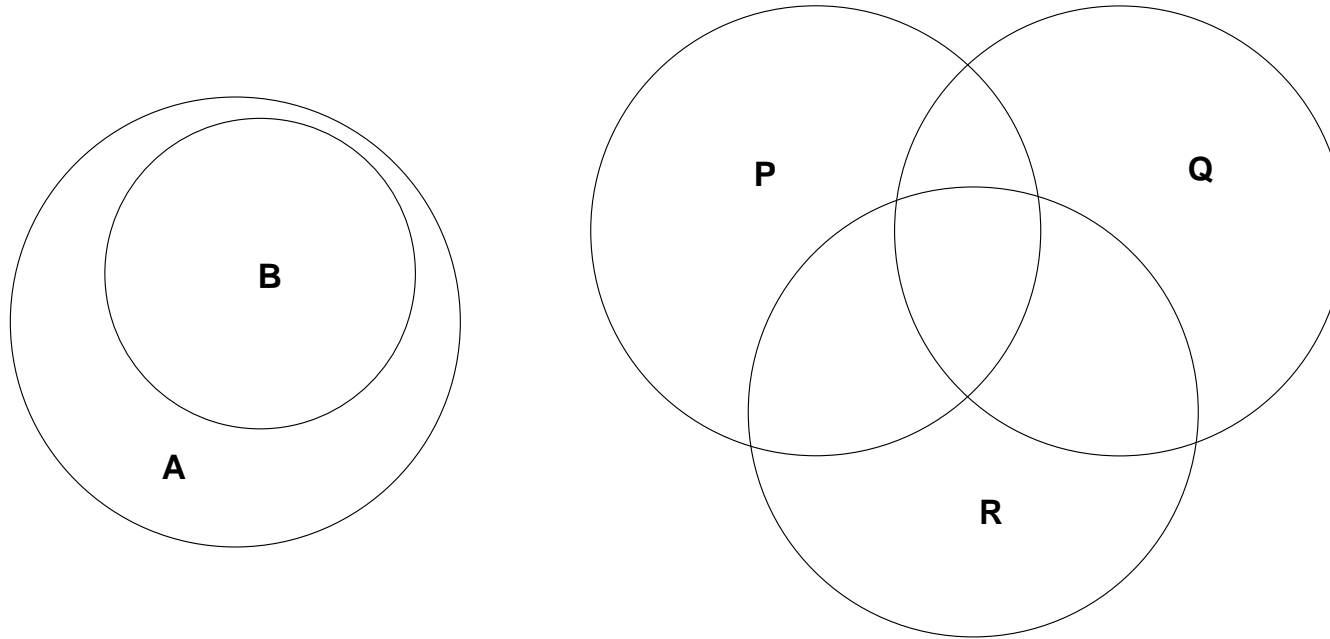# Higraphs: Visualising Information

- complex

- non-quantitative, structural

- topological, not geometrical

- Euler

  - Venn diagrams (Jordan curve: inside/outside): enclosure, intersection

  - graphs (nodes, edges: binary *relation*); hypergraphs
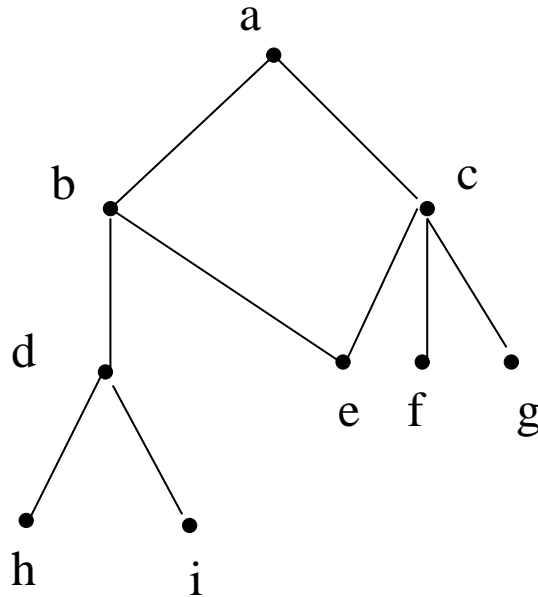
David Harel. **On Visual Formalisms.**

Communications of the ACM. Volume 31, No. 5. 1988. pp. 514 - 530.
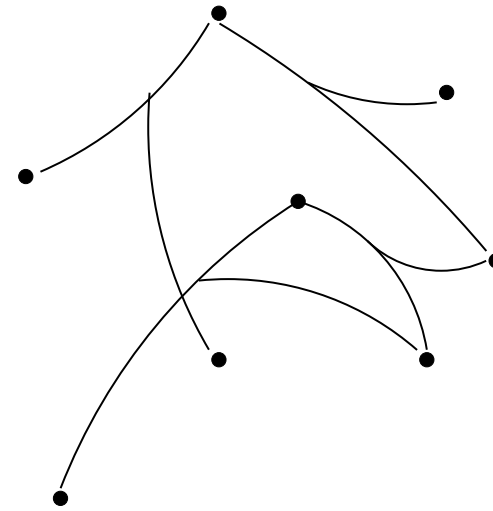
Venn diagrams, Euler circles

- *topological* notions (syntax):
  enclosure, exclusion, intersection

- Used to represent (denote) *mathematical* set operations:
  union, difference, intersection
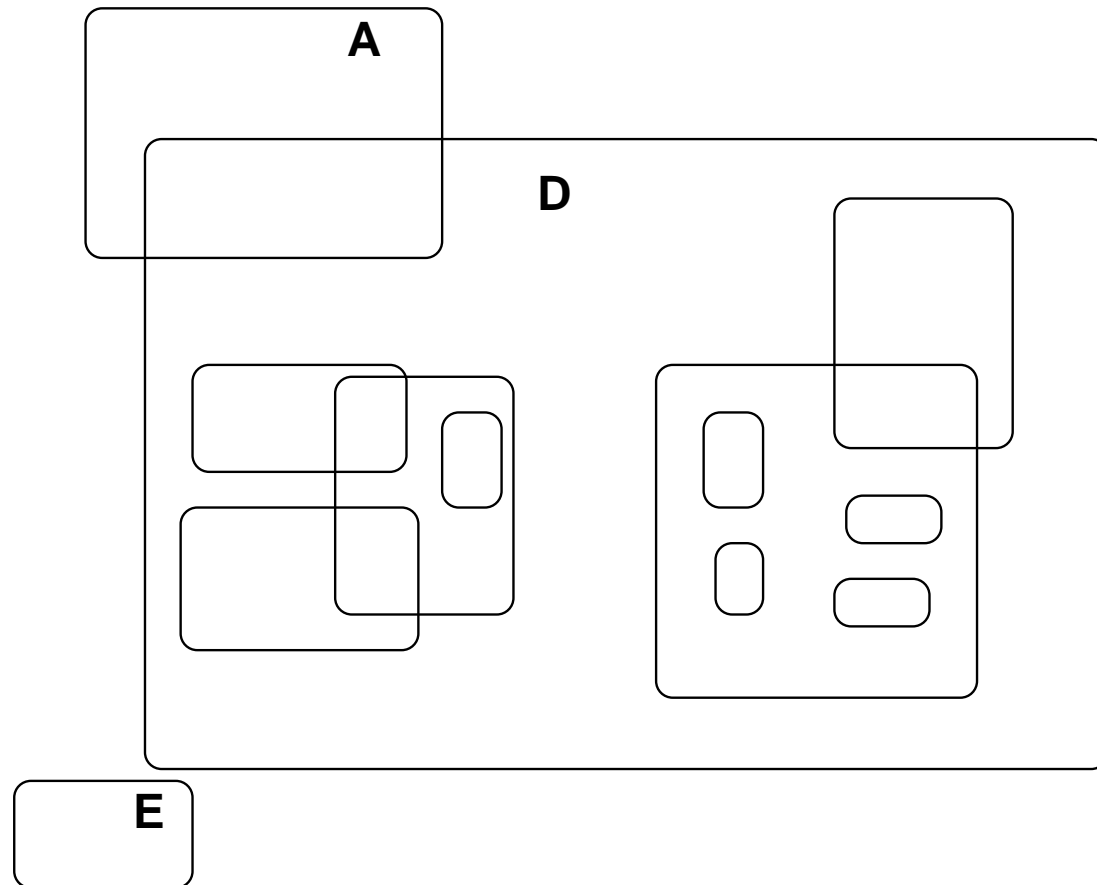
Hypergraphs

a graph

a hypergraph

- *topological* notion (syntax): connectedness

- Used to represent (denote) *relations* between sets.

- Hyperedges: non longer binary relation ($\subseteq X \times X$): $\subseteq 2^X$ (undirected), $\subseteq 2^X \times 2^X$ (directed).
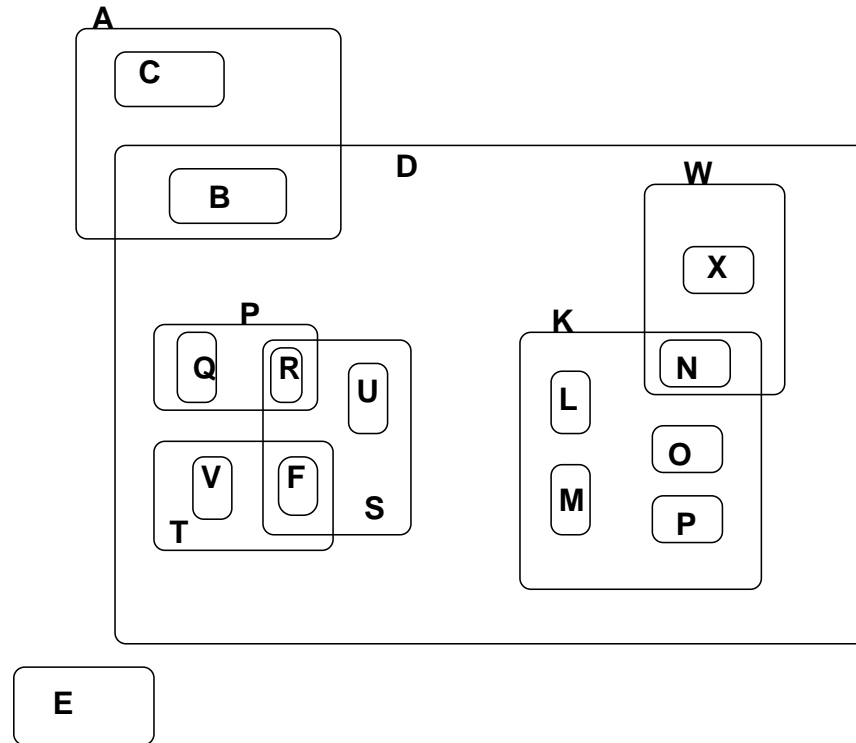
Higraphs: combining graphs and Venn diagrams

- sets + cartesian product

- hypergraphs

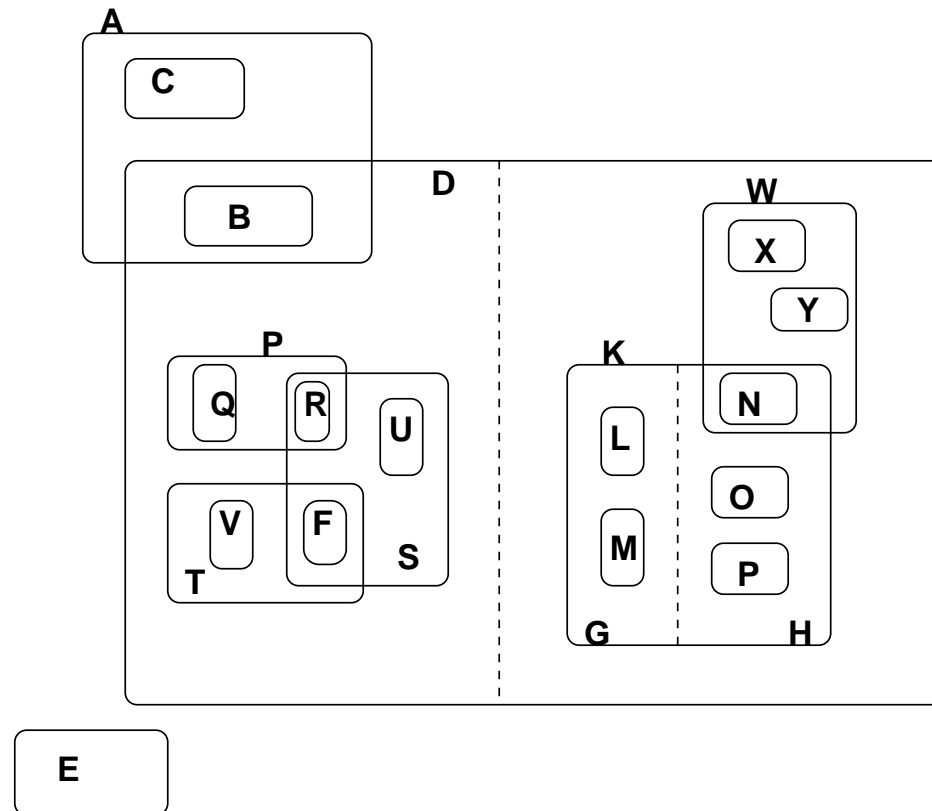# Blobs: set *inclusion, not membership*

Unique Blobs (atomic sets, no intersection)

- atomic blobs are identifiable sets
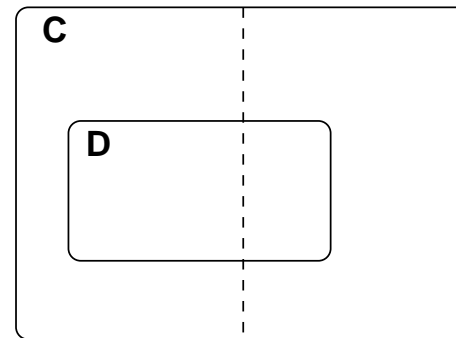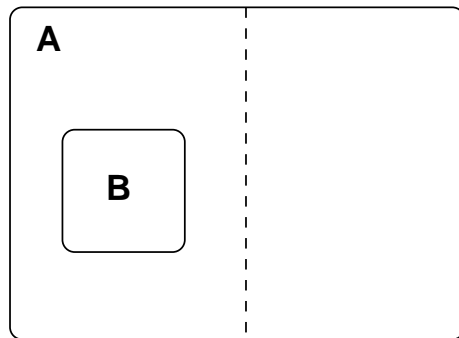
- other blobs are union of enclosed sets (*e.g.,*
  $K = L \cup M \cup N \cup O \cup P$)

- empty space meaningless, identify intersection (*e.g.*, $N = K \cap W$)

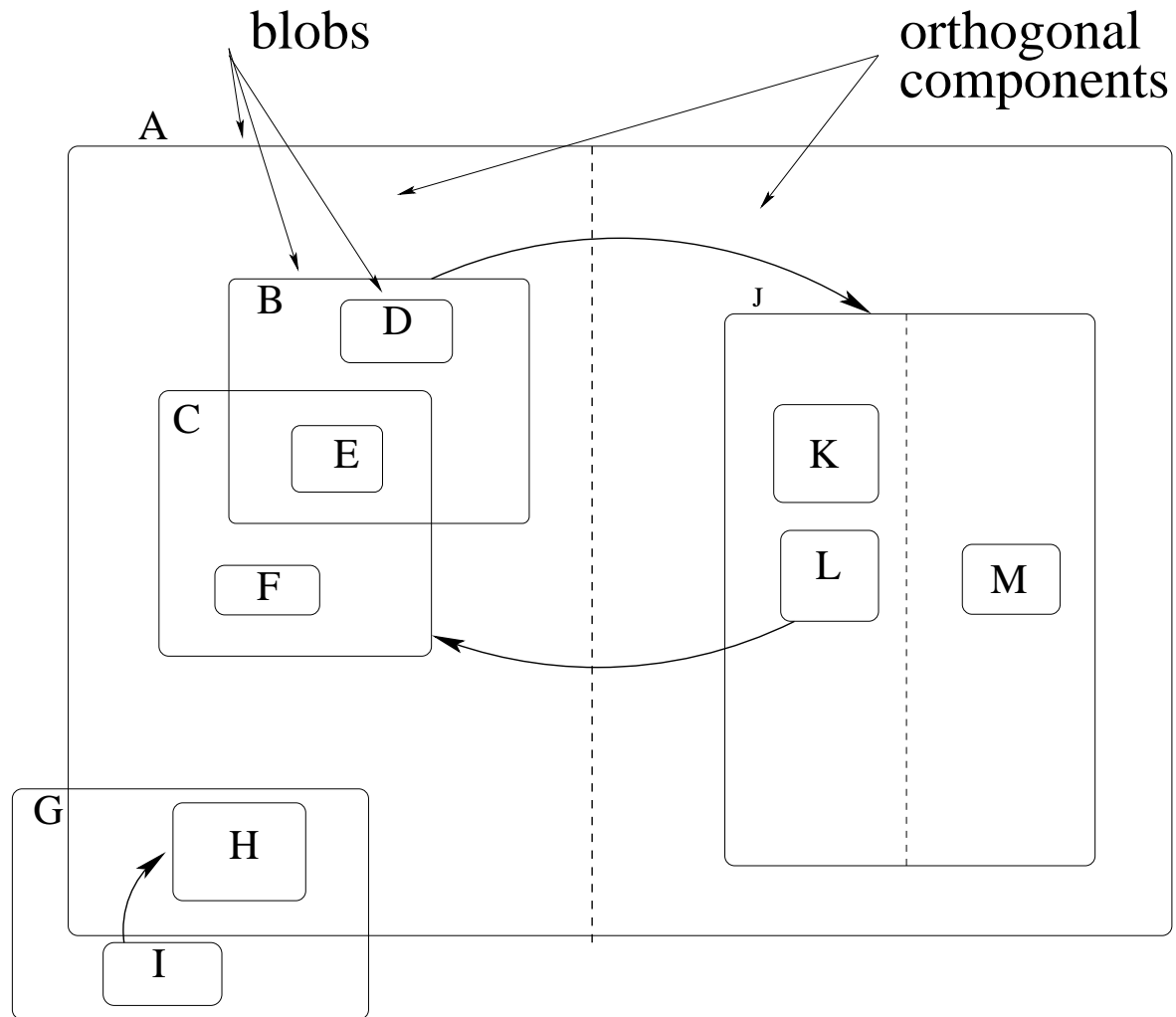# *Unordered* Cartesian Product: *Orthogonal* Components

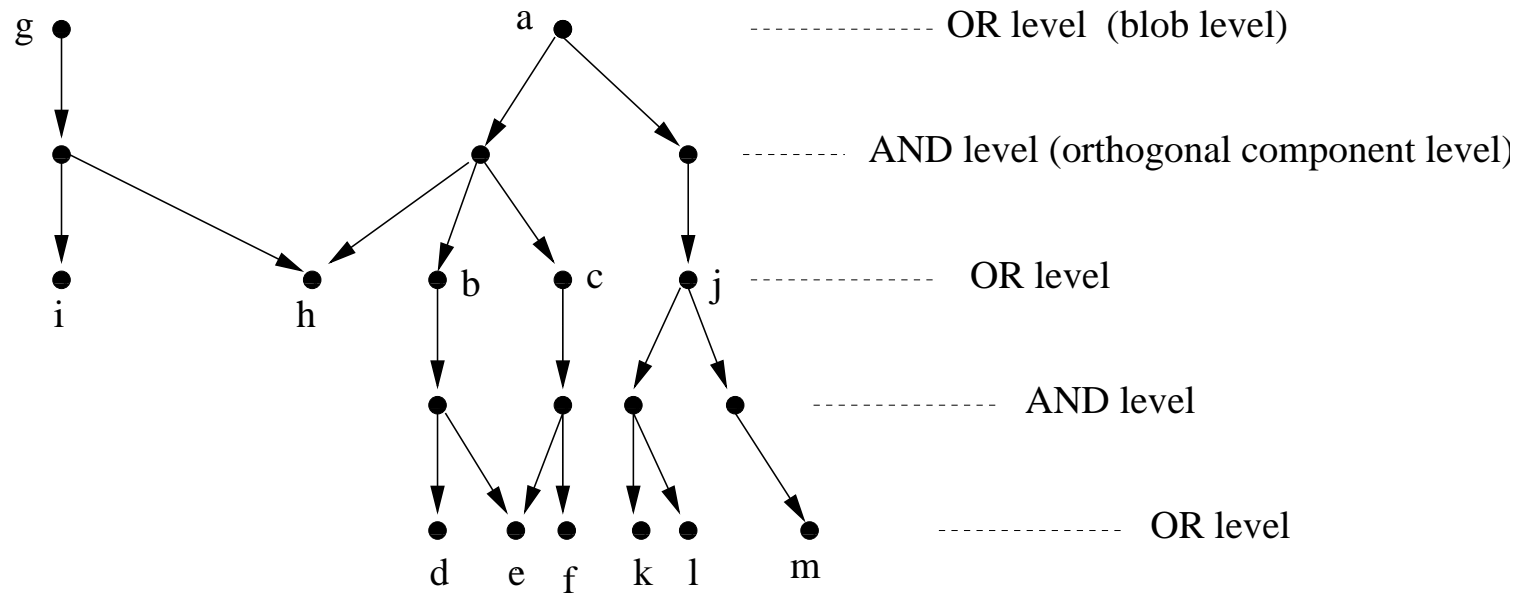
$$K = G \times H = H \times G = (L \cup M) \times (N \cup O \cup P)$$

Meaningless syntactic constructs

Simple Higraph

blobs

orthogonal components

A

B

D

C

E

F

J

K

L

M

G

H
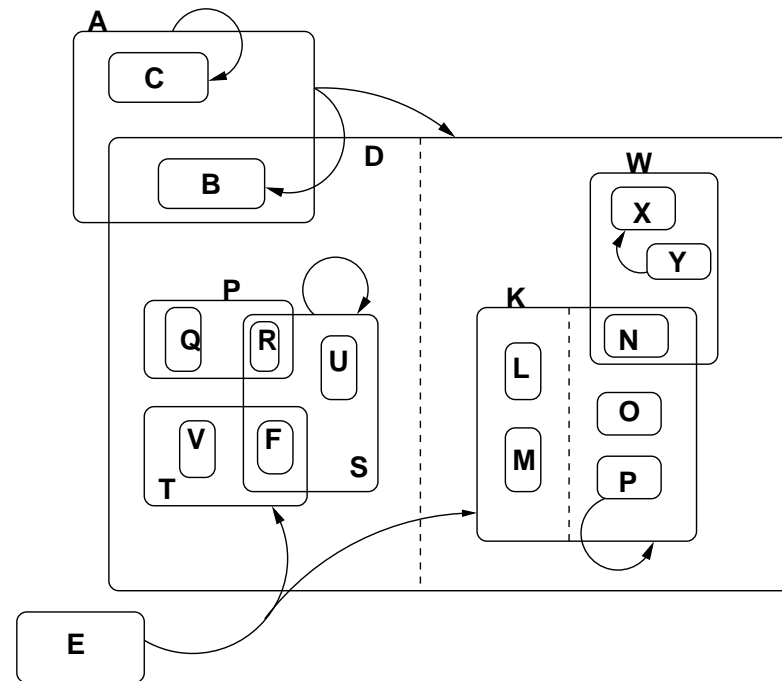
I

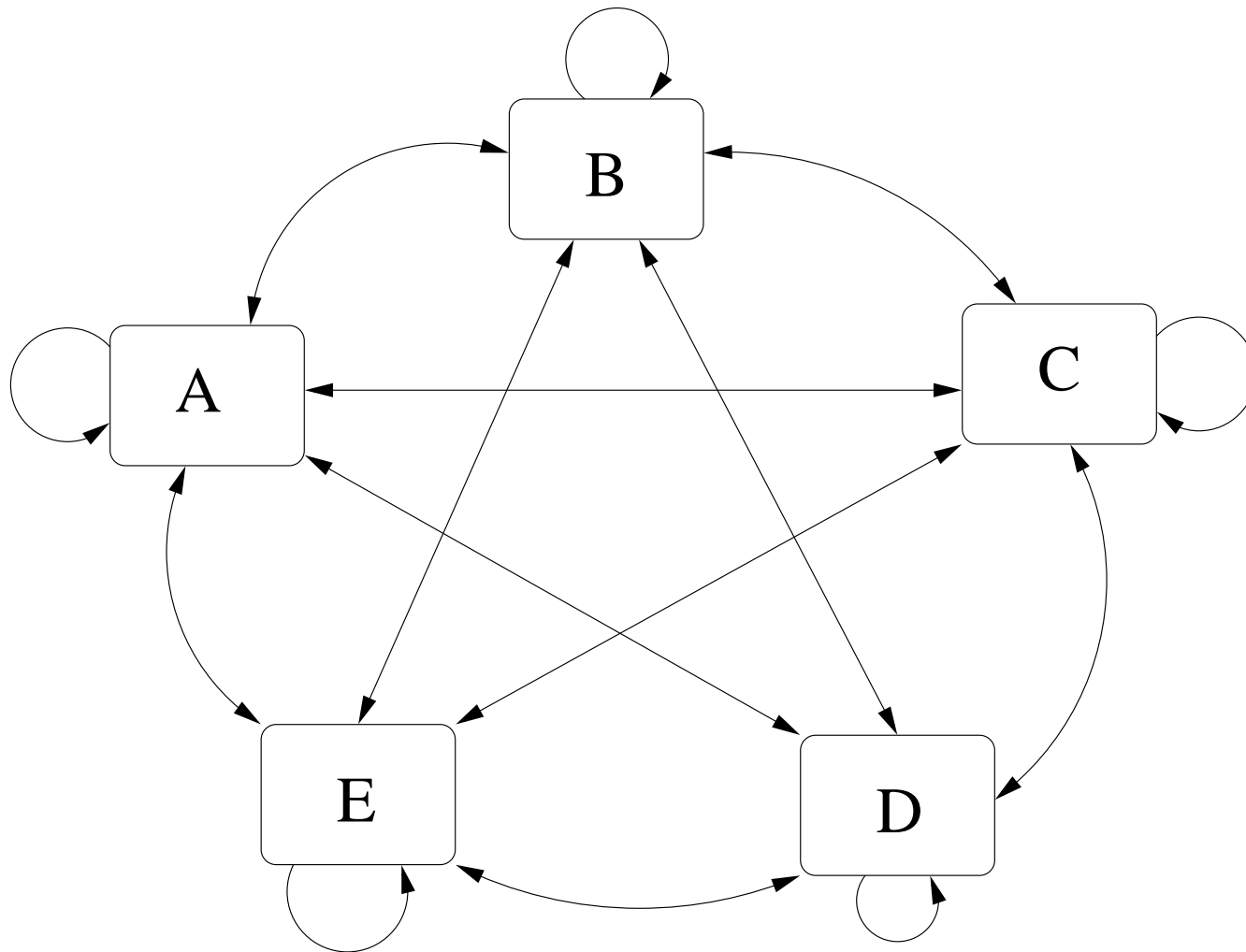# Induced Acyclic Graph (blob/orth comp alternation)
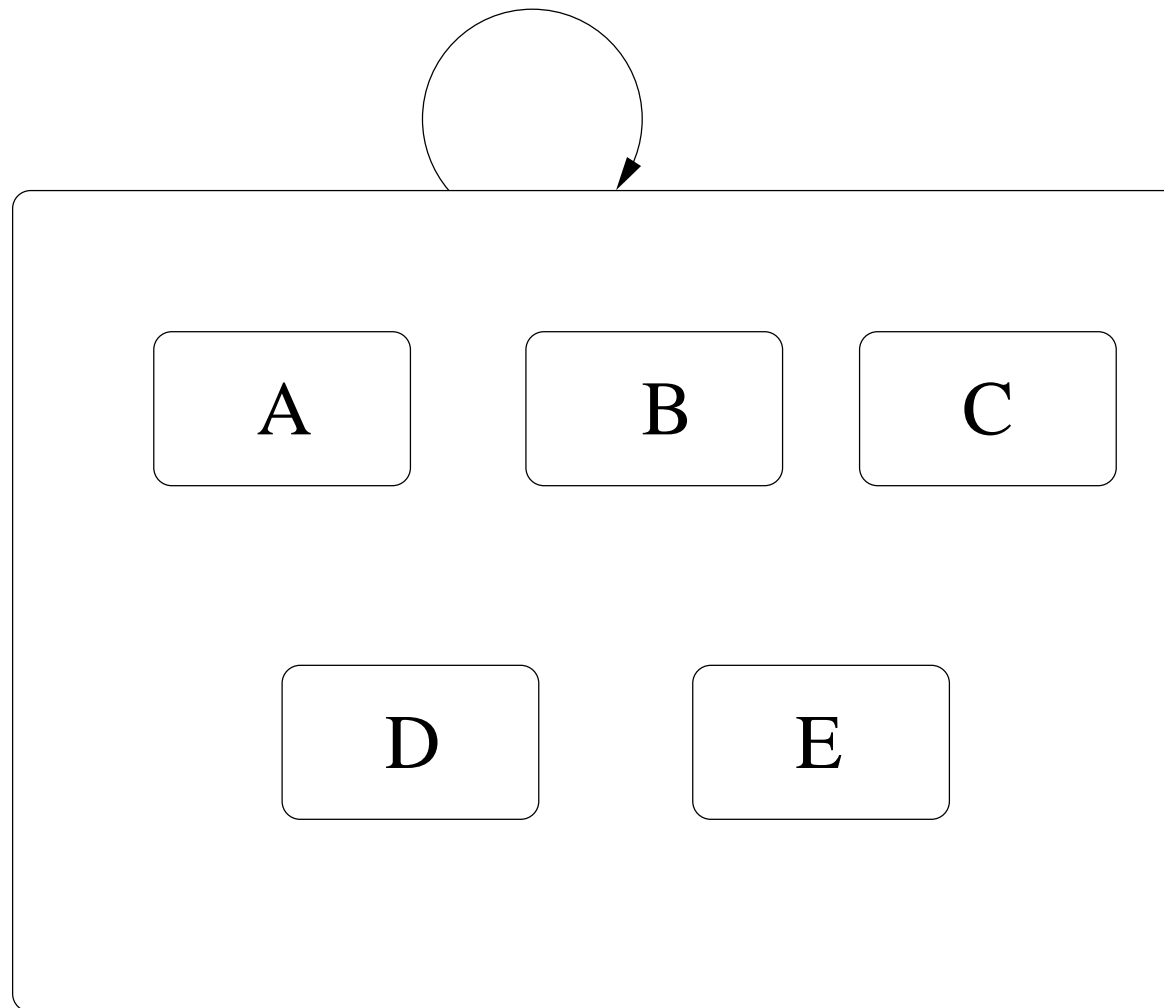
Adding (hyper) edges

- *hyper*edges

- attach to contour of *any* blob

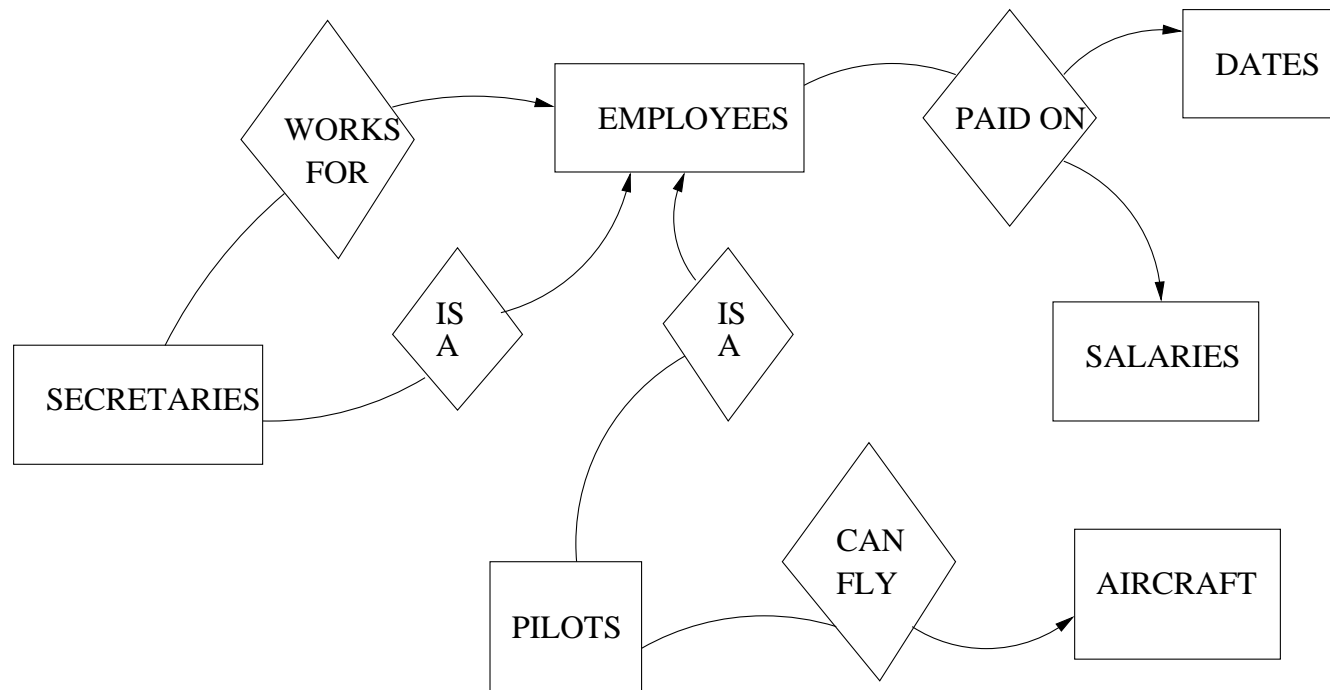- inter-level possible (*e.g.,* denote global variables binding)
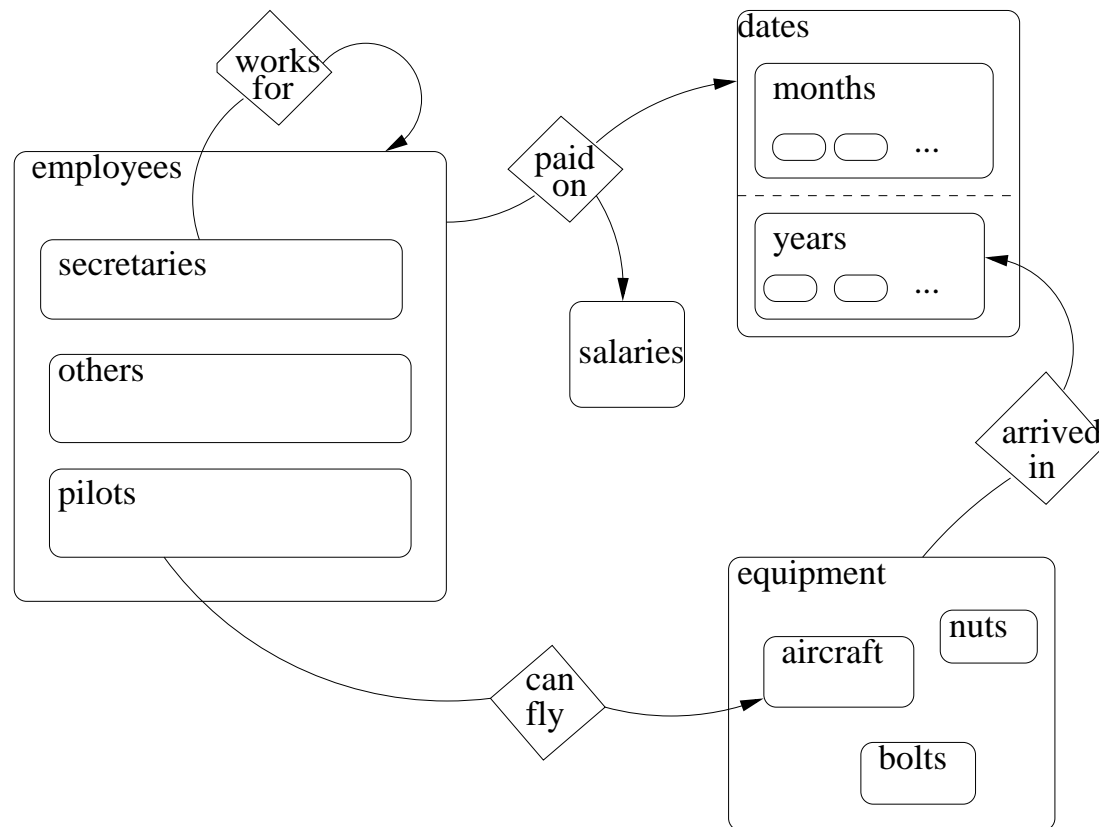
Clique Example

Clique: fully connected semantics

Entity Relationship Diagram (is-a)

Higraph version of E-R diagram

Extending the E-R diagram

Formally (syntax)

A higraph H is a quadruple

$$H = (B, E, \sigma, \pi)$$

B: finite set of all unique *blobs*

E: set of hyperedges

$$\subseteq X \times X, \quad \subseteq 2^X, \quad \subseteq 2^X \times 2^X$$

The subblob (direct descendants) function σ

$$\sigma : B \to 2^B$$

$$\sigma^0(x) = \{x\}, \; \sigma^{i+1} = \bigcup_{y \in \sigma^i(x)} \sigma(y), \; \sigma^+(x) = \bigcup_{i=1}^{+\infty} \sigma^i(x)$$

Subblobs$^+$ cycle free

$$x \notin \sigma^+(x)$$

The partitioning function π associates *equivalence relationship* with x

$$\pi : B \to 2^{B \times B}$$

Equivalence classes π_i are *orthogonal components* of x

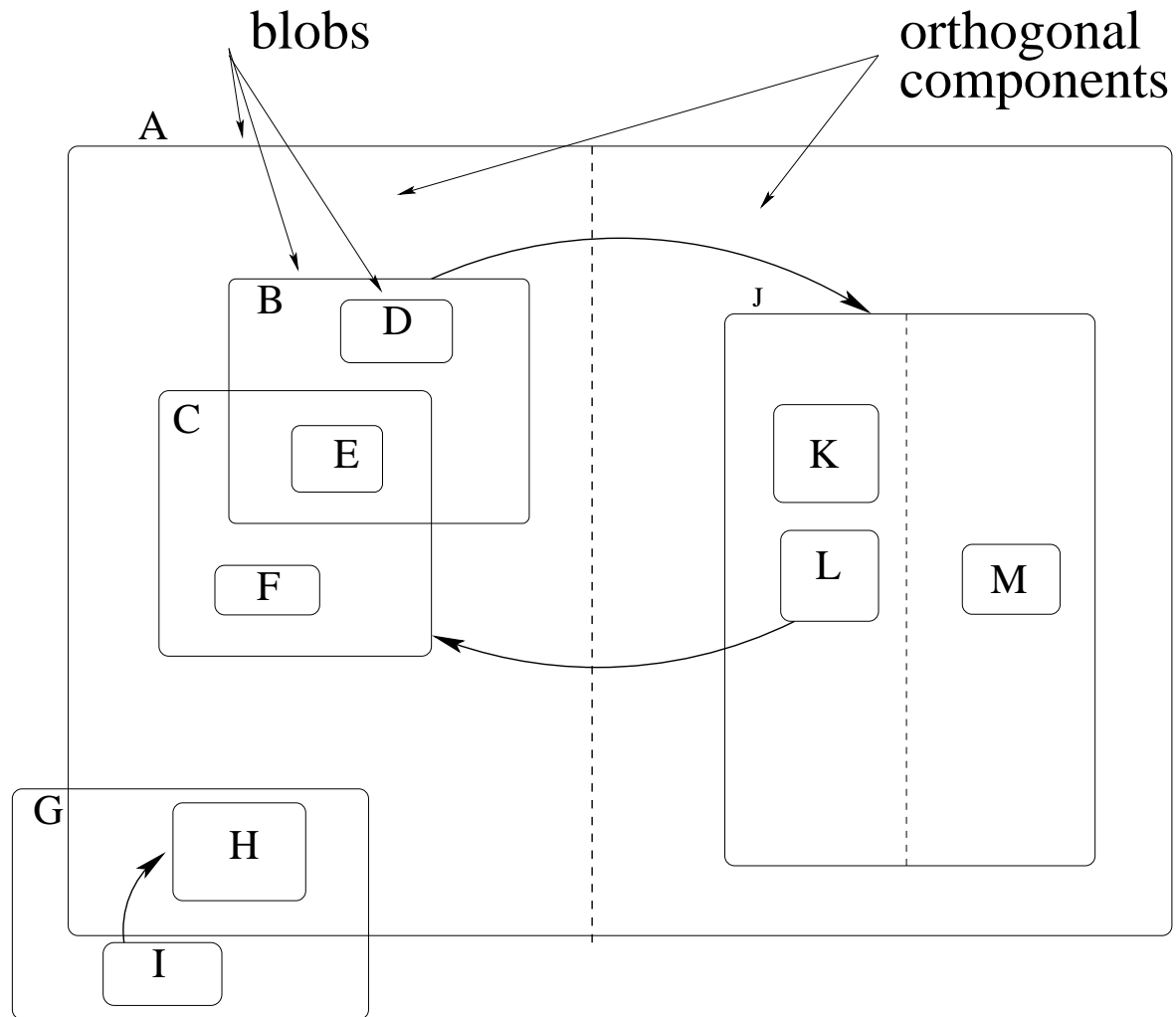$$\pi_1(x), \pi_2(x), \ldots, \pi_{k_x}(x)$$

$k_x = 1$ means a single orthogonal component (no partitioning)

Blobs in different orthogonal components of x are *disjoint*

$$\forall y, z \in \sigma(x) : \sigma^+(y) \cap \sigma^+(z) = \emptyset$$

unless in the same equivalence class

Simple Higraph

blobs

orthogonal components

A

B

D

C

E

F

J

K

L

M

G

H

I

Induced Orthogonal Components

$$B = \{A, B, C, D, E, F, C, G, H, I, J, K, L, M\}$$

$$E = \{(I, H), (B, J), (L, C)\}$$

$$\rho(A) = \{B, C, H, J\}, \rho(G) = \{H, I\}, \rho(B) = \{D, E\}, \rho(C) = \{E, F\},$$

$$\rho(J) = \{K, L, M\}$$

$$\rho(D) = \rho(E) = \rho(F) = \rho(H) = \rho(I) = \rho(K) = \rho(L) = \rho(M) = \emptyset$$

$$\pi(J) = \{(K, K), (K, L), (L, L), (L, K), (M, M)\}$$

Induces *equivalence classes* $\pi_1(J) = \{K, L\}$ and $\pi_2(J) = \{M\}$, ...
These are the *orthogonal components*

Higraph applications (add specific meaning)

1. E-R diagrams

2. data-flow diagrams (activity diagrams)
   edges represent (flow of) data

3. inheritance

4. **Statecharts**