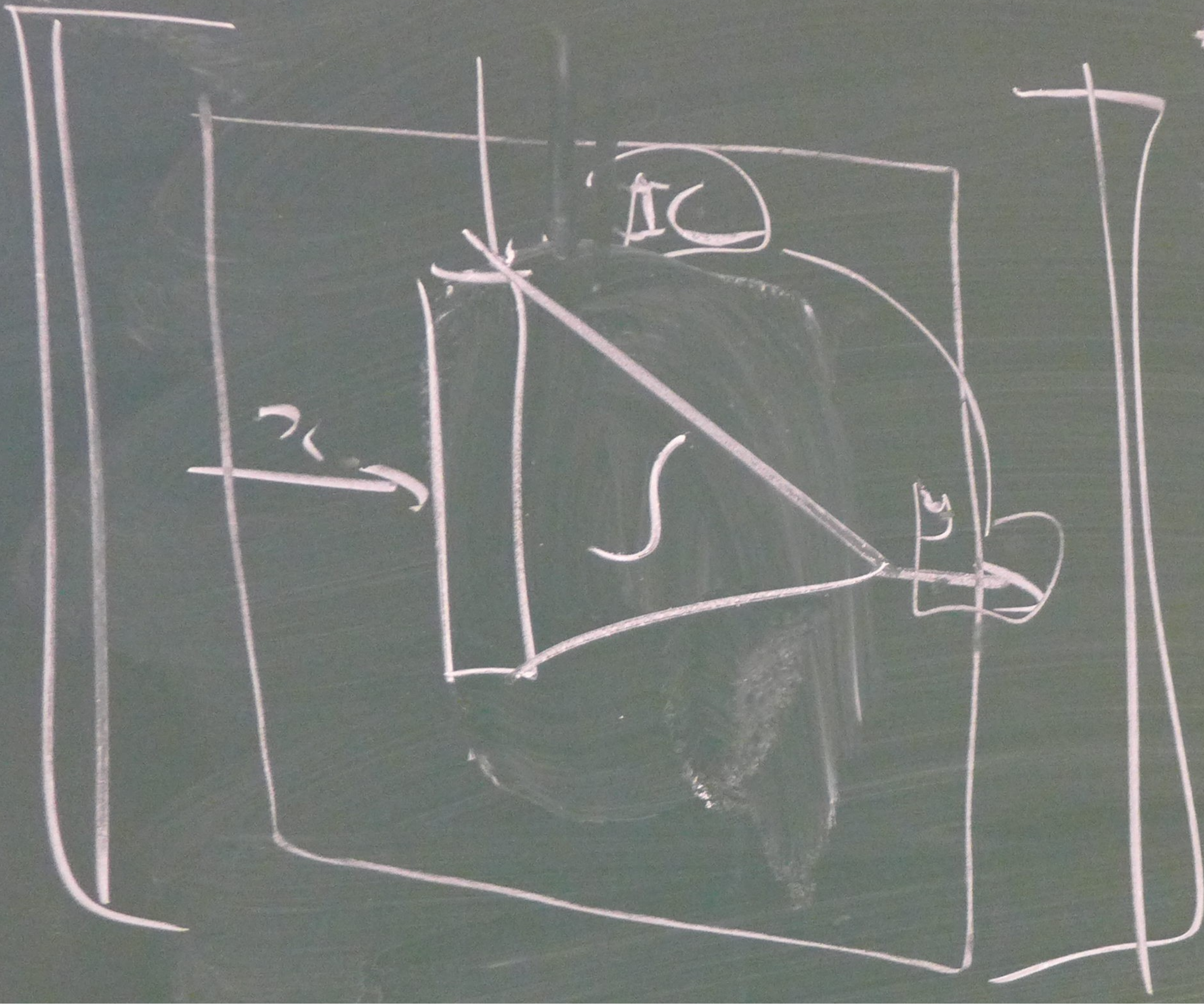
$$DELAY_{IC} = x(0)_{APPROX} - \Delta t \cdot IC_{APPROX}$$


X
X APPROX
ERROR
0
Δt
2Δt
3Δt
t
0
1. Δt
2
3
4
Δt
IC
x
d/dt
y
APROX
IC APPROX
X
+
D
x APPROX(i)
x APPROX(i-1)
y APPROX
DISCRETE


X
X APPROX
ERROR
0
Δt
2Δt
3Δt
t
0 1·Δt 2 3 4
IC
d/dt
x
y
Δt
APROX
IC APPROX
D
X APPROX
X APPROX (i-1)
y APPROX
DISCRETE-TIME SIGNALS
φ
t APPROX (i), y APPROX (i)



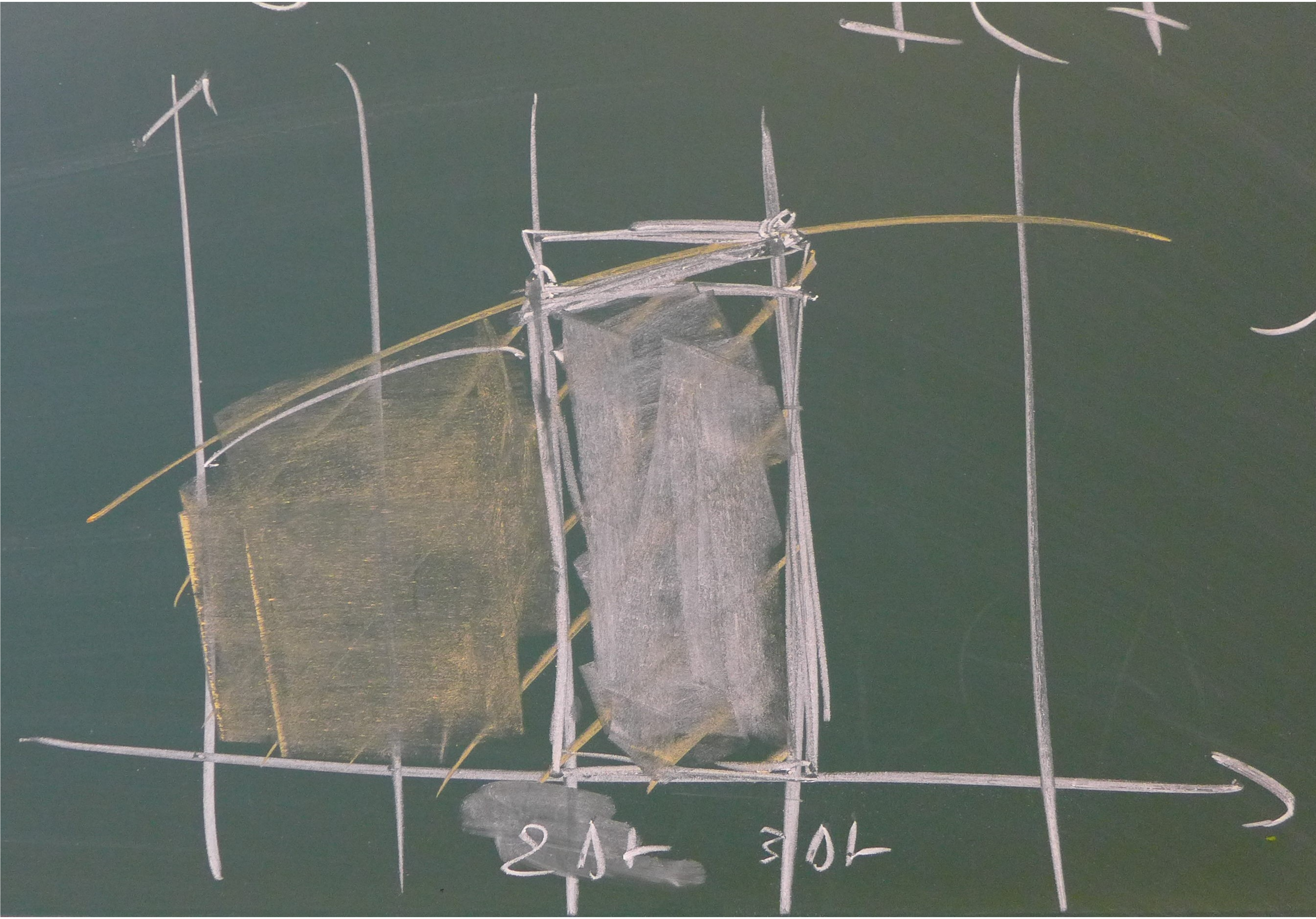
$$ic_{Approx} = y(0)_{Approx} = \frac{x(0)_{Approx} - \boxed{Dev_{ic}}}{\Delta t}$$



Δr

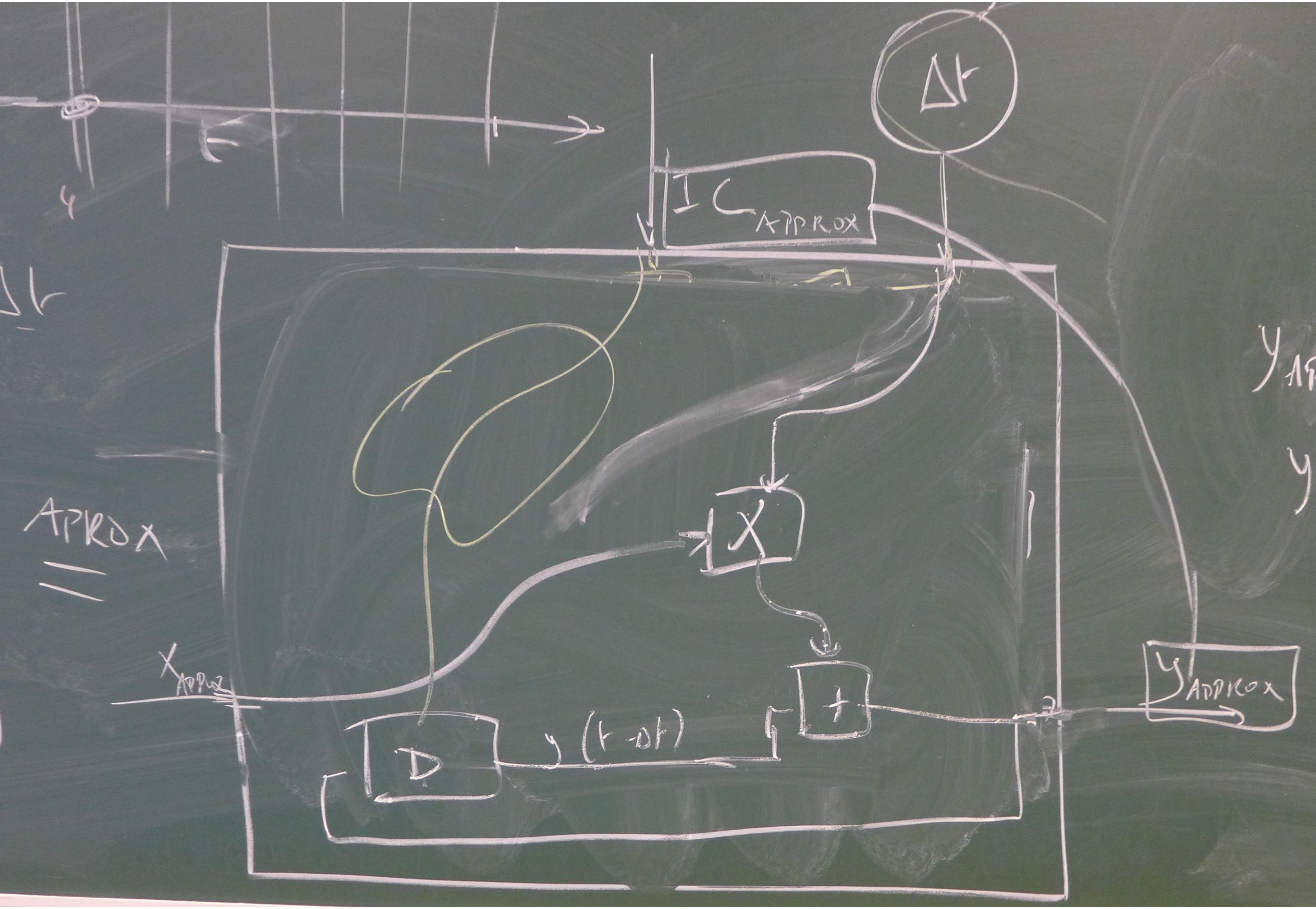
APROX

X_{APR}

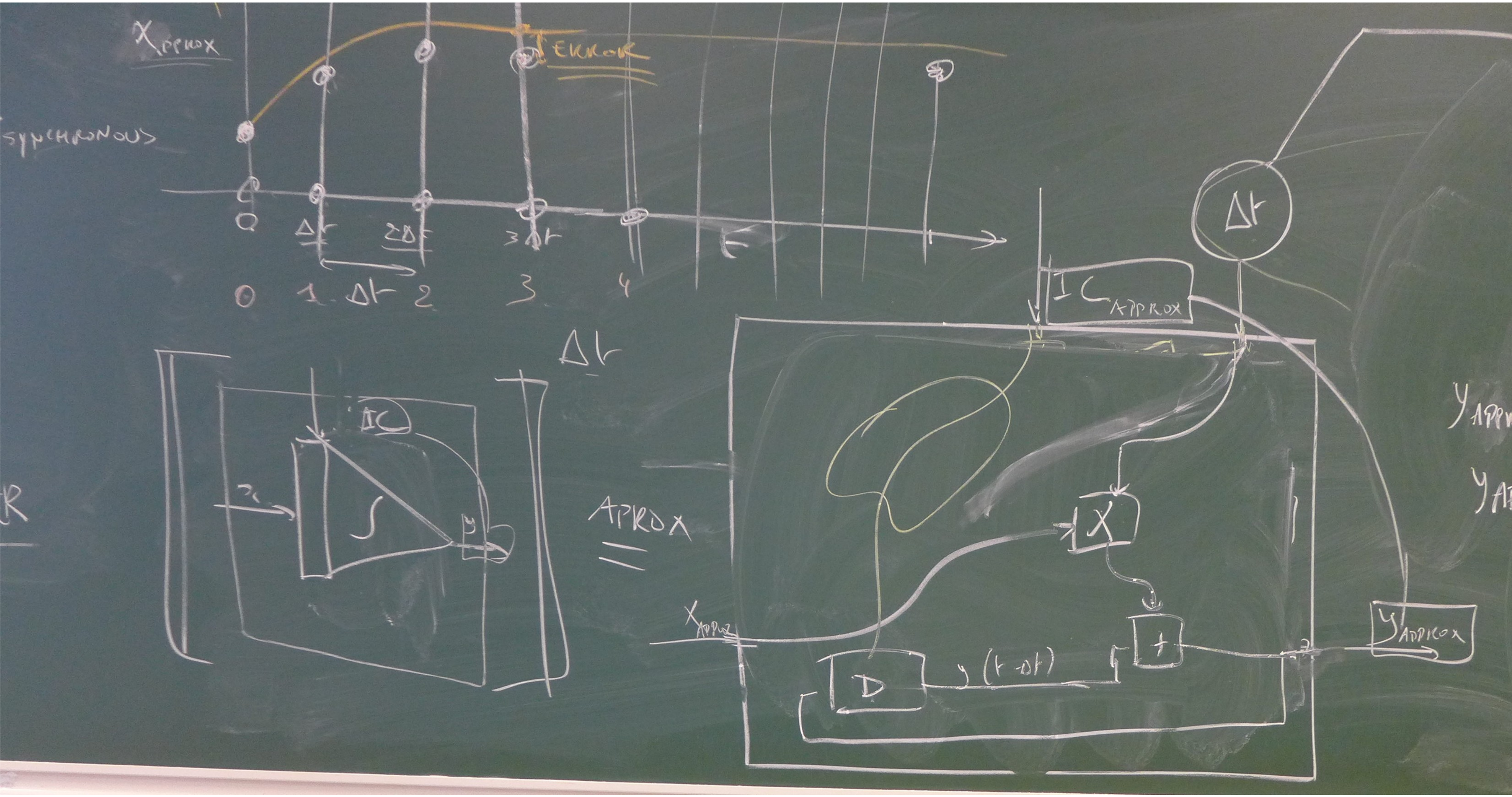

2Δr
3Δr

$$y_{\text{APPROX}}\left(t_{\text{APPROX}}\right) =$$

$$y_{\text{APPROX}}\left(t_{\text{APPROX}} - \Delta t\right) + \Delta t \cdot x\left(t_{\text{APPROX}} \;\; {}_{\text{APPROX}}\right)$$



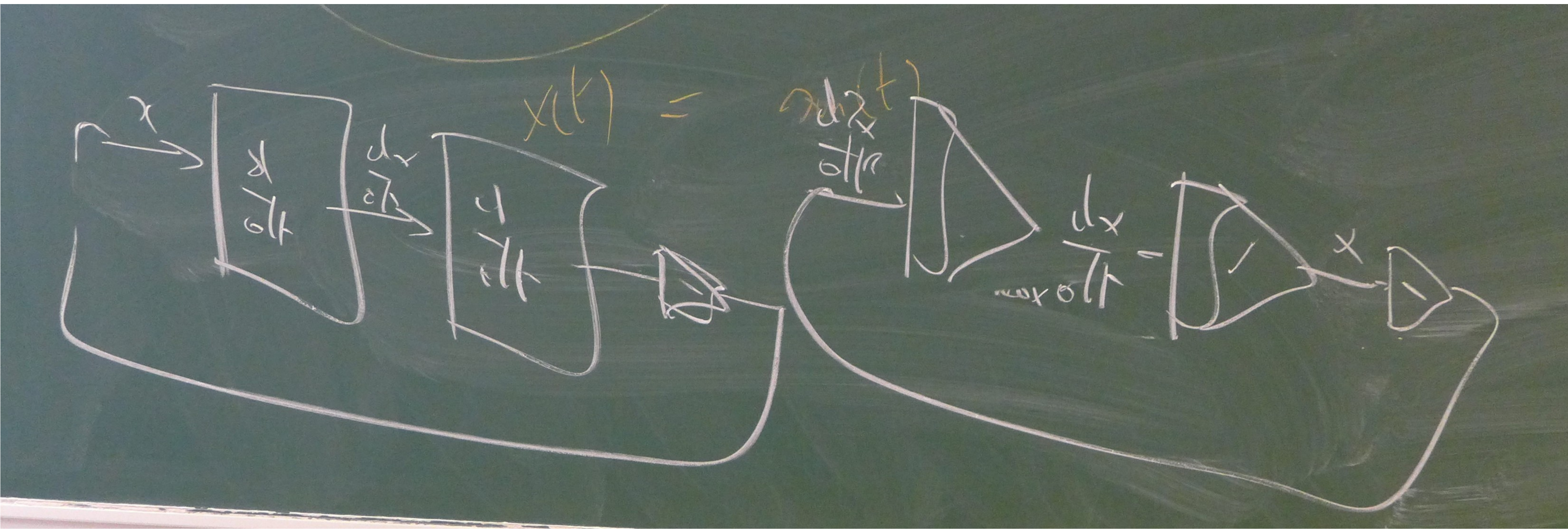
Δt
IC APPROX
APROX
D
y (t-Δt)
y APPROX


X APPROX
SYNCHRONOUS
ERROR
0
Δt
2Δt
3Δt
t
0
1 Δt
2
3
4
IC
S
y
Δt
APROX
IC APPROX
X
+
D
y (t-Δt)
X APPROX
Y APPROX

$$\boxed{\frac{d^2x}{dt^2} = -x}$$

$$x(0) = p$$

$$\frac{dx}{dt}(0) = 1$$

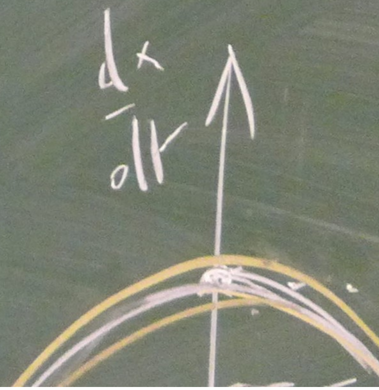


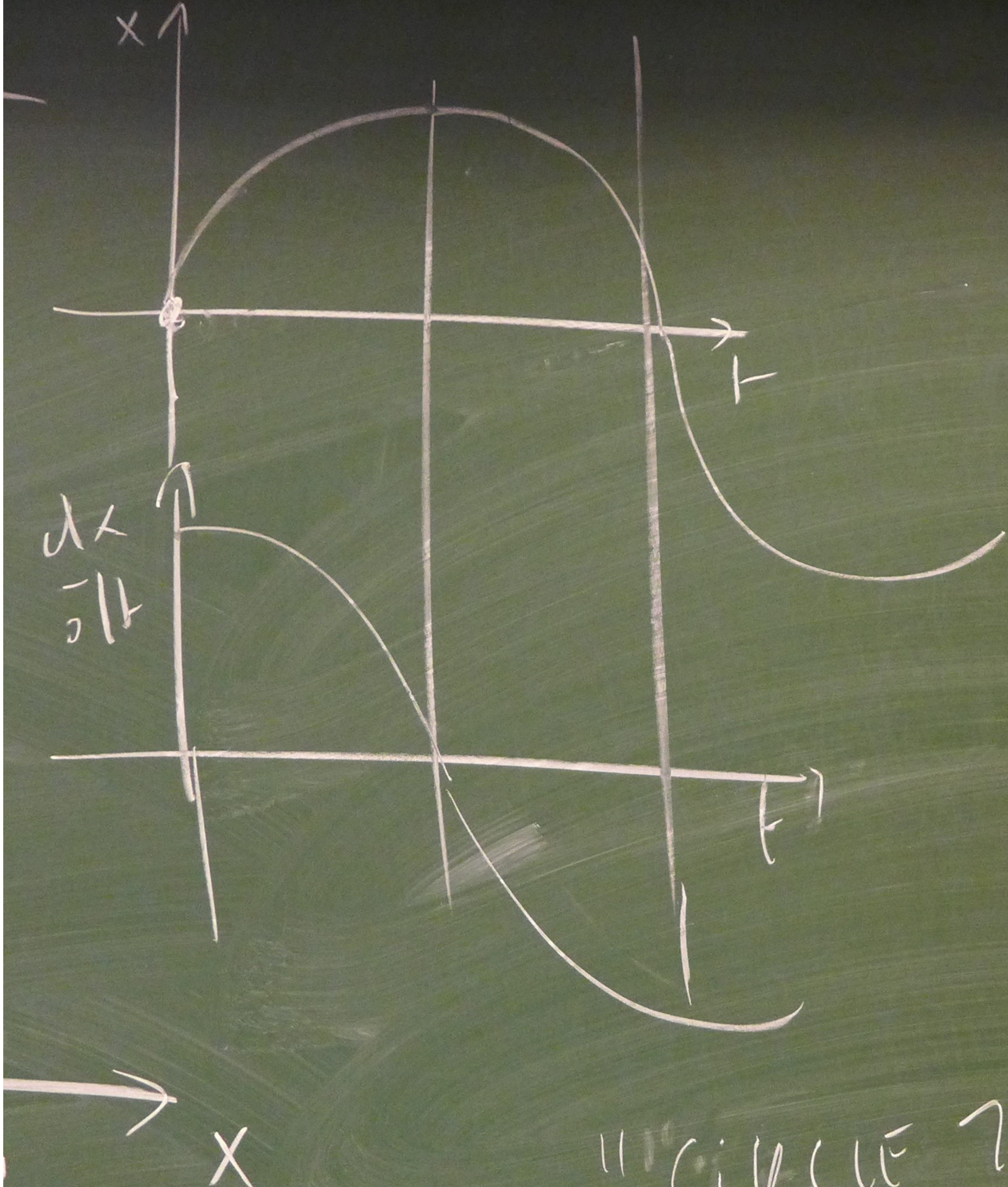
x
x(t) =
x

$$x(t) = A \sin t + \underline{B} \cos t \qquad \underline{B} = 0$$

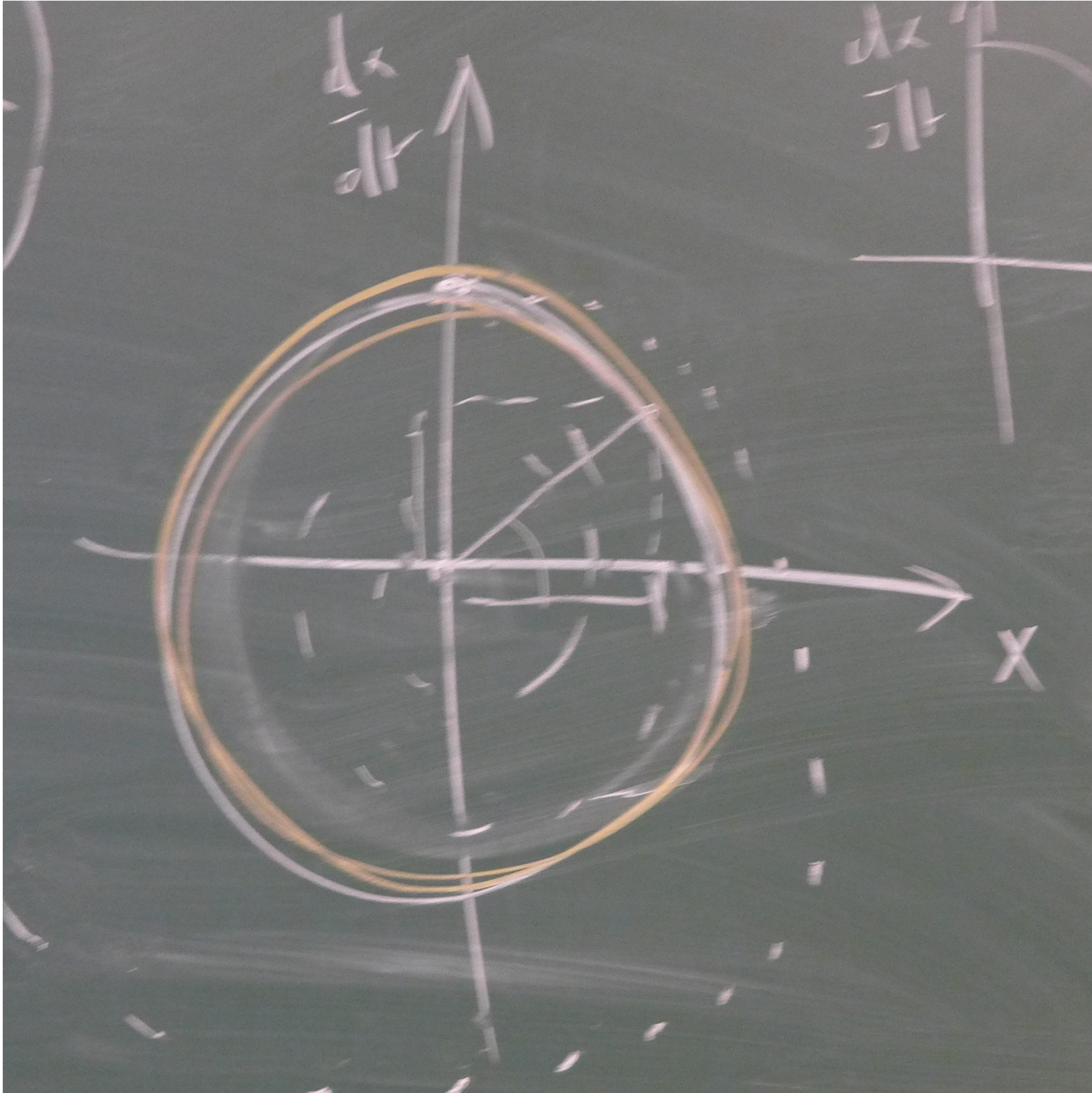
$$\frac{dx}{dt}(t) = A \cos t - \cancel{B \sin t} \qquad A = 1$$

$$\frac{d}{dt}\left(\frac{d}{dt}x\right)(t) = -\underbrace{\left(A \sin t + B \cos t\right)}_{x}$$





"CIRCLE TEST"


dx/dt
x
dx/dt

CAUSAL BLOCK DIAGRAMS (CBD)

SEMANTICS

	T	DENOTATIONAL SEM. DOM.		OPERATIONAL
ALG	{NOW}	ALG EQNS	INCL LOOPS	INCL LOOPS
DT	$\mathbb{N}$	DIFFERENCE EQNS		
CT	$\mathbb{R}$	DIFFERENTIAL INTEGRAL EQNS TERM. COND × TIME × STATE	APPROX	II

RUN FOREVER:

- ONLY RECUR DELAY
  → NO MEM OVFLOW
- TIME – TRANSLATION INV.
  → NO NUMERICAL OVERFLOW