# Controller Design and Tuning

### Hans Vangheluwe and Claudio Gomes

Modelling, Simulation and Design Lab (MSDL)

University of Antwerp, Belgium and McGill University, Canada

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- Our example (closed loop): velocity control in rail car

# Moving Car (the physical "plant") Model

$$F_{res} = F_{traction} + F_{drag}$$

$$F_{drag} = -\frac{1}{2} \cdot p \cdot v^2 \cdot C_D \cdot A$$

$$F_{res} = M \cdot a = M \cdot \frac{dv}{dt}$$

$$F_{drag} = \frac{1}{2} \cdot p \cdot v^2 \cdot C_D \cdot A$$

$$F_{res} = \frac{1}{2} \cdot p \cdot v^2 \cdot C_D \cdot A$$

$$\frac{dv}{dt} = \frac{1}{M} \left( F_{traction} - \frac{1}{2} \cdot p \cdot v^2 \cdot C_D \cdot A \right)$$
$$v(0) = 0$$

A Proportional-Integral-Derivative (PID) controller takes as input the error (deviation of the measured/sensed value from the ideal or "setpoint" value)  $v_i - v$  and produces an output to be sent to the plant via the actuator.

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This is done by combining three different controllers:

Proportional Controller – outputs  $K_p \cdot (v_i - v)$ , with  $K_p$  an appropriate constant;

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Integral Controller – outputs 
$$K_i \cdot \int (v_i - v)dt$$
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Integral Controller – outputs  $K_i \cdot \int (v_i - v)dt$ , with  $K_i$  an appropriate constant;

Derivative Controller – outputs  $K_d \cdot \frac{d(v_i - v)}{dt}$ , with  $K_d$  an appropriate constant;

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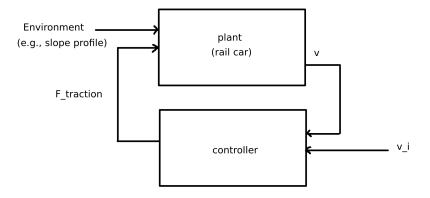
Derivative Controller – outputs  $K_d \cdot \frac{d(v_i - v)}{dt}$ , with  $K_d$  an appropriate constant;

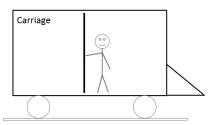
A PID controller produces a control output:

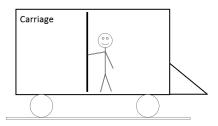
$$K_p \cdot (v_i - v) + K_i \cdot \int (v_i - v) dt + K_d \cdot \frac{d(v_i - v)}{dt}$$



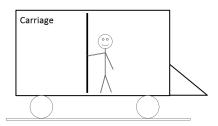
# Closed-Loop PID Controller for Velocity Control



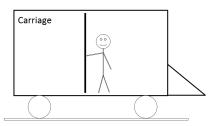




▶ Build the controller for a driverless rail car.

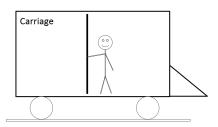


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- ▶ Passengers should not fall (i.e., accelerate too much).
- Other requirements such as minimizing total energy consumption could be added.

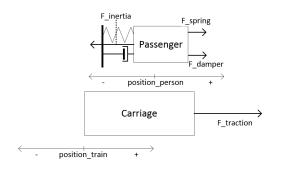


# Abstracting the Passenger: Mass-Spring-Damper System

$$\begin{cases} F_{ext} & = -f \\ F_{spring} & = -k(-x) \\ F_{damper} & = -c(-v) \\ M \cdot a & = F_{ext} + F_{spring} + F_{damper} \\ \frac{dv}{dt} & = a \\ \frac{dx}{dt} & = v \end{cases}$$

$$\begin{cases} \frac{dv}{dt} &= \frac{1}{M}(-f + k \cdot x + c \cdot v) \\ \frac{dx}{dt} &= v \end{cases}$$

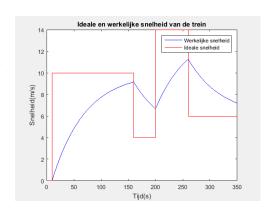
# Abstracting Train-and-Passenger ("Plant" model)



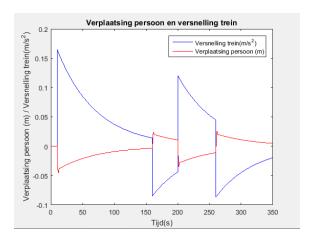
$$\begin{cases} m_{passger} * a_{passger} &= k(-x_{passger}) + c(-v_{passger}) - m_{passger} * a_{train} \\ F_{traction} &= (m_{train} + m_{passger}) * a_{train} \\ a_{passger} &= \frac{dv_{passger}}{dt} \\ v_{passger} &= \frac{dx_{passger}}{dt} \\ a_{train} &= \frac{dv_{train}}{dt} \\ v_{train} &= \frac{dx_{train}}{dt} \end{cases}$$

# Some Results - Train Velocity

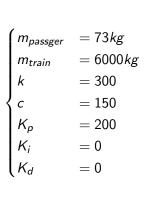
$$\begin{cases} m_{passger} &= 73 kg \\ m_{train} &= 6000 kg \\ k &= 300 \\ c &= 150 \\ K_p &= 100 \\ K_i &= 0 \\ K_d &= 0 \end{cases}$$

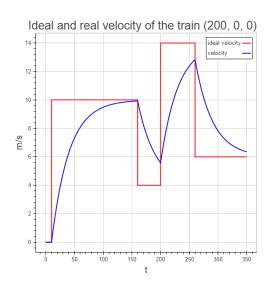


# Some Results - Passenger Displacement and Acceleration

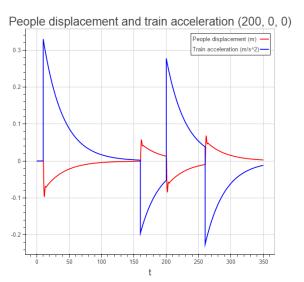


## Some Results - Train Velocity



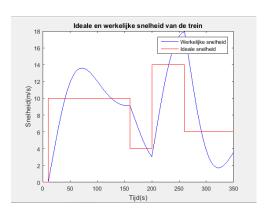


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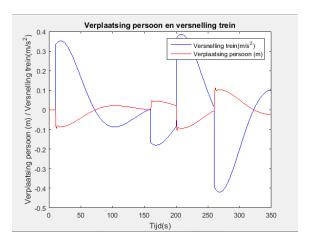


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\begin{cases} m_{passger} &= 73 kg \\ m_{train} &= 6000 kg \\ k &= 300 \\ c &= 150 \\ K_p &= 200 \\ K_i &= 10 \\ K_d &= 0 \end{cases}
```

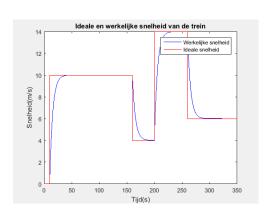


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$$\begin{cases} m_{passger} &= 73 kg \\ m_{train} &= 6000 kg \\ k &= 300 \\ c &= 150 \\ K_p &= 1500 \\ K_i &= 0 \\ K_d &= 2000 \end{cases}$$



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