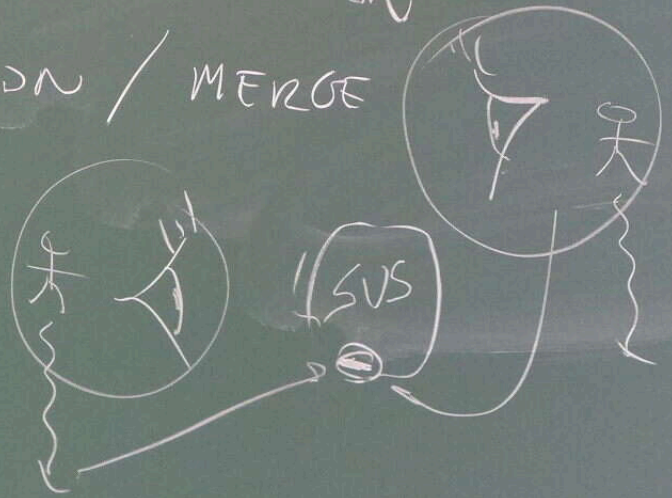
APPROPRIATE FORMALISM

TRANSFORMATION

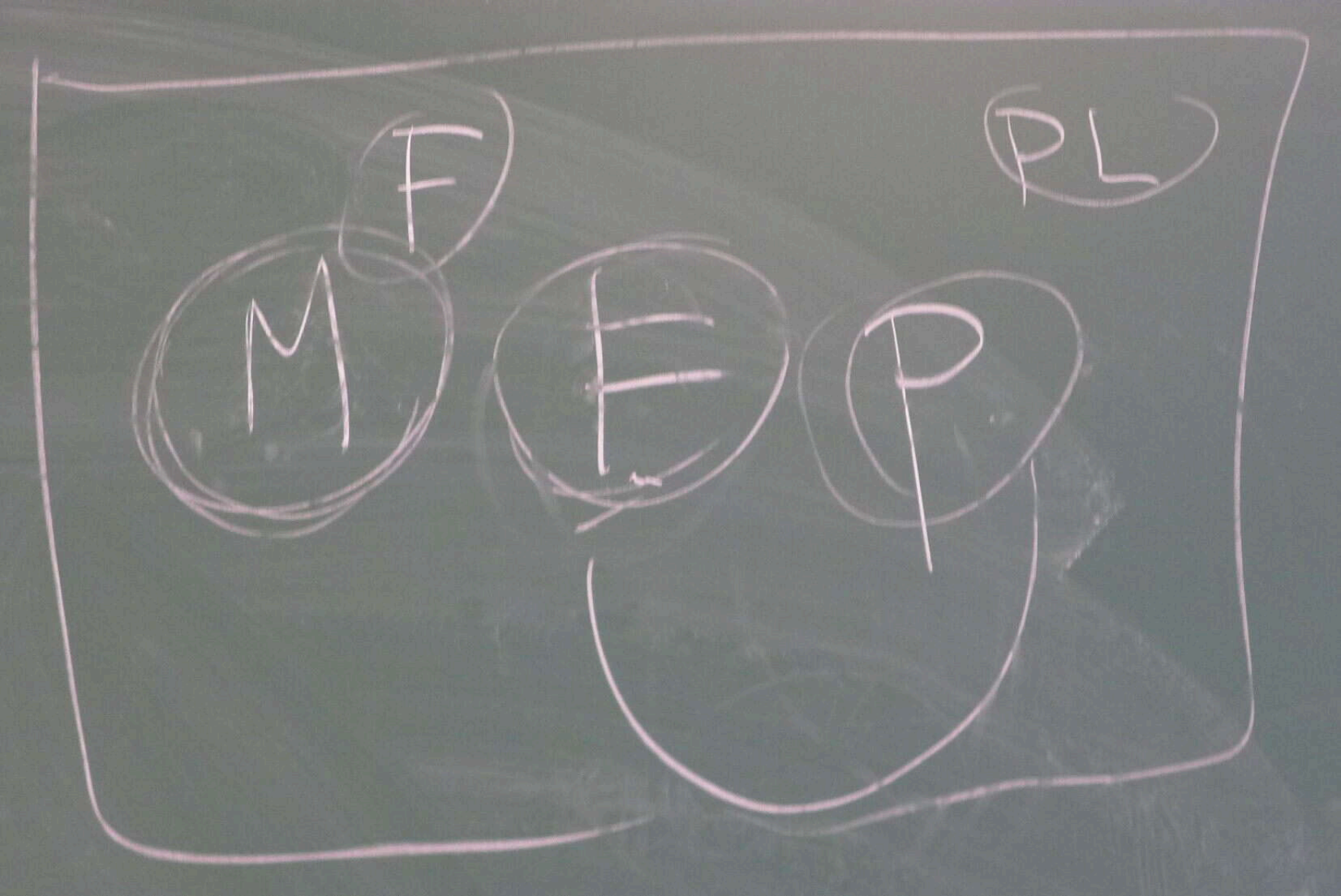
MODULARITY : RE-USE / EVOLUTION

- ABSTRACTION / REFINEMENT
- ARCHITECTURAL (DE-) COMPOSITION
- VIEW (DE-) COMPOSITION / MERGE

WORKFLOW



CONCURRENT ENGINEERING


F
PL
M
E
P

OMG

OBJECT MANAGEMENT GROUP

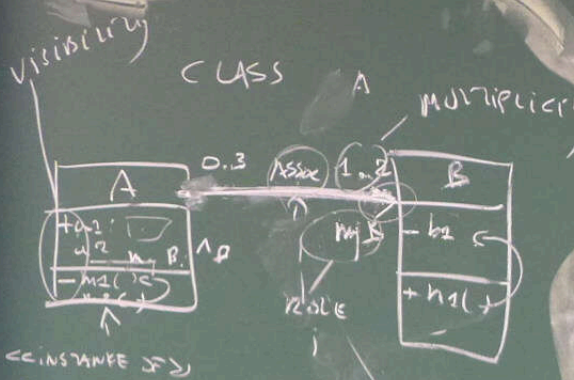
IDL

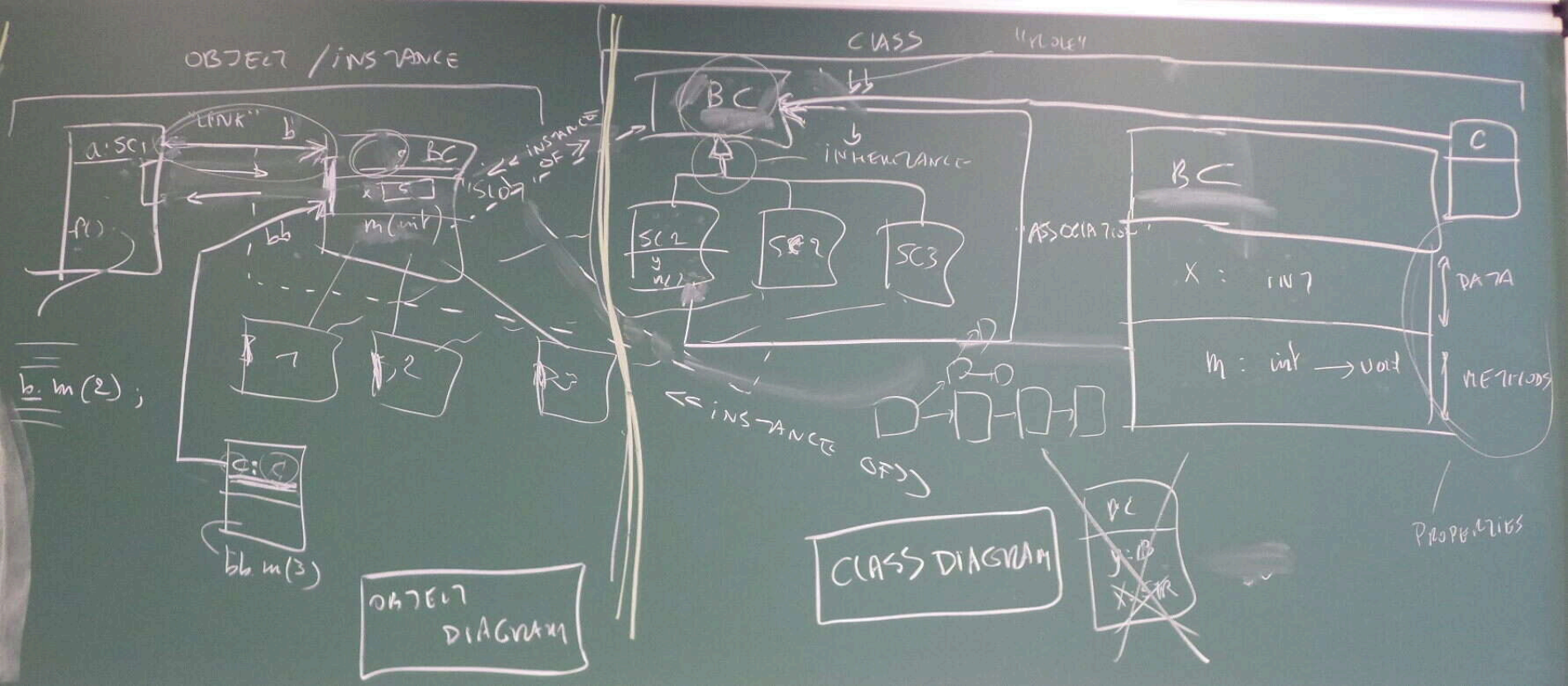
CORBA

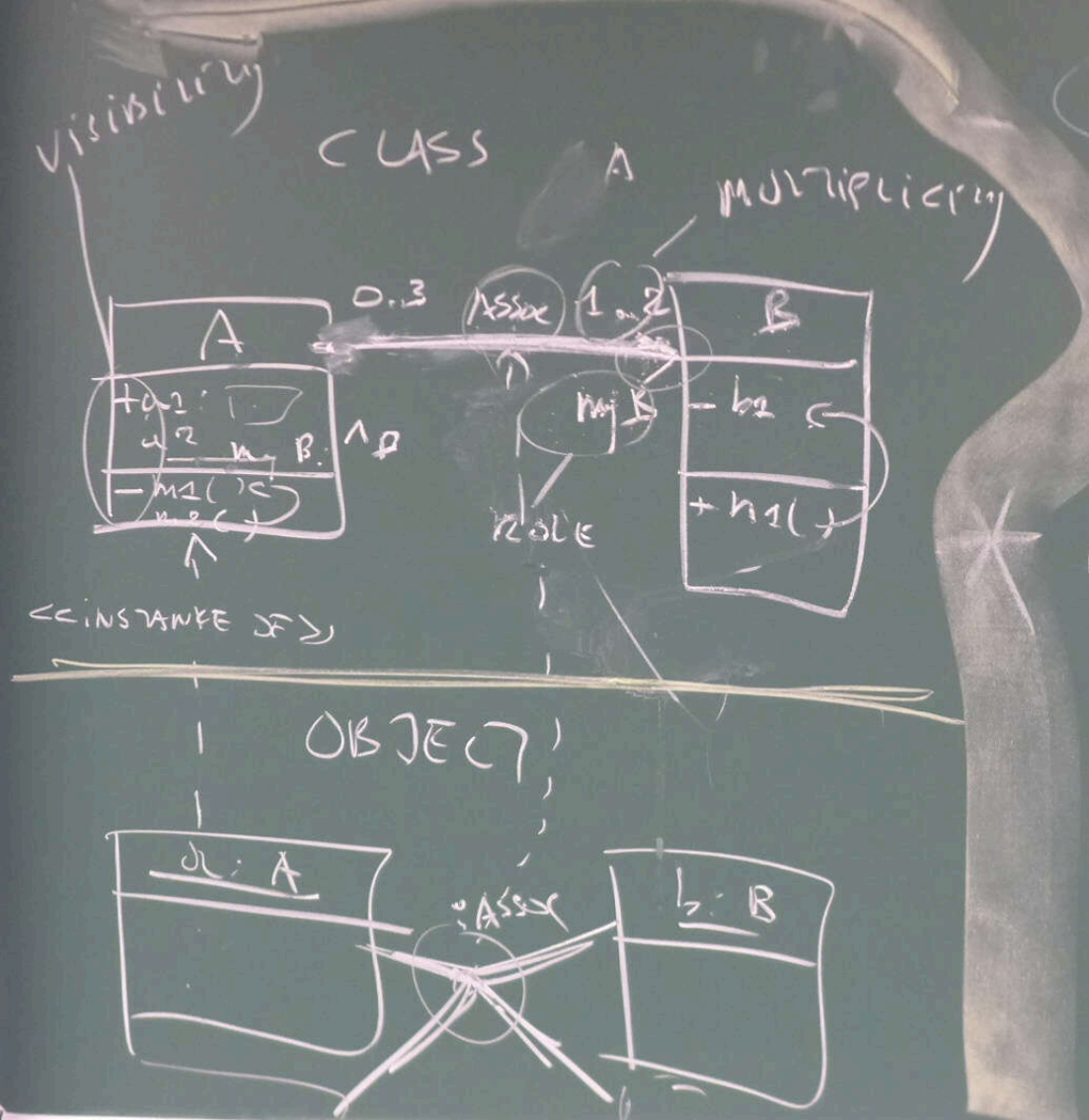
OO:

- OBJECT [- DATA, - METHODS]
- ENCAPSULATION
- CLASSES / CLASSIFY
- INHERITANCE — SUB-TYPING
- POLYMORPHISM = SUBSTITUTABILITY
- MESSAGE PASSING
  - ~ NAVIGATE
  - ~ ASSOCIATIONS / LINKS

OBJECT INTERACTION

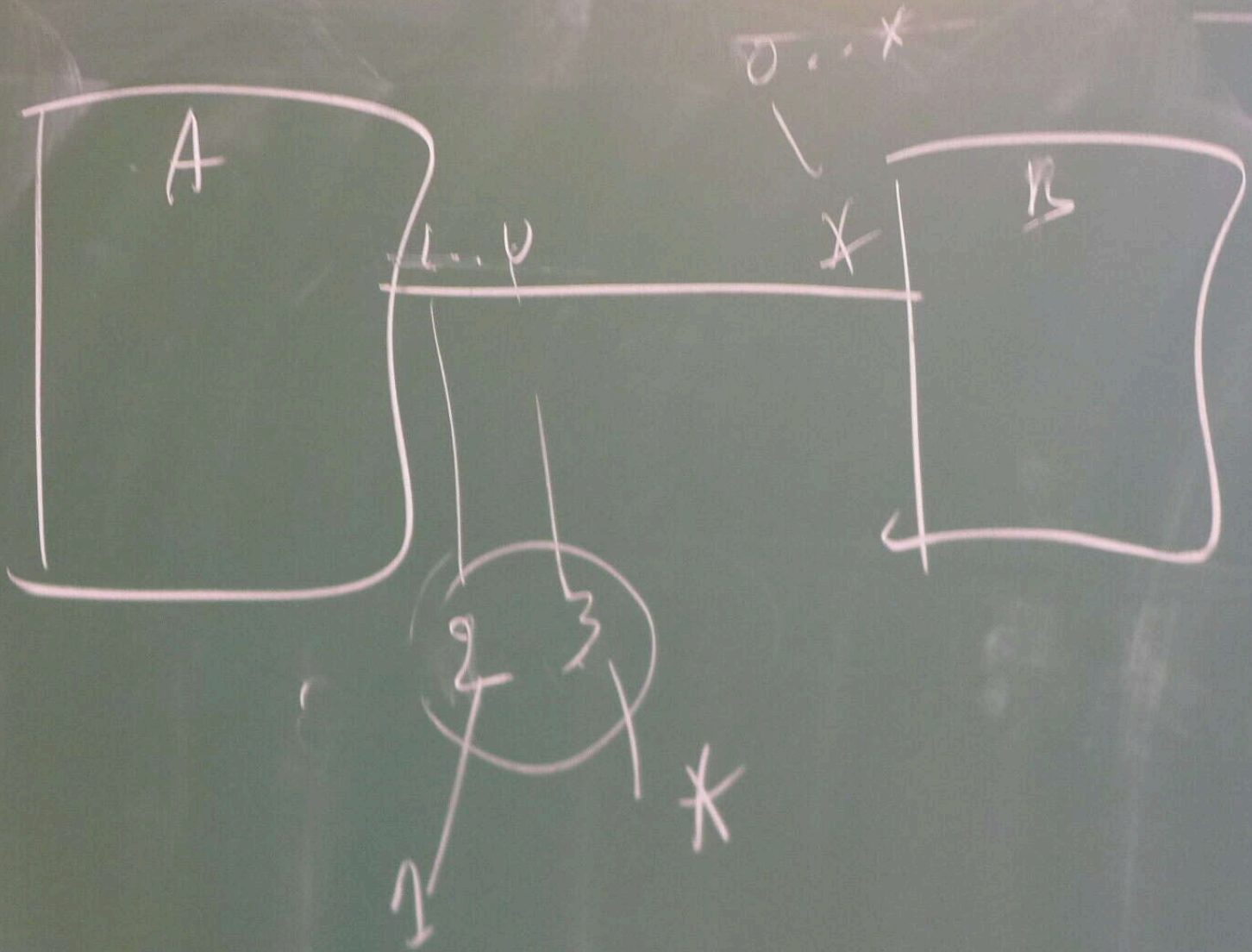



OBJECT / INSTANCE
"LINK"
a : SC1
b
f()
BC
x : 5
m(int)
INSTANCE OF
"SLOT"
bb
b. m(2);
bb. m(3)
c : C
OBJECT DIAGRAM
CLASS
"ROLE"
BC
INHERITANCE
SC2
SC3
ASSOCIATION
INSTANCE OF
C
X : INT
m : int → void
DATA
METHODS
PROPERTIES
CLASS DIAGRAM

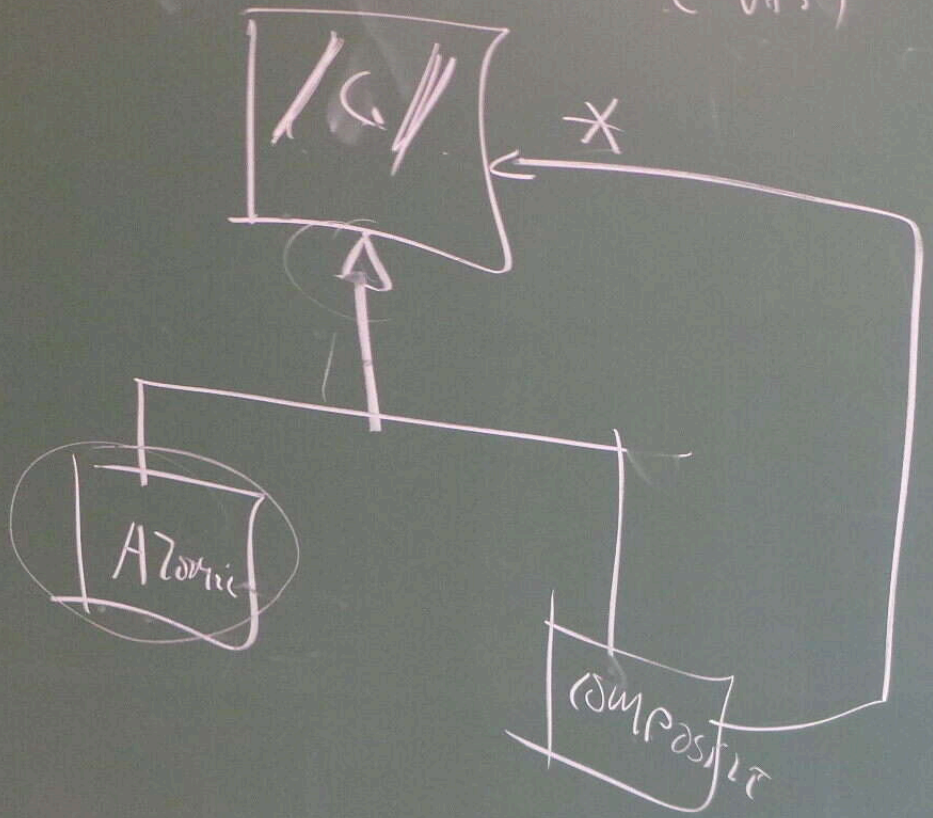

VISIBILITY
CLASS A
MULTIPLICITY
A
0..3
Assoc
1..2
B
+a1 : B
+a2 : B
-m1()
+m2()
myB
-b1
+m1()
ROLE
<<INSTANCE OF>>
OBJECT
a1 : A
:Assoc
b1 : B

- MoSIS
- 7 Model Everything 7

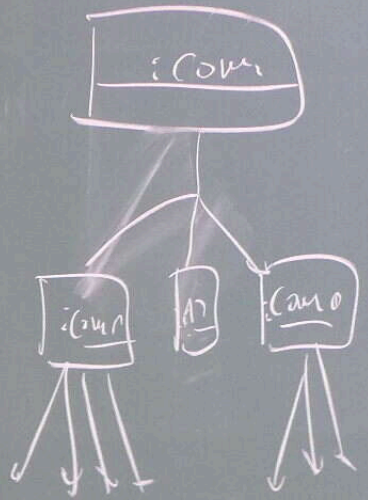
MDE


A
1..1
0..*
B
1
*

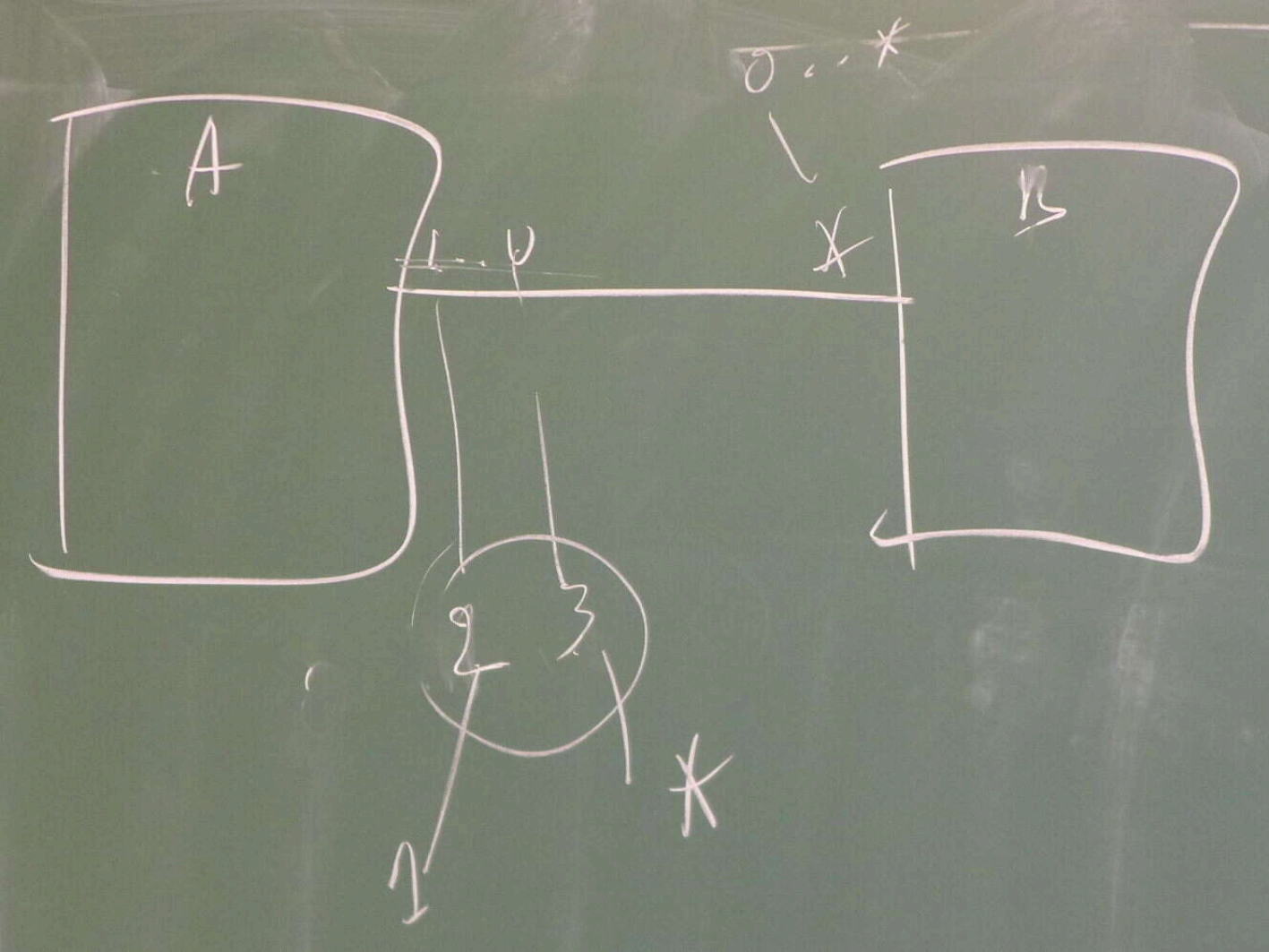
CLASS

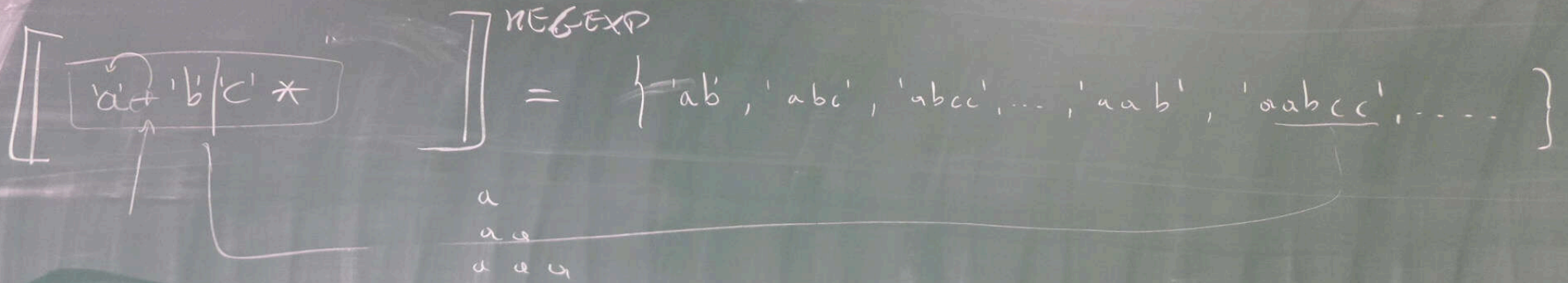


OBJECT



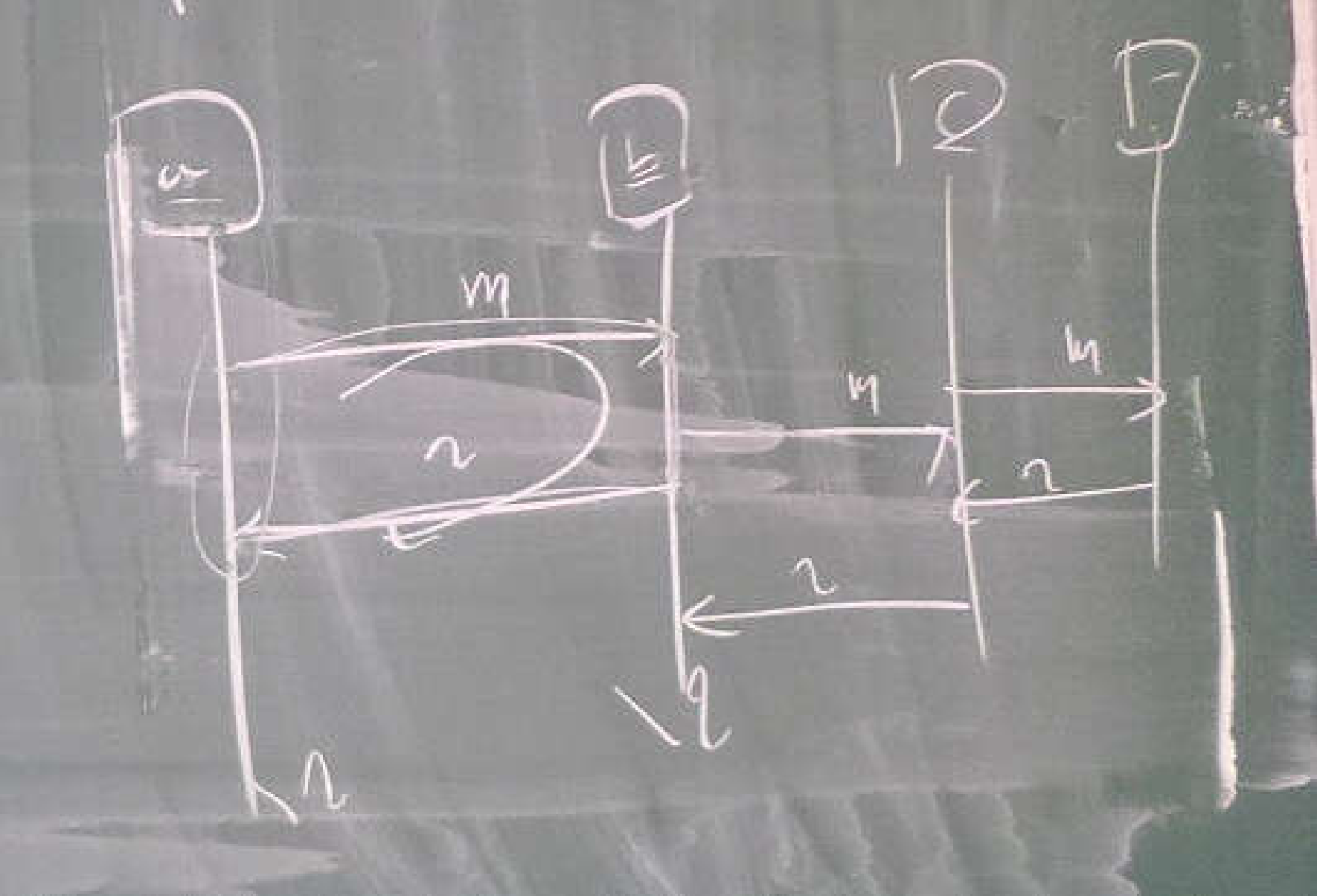
COMPOSITE PATTERN


A
1..1
0..*
B
1
*


'a' + 'b' | 'c' *
REGEXP
= { 'ab', 'abc', 'abcc', ..., 'aab', 'aabcc', ... }
a
a a
a a a

$\{(a+b)\,c\,(d*)\,\backslash 1\,\backslash 2\,\backslash 1\}*$ REGEXP

"$\underbrace{aab}_{\backslash 1}\,c\,\underbrace{d\,d\,d\,d}_{\backslash 2}\,aab\,dddd\,aab$" $\in$ $[\![(a+b)\,c\,(d*)\,\backslash 1\,\backslash 2\,\backslash 1]\!]$ REGEX

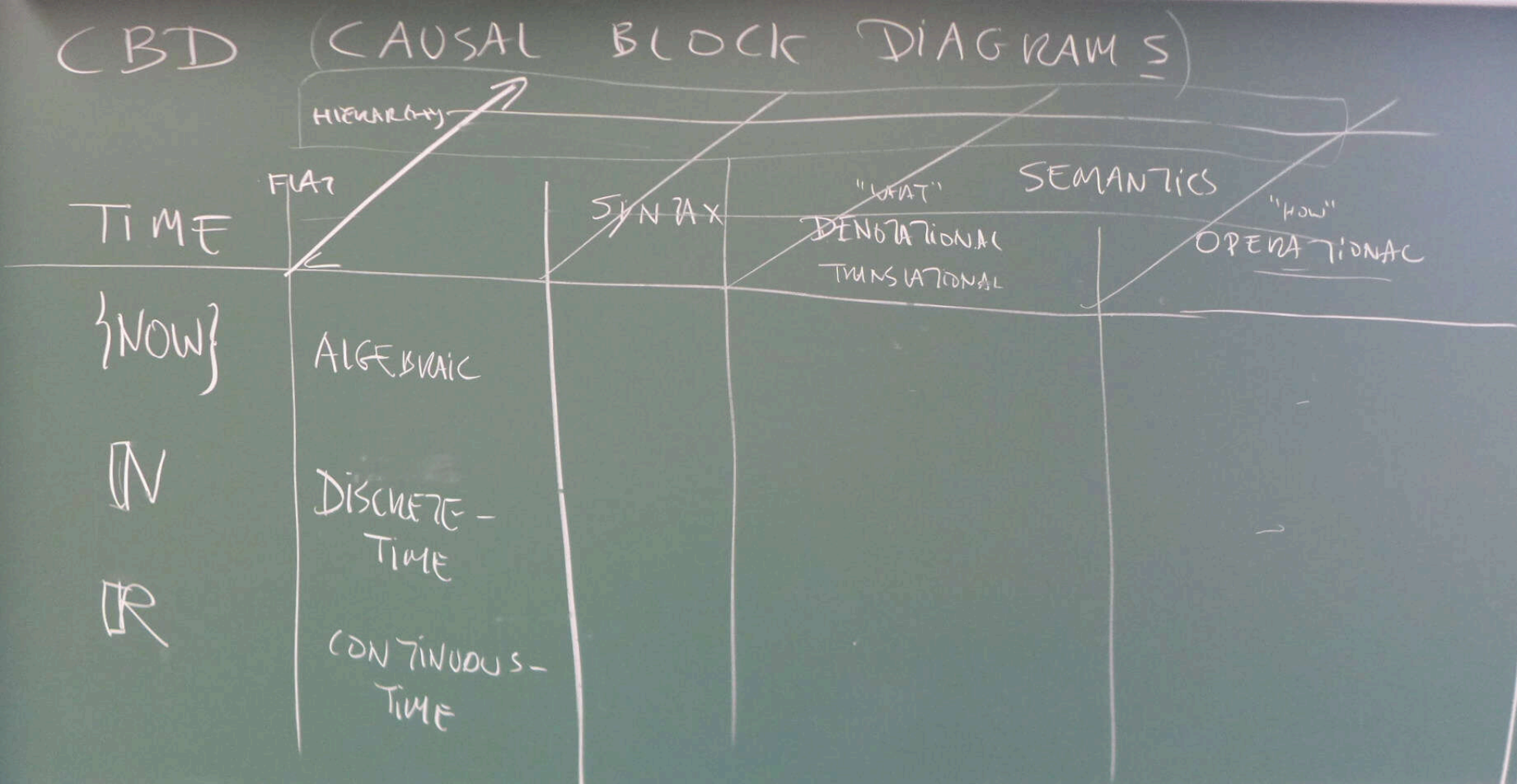


CHECK REGEXP ('a+b c*', 'aabcc')

re = REGEXP.COMPILE('a+b c*')

ws = re.check('aabcc')

CBD (CAUSAL BLOCK DIAGRAMS)

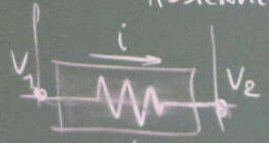


$V_2 := R_i + V_2$

$V_1 := V_2 - R_i$

$i := \frac{V_2 - V_1}{R}$

CAUSALITY ASSIGNMENT



Ohm's Law

$(V_2 - V_1) - R i = 0$

EXAMPLE

(ALG) CBD

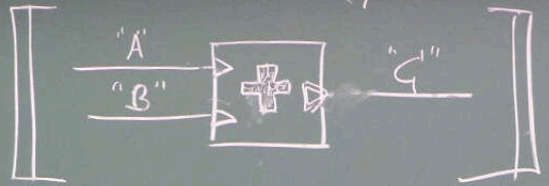
(ALG) ABD (RE-USE)

CAUSAL

CAUSAL

COMPUTATIONAL CAUSALITY

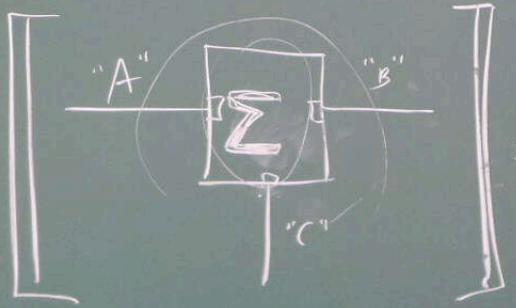
~ SDF (SYNCHRONOUS DATA FLOW)



$: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$

$(a, b) \to c := a + b$

$a, b, c \in \mathbb{R}$



$a, b, c \in \mathbb{R}$

$a + b + c = 0$

DECLARATIVE

IMPLICIT

CONSTRAINT

CBD CAUSAL BLOCK DIAGRAMS

TIME	FLAT	SYNTAX	SEMANTICS "WHAT" DENOTATIONAL TRANSITIONAL	SEMANTICS "HOW" OPERATIONAL
{NOW}	ALGEBRAIC			
ℕ	DISCRETE-TIME			
ℝ	CONTINUOUS-TIME			

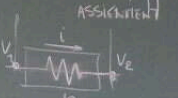
HIERARCHY

$V_2 := R_1 + V_a$

$V_1 := V_2 - R_1$

$i := \frac{V_2 - V_1}{R}$

CAUSALITY ASSIGNMENT



Ohm's Law

$(V_2 - V_1) - R\,i = 0$

EXAMPLE

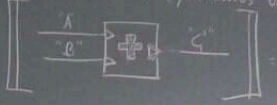
(ALG) CBD

(ALG) ABD (RE-USE)

CAUSAL

COMPUTATIONAL CAUSALITY

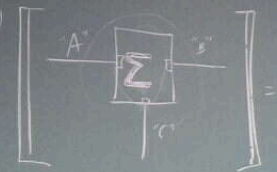
USDF (SYNCHRONOUS DATA FLOW)



$+ : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$

$(a, b) \to \text{result}$

$a, b, c \in \mathbb{R}$



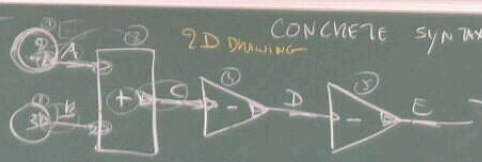
$a, b, c \in \mathbb{R}$

$a + b + c = 0$

DECLARATIVE IMPLICIT CONSTRAINT

CONCRETE SYNTAX

2D DRAWING

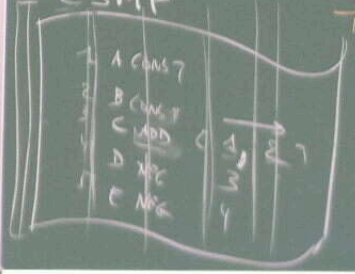


TOPOLOGICAL ≠ GEOMETRIC

PARSE

PRETTY-PRINT RENDER

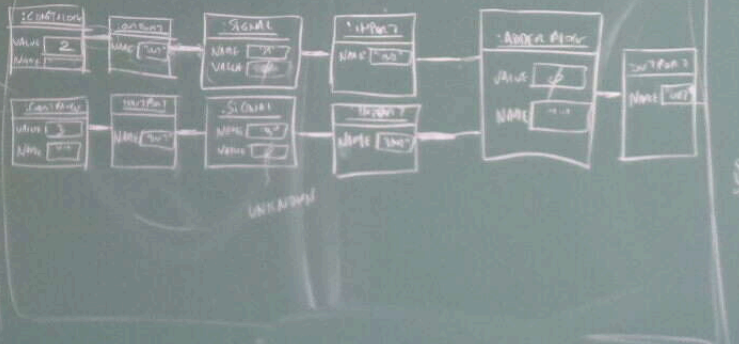
CSMP



7X7

ABSTRACT SYNTAX

OBJECT DIAGRAM



UNKNOWN

SEMANTICS

$a, b, c, d, e \in \mathbb{R}$

SET OF OPS.

SET OF ALG EQNS

$a := 2$

$b := 3$

$c := a + b$

$d := -c$

$e := -d$

$b := 2$

$d := -c$

$e := d$

$c := a + b$

$a := 2$

SET

SEQ

ALL VALUES

$a, b, c, d, e \in \mathbb{R}$

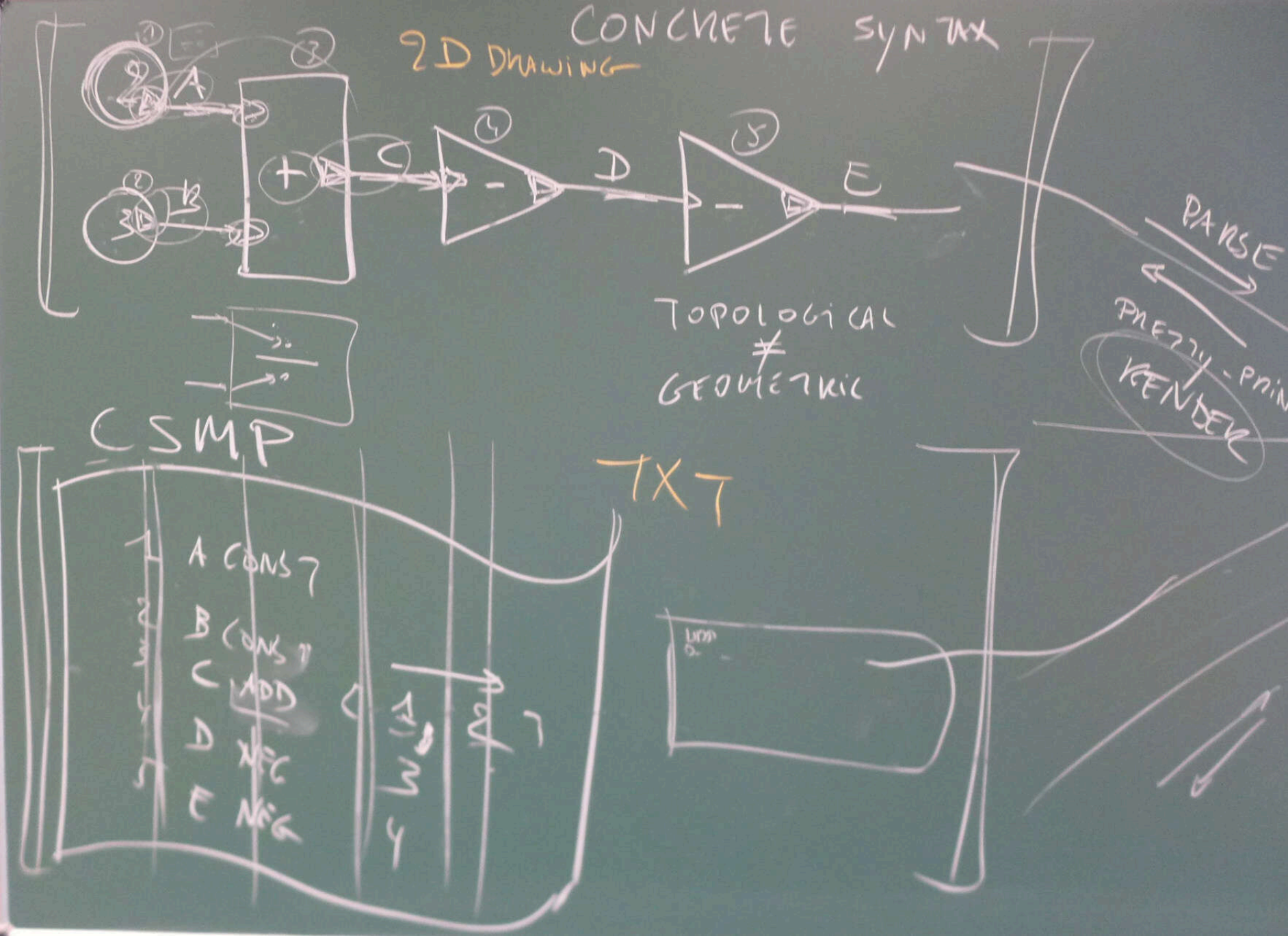
$(2, 3, 5, -5, 5)$

UNDERSPECIFIED

HOW?

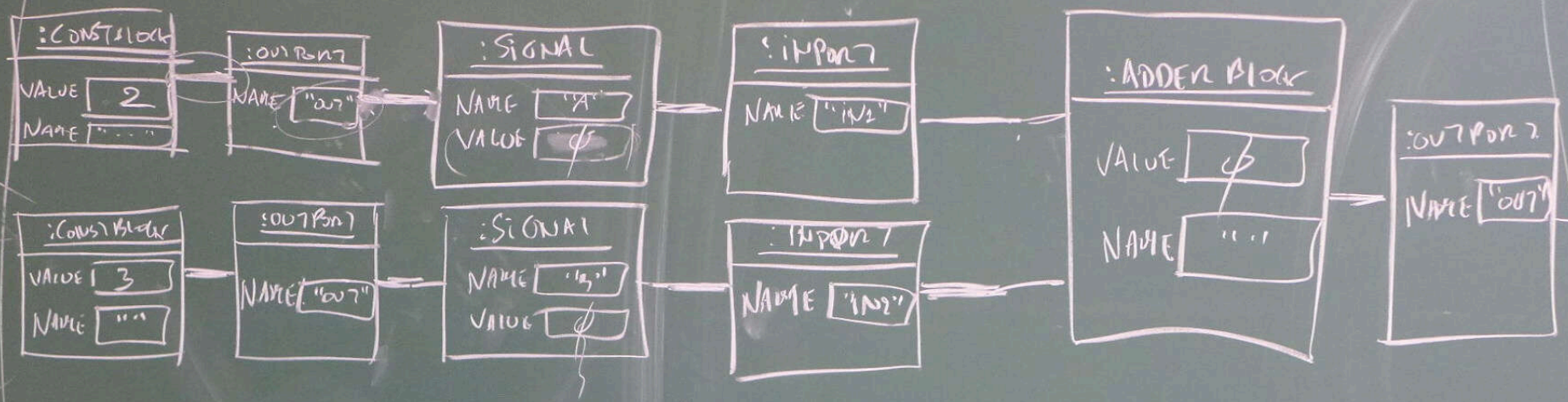
OVERSPECIFIED

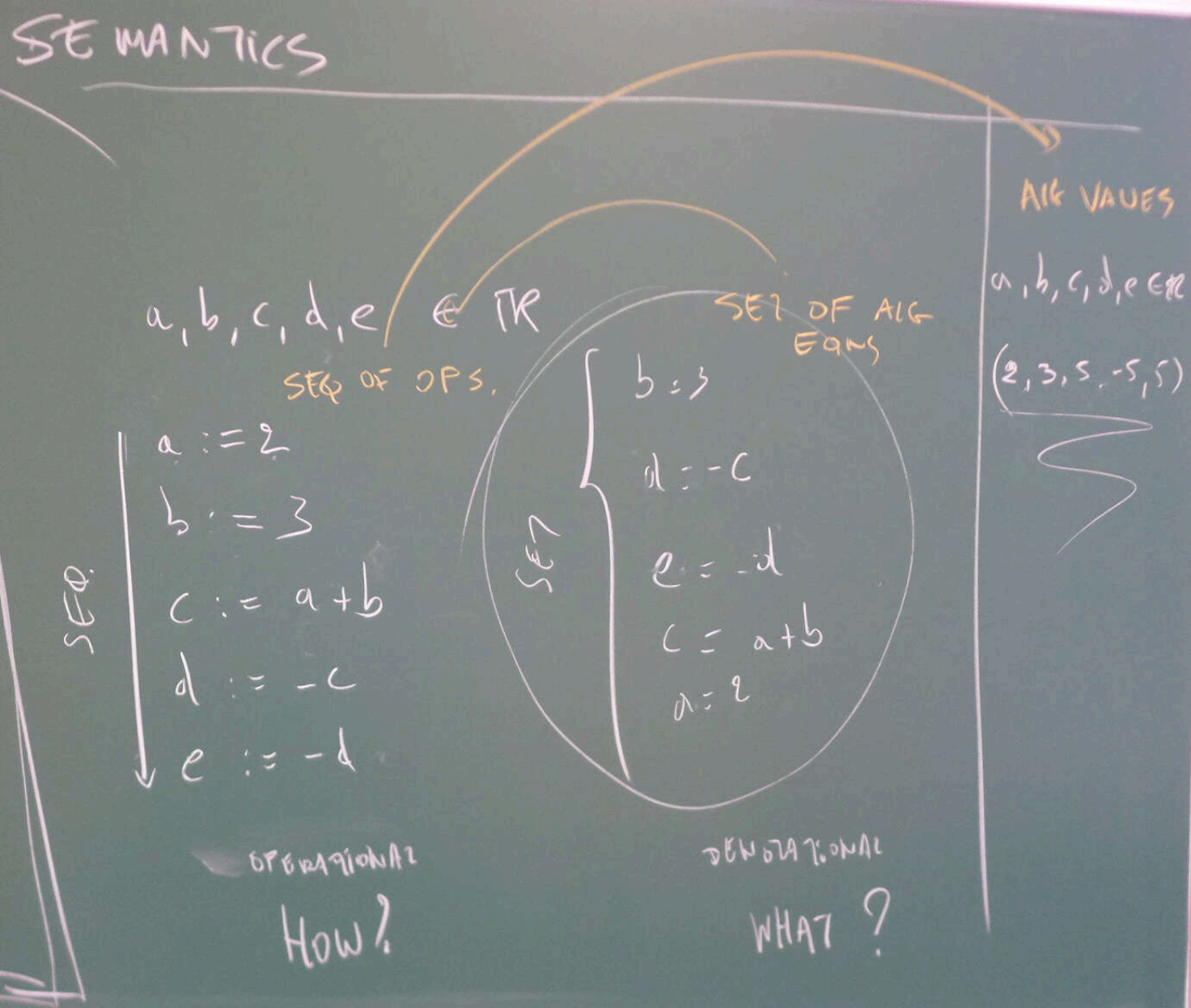
WHAT?


CONCRETE SYNTAX
2D DRAWING
A
B
C
D
E
TOPOLOGICAL
≠
GEOMETRIC
PARSE
PRETTY-PRINT
RENDER
CSMP
TXT
A CONST
B CONST
C ADD
D NEG
E NEG

ABSTRACT SYNTAX

OBJECT DIAGRAM

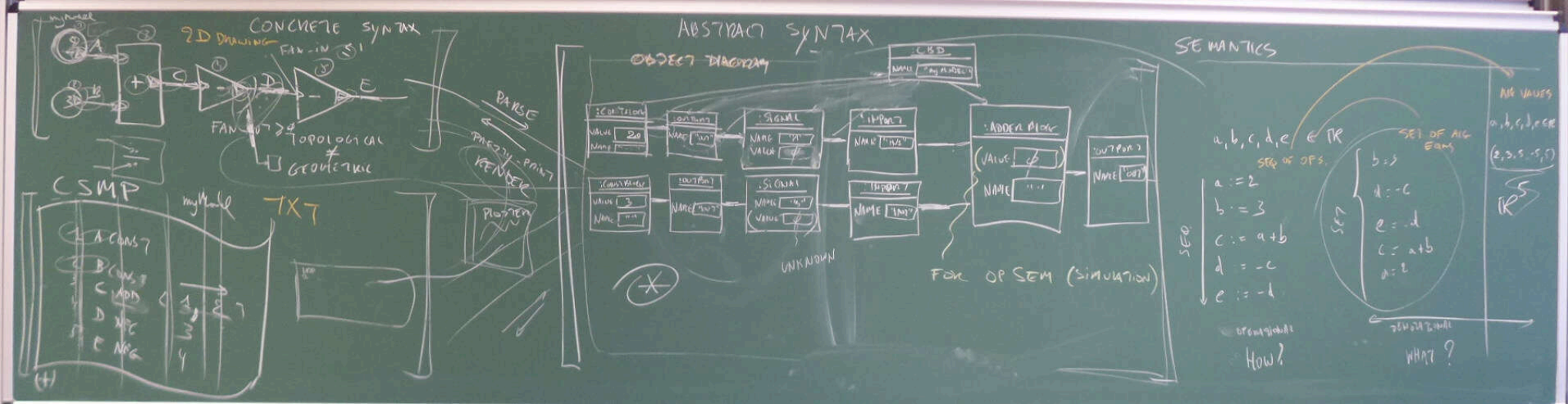


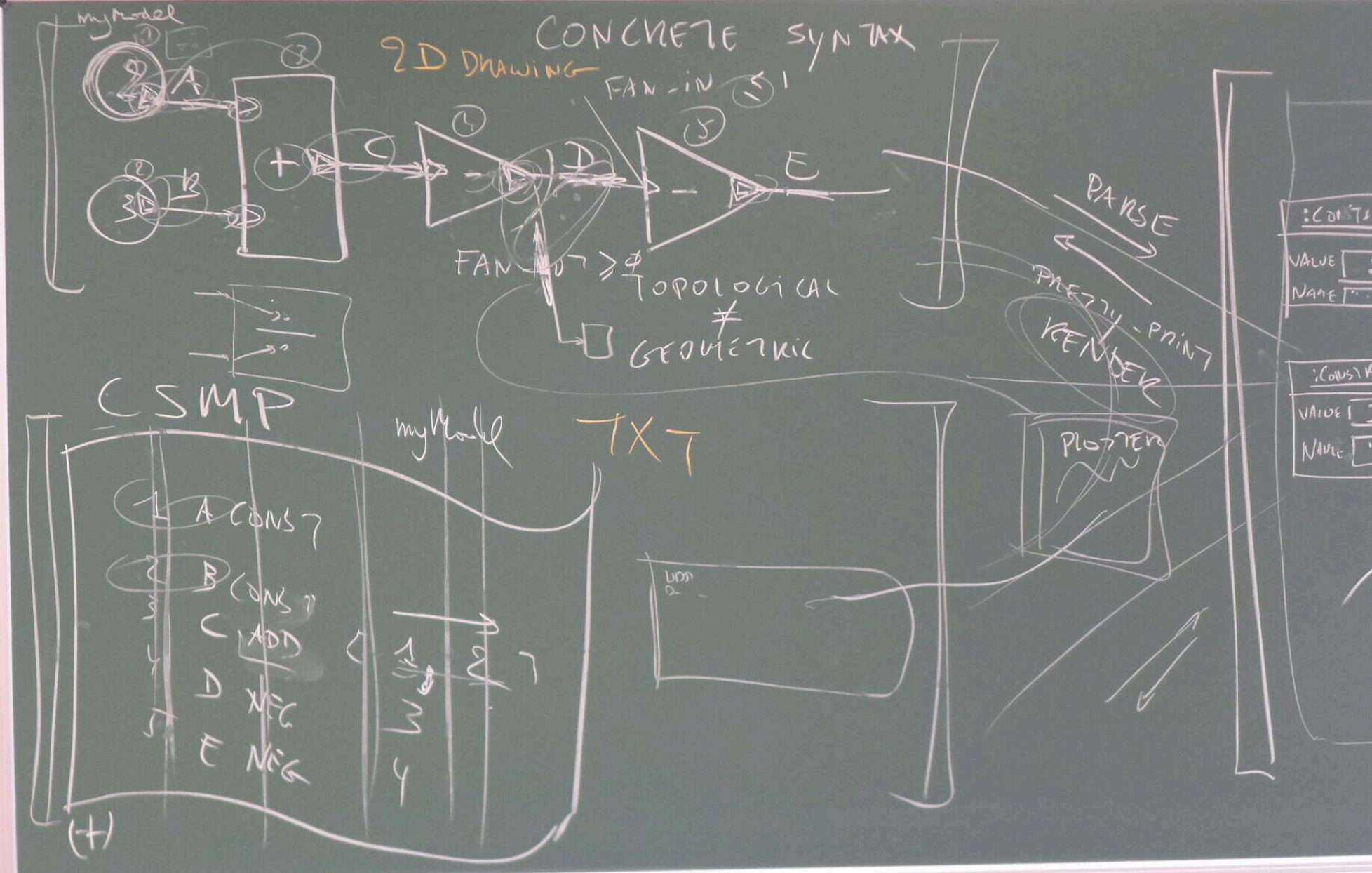

SEMANTICS
ALG VALUES
a, b, c, d, e ∈ ℝ
a, b, c, d, e ∈ ℝ
(2, 3, 5, -5, 5)
SEQ OF OPS.
SEQ
a := 2
b := 3
c := a + b
d := -c
e := -d
SET OF ALG EQNS
SET
b := 3
d := -c
e := -d
c := a + b
a := 2
OPERATIONAL
HOW?
DENOTATIONAL
WHAT ?

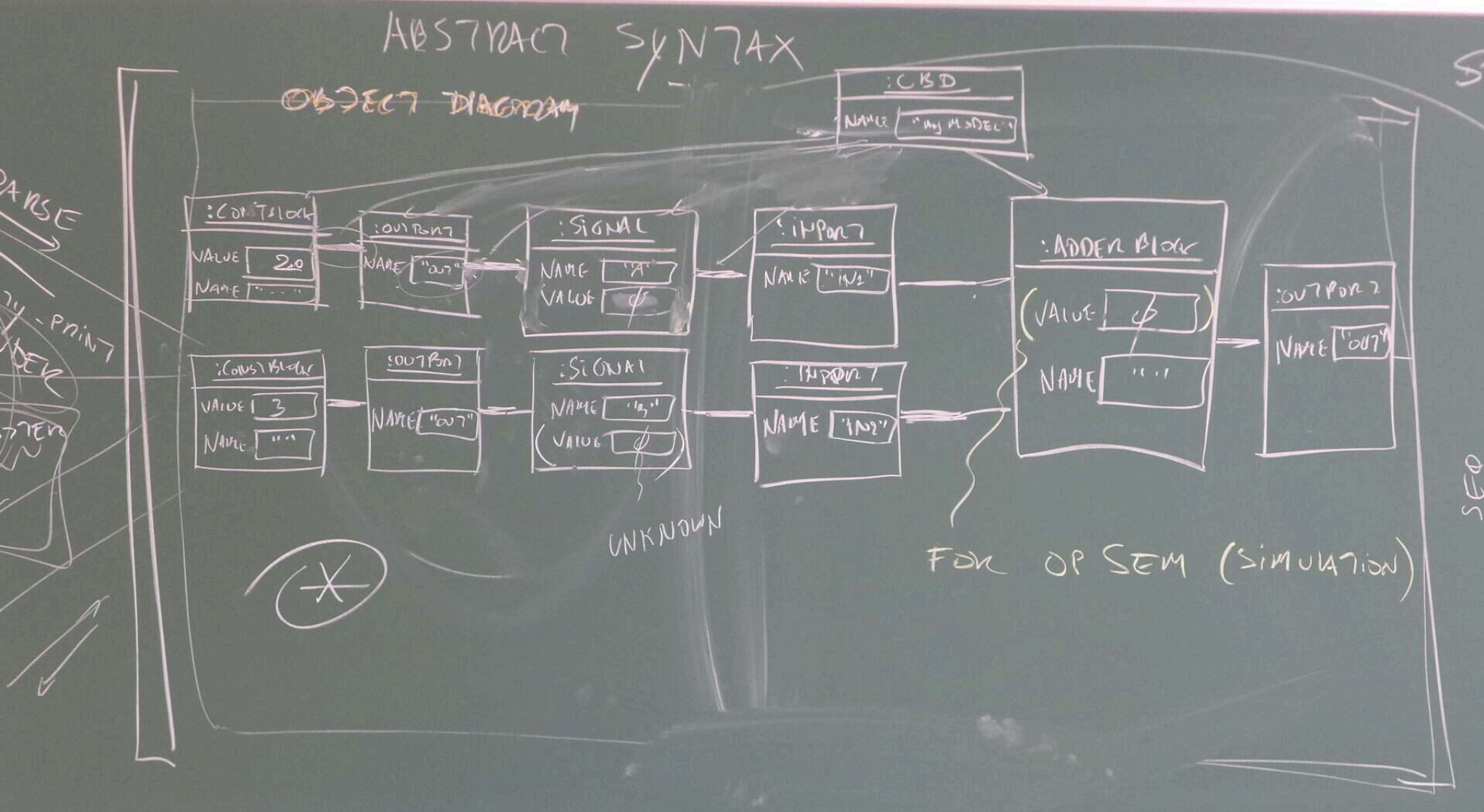
CBD (CAUSAL BLOCK DIAGRAMS)

HIERARCHY

TIME	FLAT	SYNTAX	SEMANTICS "WHAT" DENOTATIONAL TRANSLATIONAL	SEMANTICS "HOW" OPERA
{NOW}	ALGEBRAIC			
ℕ	DISCRETE-TIME			
ℝ	CONTINUOUS-TIME			


CONCRETE SYNTAX
2D DRAWING
FAN-IN
FAN-OUT
TOPOLOGICAL
GEOMETRIC
CSMP
myModel
7X7
PARSE
PRETTY-PRINT
RENDER
PLOTTER
ABSTRACT SYNTAX
OBJECT DIAGRAM
:CONSTANT
:OUTPORT
:SIGNAL
:IMPORT
:ADDER BLOCK
:CBD
UNKNOWN
FOR OP SEM (SIMULATION)
SEMANTICS
a, b, c, d, e ∈ ℝ
SEQ OF OPS.
a := 2
b := 3
c := a+b
d := -c
e := -d
SEQ
SET OF ALG EQNS
SET
b := 3
d := -c
e := d
c := a+b
a := 2
ALG VALUES
(2, 3, 5, -5, -5)
ℝ^5
ORDERABLE
HOW?
WHAT?


myModel
A
B
C
D
E
CONCRETE SYNTAX
2D DRAWING
FAN-IN
FAN-OUT
TOPOLOGICAL
GEOMETRIC
PARSE
PRETTY-PRINT
RENDER
PLOTTER
CSMP
myModel
TXT
A CONST
B CONST
C ADD
D NEG
E NEG
(+)
:CONST
VALUE
NAME


ABSTRACT SYNTAX
OBJECT DIAGRAM
:CBD
NAME "MY MODEL"
:CONSTBLOCK
VALUE 2.0
NAME "..."
:OUTPORT
NAME "OUT"
:SIGNAL
NAME "A"
VALUE
:INPORT
NAME "IN1"
:ADDER BLOCK
VALUE
NAME "..."
:OUTPORT
NAME "OUT"
:CONSTBLOCK
VALUE 3
NAME "..."
:OUTPORT
NAME "OUT"
:SIGNAL
NAME "B"
VALUE
:INPORT
NAME "IN2"
UNKNOWN
FOR OP SEM (SIMULATION)

SEMANTICS

$a, b, c, d, e \in \mathbb{R}$

SEQ OF OPS.

SEQ.

$a := 2$

$b := 3$

$c := a + b$

$d := -c$

$e := -d$

OPERATIONAL

HOW?

SET OF ALG EQNS

SET

$b = 3$

$d = -c$

$e = -d$

$c = a + b$

$a = 2$

DENOTATIONAL

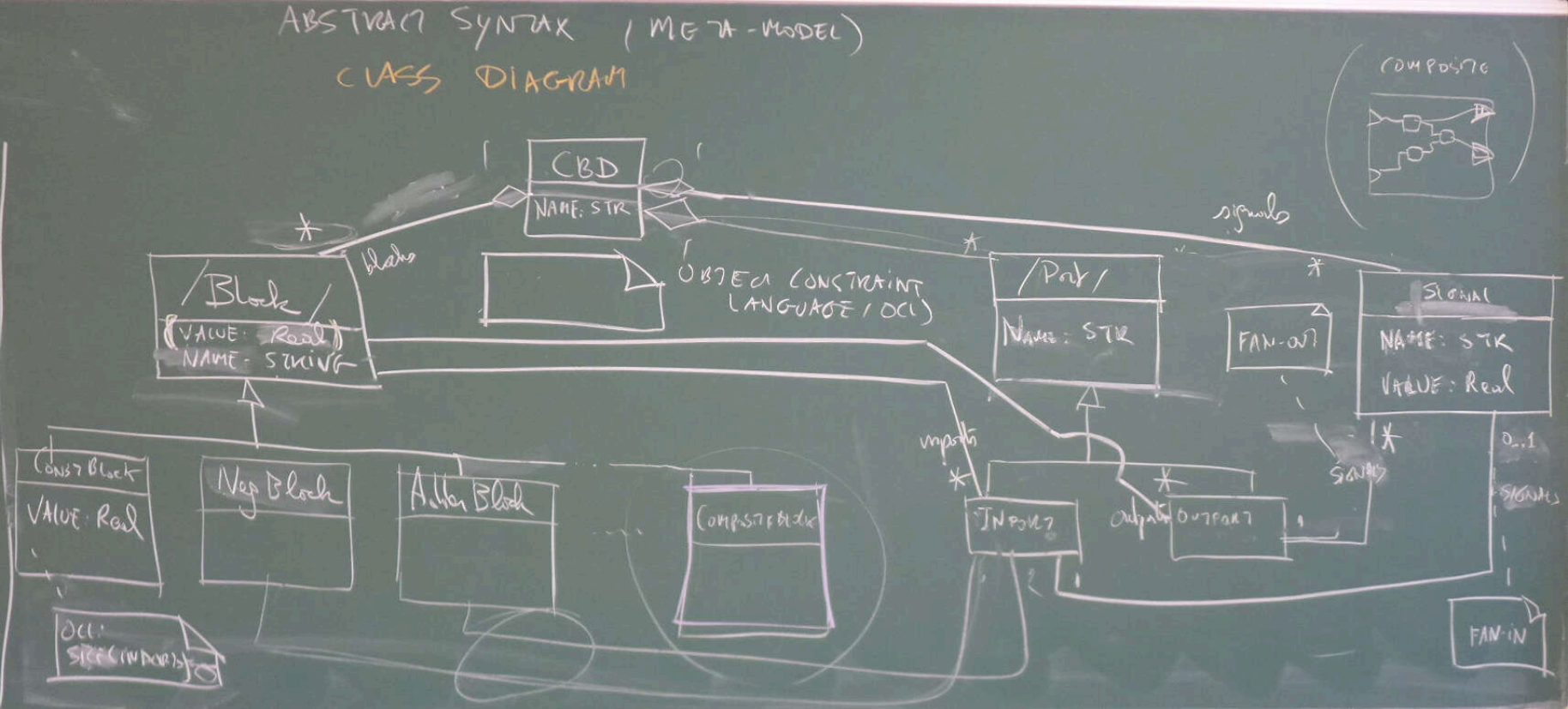
WHAT ?

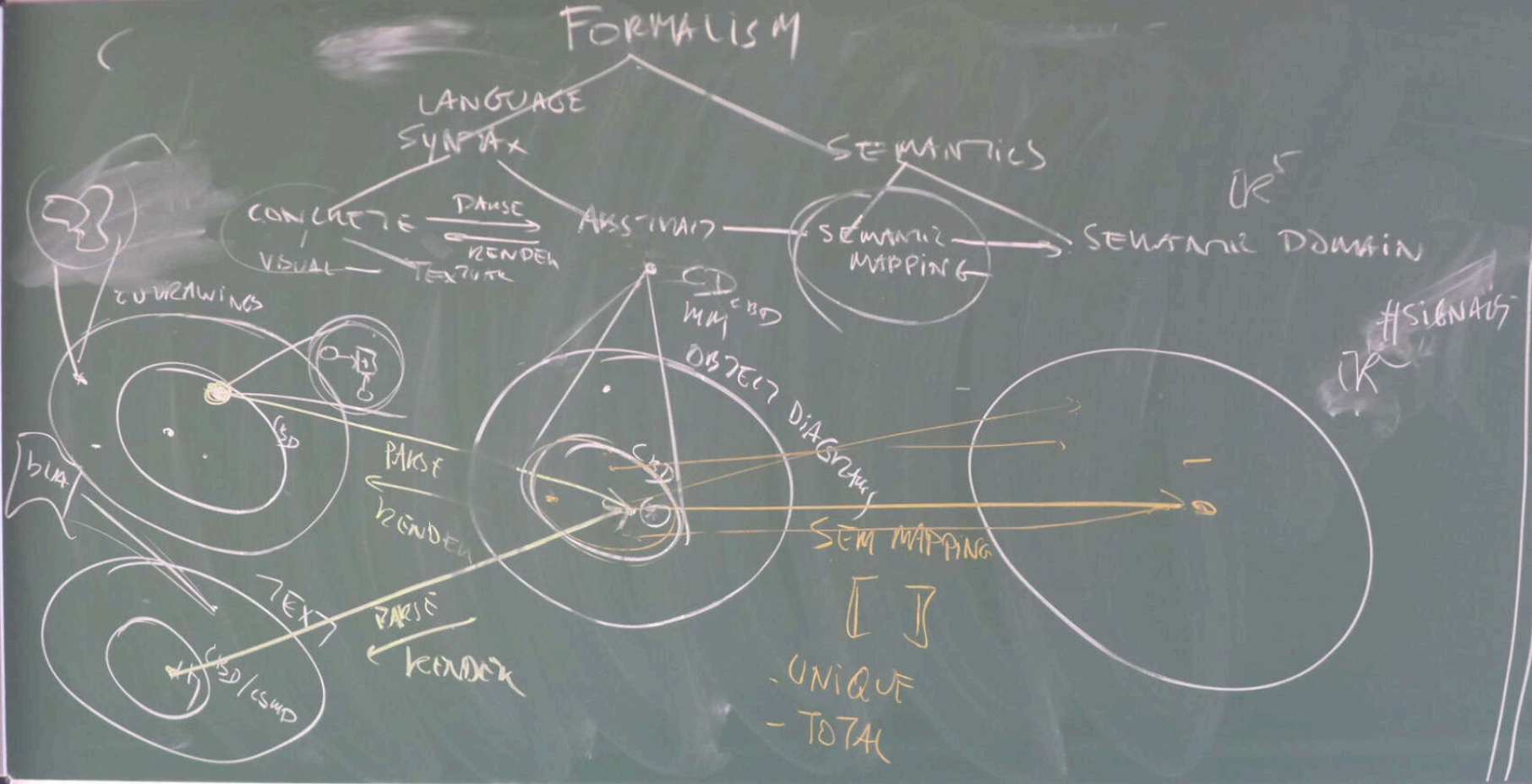
ALG VALUES

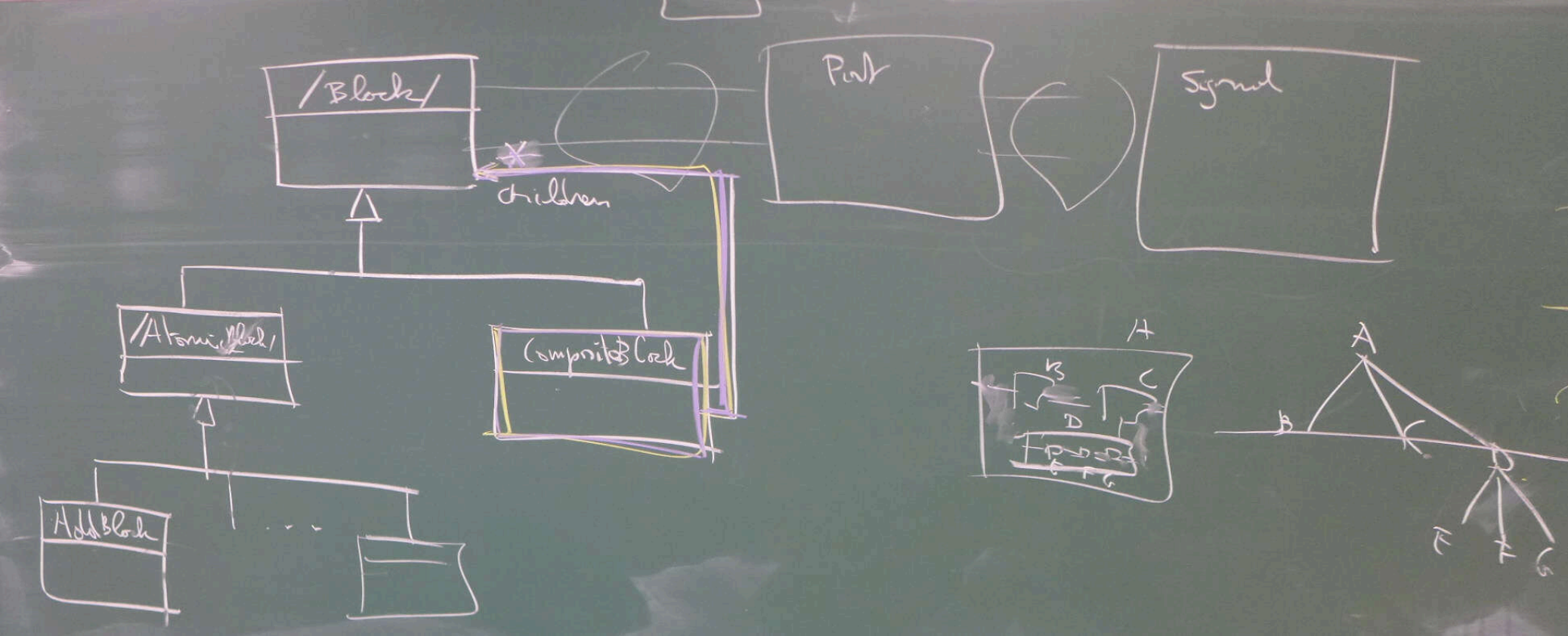
$a, b, c, d, e \in \mathbb{R}$

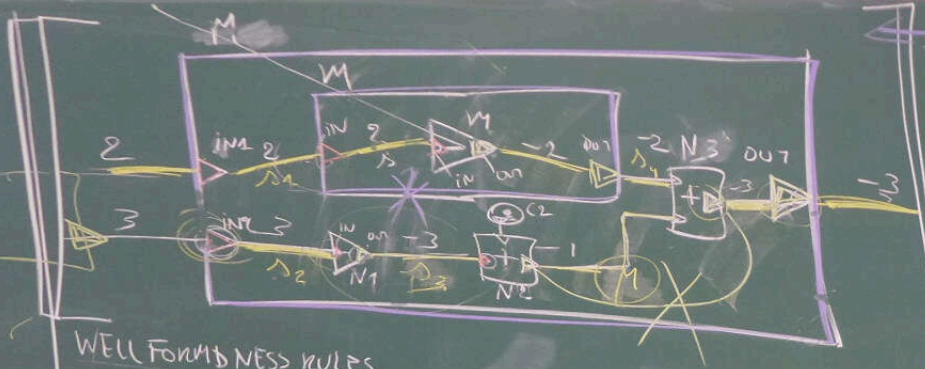
$(2, 3, 5, -5, 5)$

$\mathbb{R}^5$


ABSTRACT SYNTAX (META-MODEL)
CLASS DIAGRAM
COMPOSITE
CBD
NAME: STR
blocks
signals
/Block/
VALUE: Real
NAME: STRING
OBJECT CONSTRAINT LANGUAGE (OCL)
/Port/
NAME: STR
FAN-OUT
SIGNAL
NAME: STR
VALUE: Real
Const Block
VALUE: Real
Neg Block
Adder Block
Composite Block
inports
INPORT
outports
OUTPORT
SIGNALS
0..1
SIGNALS
OCL: SPECIFICATIONS
FAN-IN


FORMALISM
LANGUAGE
SYNTAX
SEMANTICS
CONCRETE
VISUAL
TEXTUAL
PARSE
RENDER
ABSTRACT
SEMANTIC MAPPING
SEMANTIC DOMAIN
$\mathbb{R}^5$
#SIGNALS
CD
MM
OBJECT DIAGRAMS
SEM MAPPING
[ ]
- UNIQUE
- TOTAL
PARSE
RENDER
BLA
TEX
CSD/CSMD


/Block/
children
Port
Signal
/AtomicBlock/
CompositeBlock
AddBlock
A
B
C
D
E
F
G



WELL FORMEDNESS RULES

UNIQUE NAMES AT EACH LEVEL

I GLOBALLY UNIQUE NAMES

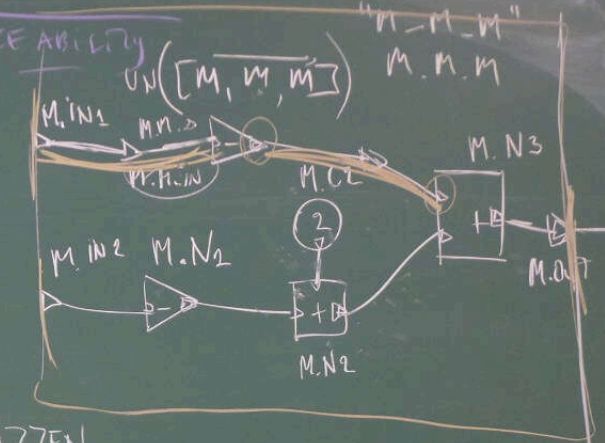
II DIRECT CONNECT

TRACEABILITY

$UN([M, M', M''])$

"M_M_M"

M.M.M



FLATTEN

TRANSFORMATIONAL / DENOTATIONAL

OPERATIONAL

CBD

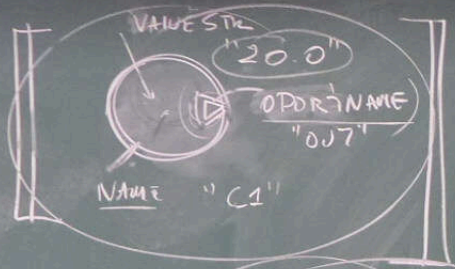
CBD

HIERARCHY

FLAT

SEMANTICS

	TIME	SYNTAX	DENOT.	≈ OPER
AG	{NOW}		✓	X
DT	IN			
CT	IR			



=

$\mathbb{R}$

$\downarrow$

$VAR(NAME) = VALUE\ (VALUESTR)$

$20 \in \mathbb{R}$

C1

20.0

$C_1.value,\ C_1.out \in \mathbb{R}$

$\begin{cases} C1.value = 20.0 \\ C1.out = C1.value \end{cases}$

C1_value

$C1.value = 20,\ C1 \in \mathbb{R}$

$20 \in \mathbb{R}$

$VAR(OPOR7NAME) = C1\_value$

"OUT"

$C1\_out \in \mathbb{R}$

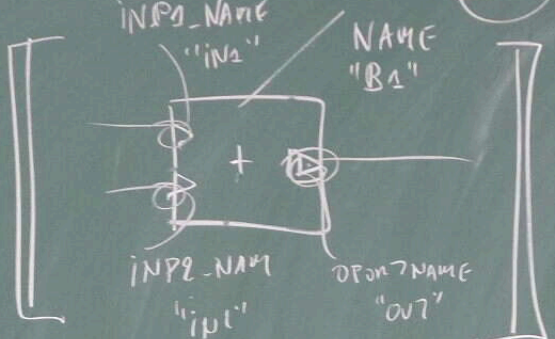
VAR(

A

"3"

$a \in \mathbb{R}$

$a = 3$



$b1\_out = b1\_in1 + b1\_in2$

$+ : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$

VAR(NAME), VAR([NAME, INP1_NAME]),

$b1\_in1 \in \mathbb{R}$

b1 exit

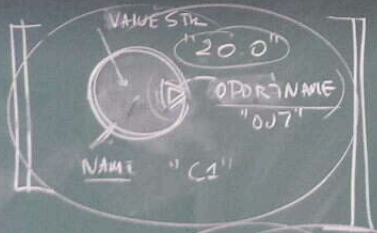
VAR([NAME, INP2_NAME]),

VAR([NAME, OPOR7NAME])

$b1\_in2 \in \mathbb{R}$

$b1\_out \in \mathbb{R}$

VAR([NAME, OPOR7NAME]) = VAR([NAME, INP1_NAME]) + VAR([NAME, INP2_NAME])



=

$\mathbb{R}$

$\vee$

$VAR(NAME) = VALUE(VALUESTR)$

$c1\_value \qquad 20 \in \mathbb{R}$

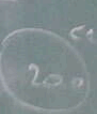
$c1\_value = 20, \quad c1 \in \mathbb{R}$

$20 \in \mathbb{R}$

$VAR(OPDR7NAME) = c1\_value$

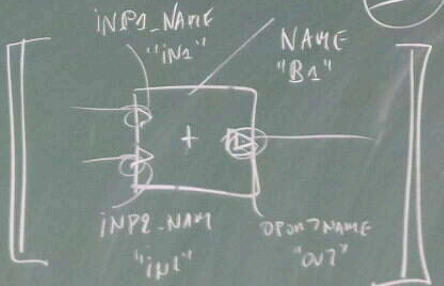
"OU7"

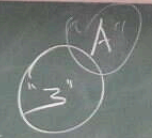
$c1\_out \in \mathbb{R}$



$c_1\_value, c_1\_out \in \mathbb{R}$

$\begin{cases} c1\_value = 20.0 \\ c1\_out = c1\_value \end{cases}$





$a \in \mathbb{R}$

$a = 3$

$b1\_out = b1\_in1 + b1\_in2$

$f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$

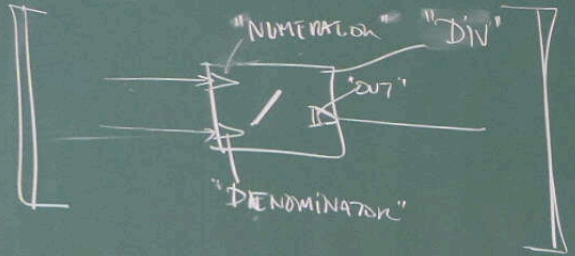
$VAR(NAME), VAR([NAME, INP1\_NAME]),$

$b1\_in1 \in \mathbb{R}$

$b1\_in2 \in \mathbb{R} \qquad VAR([NAME, INP2\_NAME]),$

$b1\_out \in \mathbb{R} \qquad VAR([NAME, OPDR7NAME])$

$VAR([NAME, OPDR7NAME]) = VAR([NAME, INP1\_NAME]) + VAR([NAME, INP2\_NAME])$



$= \begin{cases} div\_out = div\_num \,/\, div\_den & \text{if } div\_den \neq 0 \\ div\_out = ERR & \text{if } div\_den = 0 \end{cases}$

$div\_out \in \mathbb{R} \cup \{ERR\}$, $div\_num \in \mathbb{R} \cup \{ERR\}$, $div\_den \in \mathbb{R} \setminus \{0\}$

$\in \mathbb{R} \cup \{ERR\}$

I

II

STATIC CHECK ✗

DYNAMIC CHECK

$\mathbb{R} \cup \{ERR\}$

++ = PROPAGATE ERR

$\begin{cases} \text{if } input \neq ERR \\ \text{if } input = ERR \end{cases}$

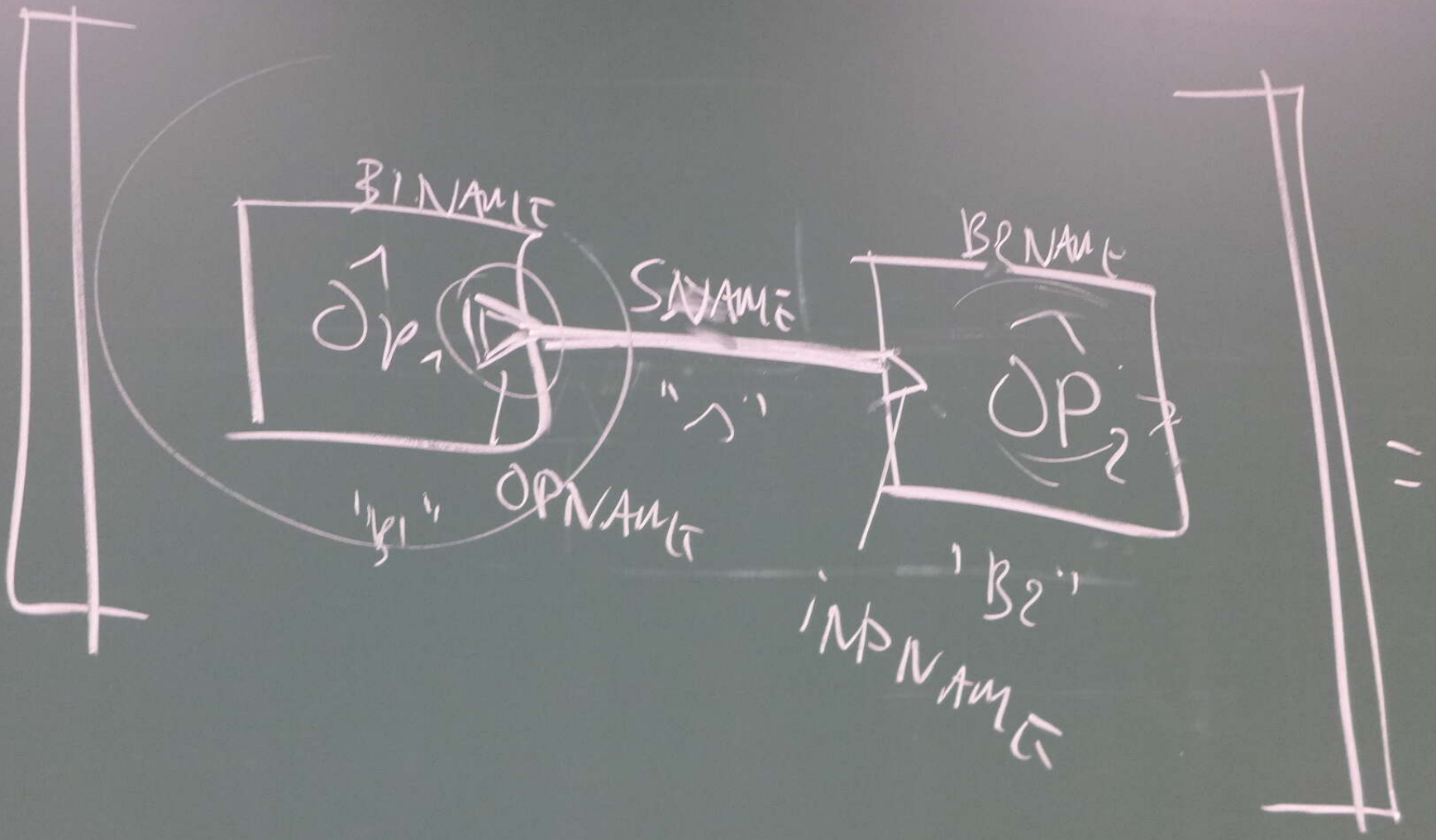
CBD

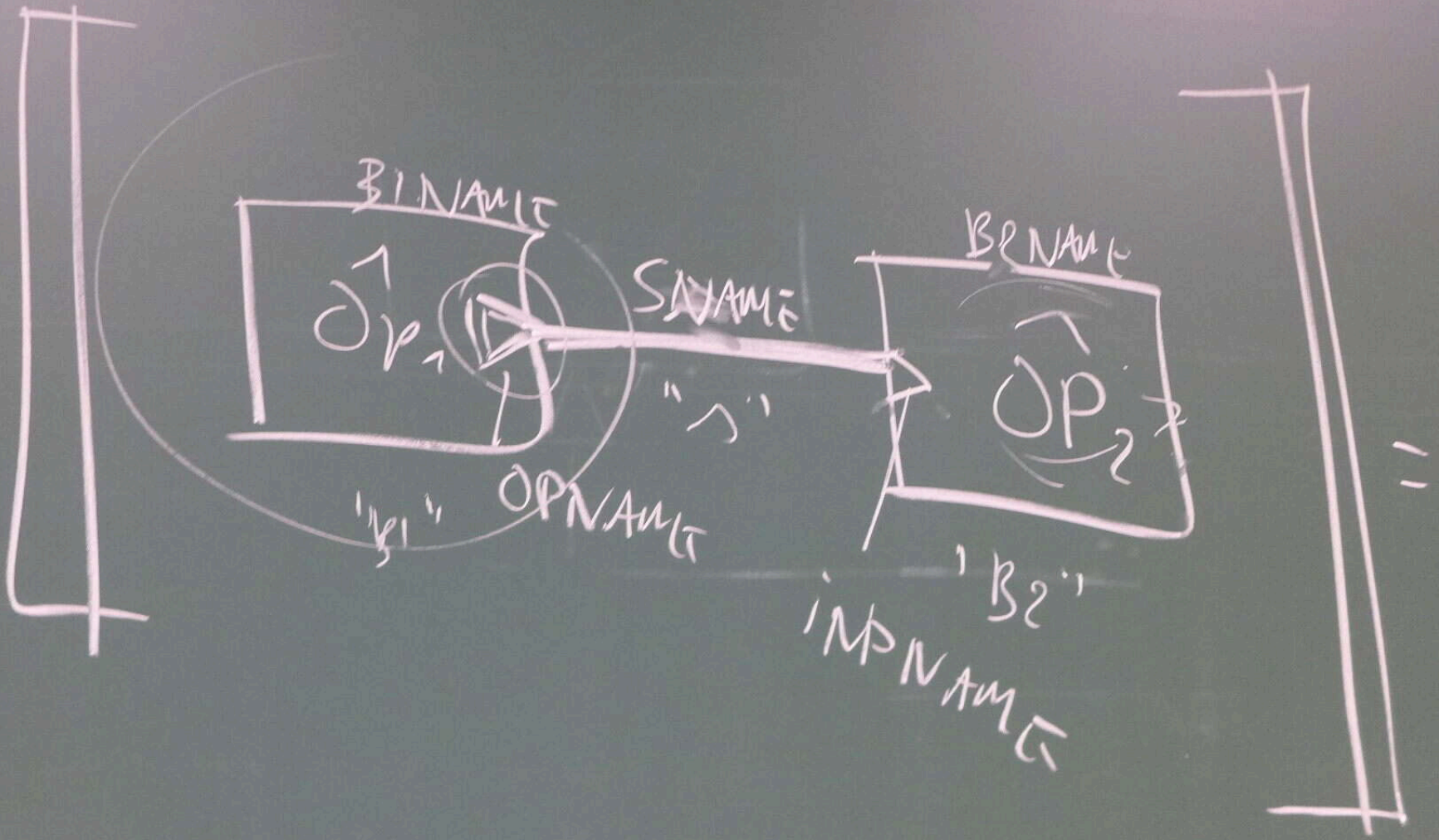
HIERARCHY

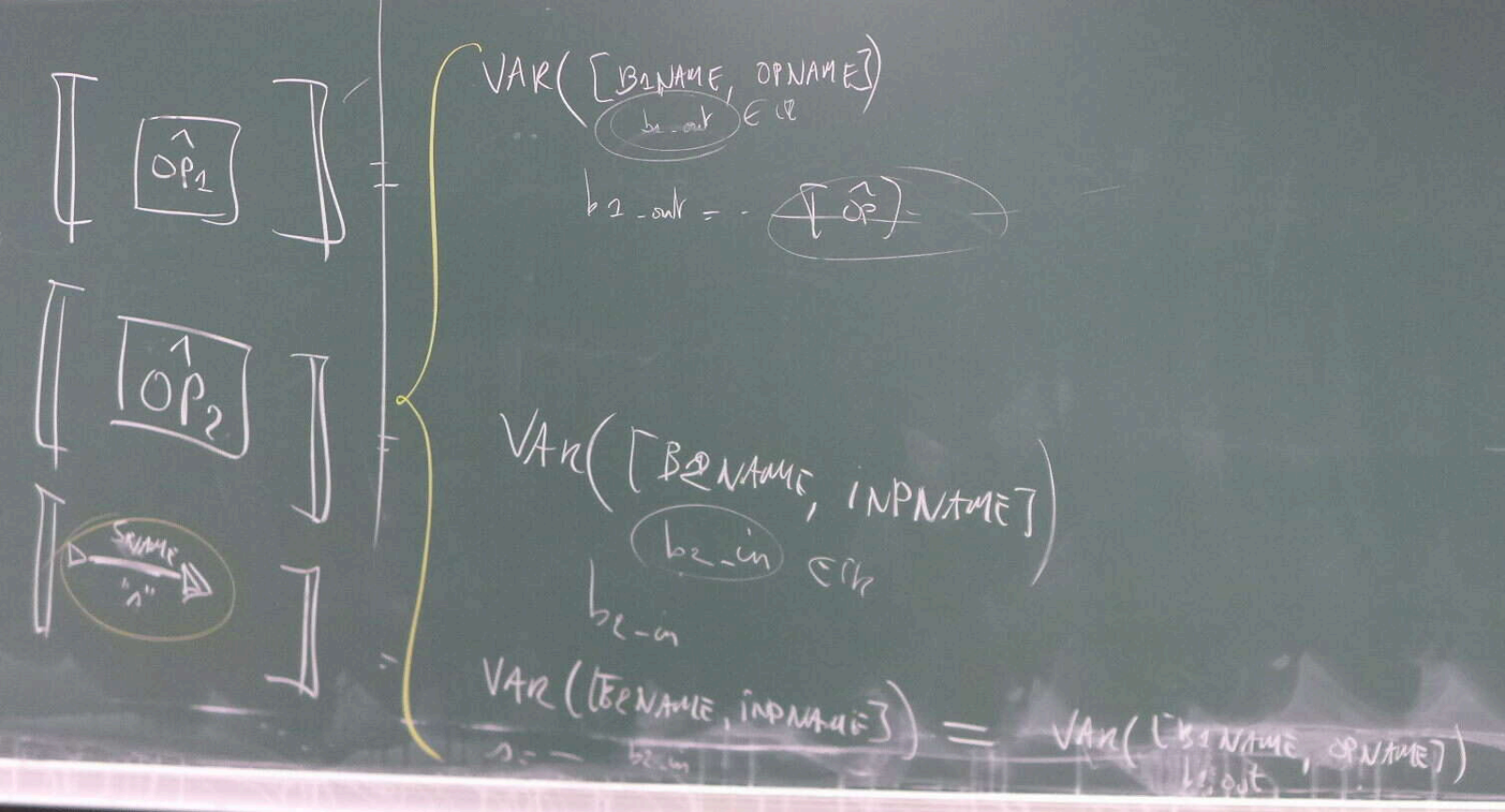
FLAT

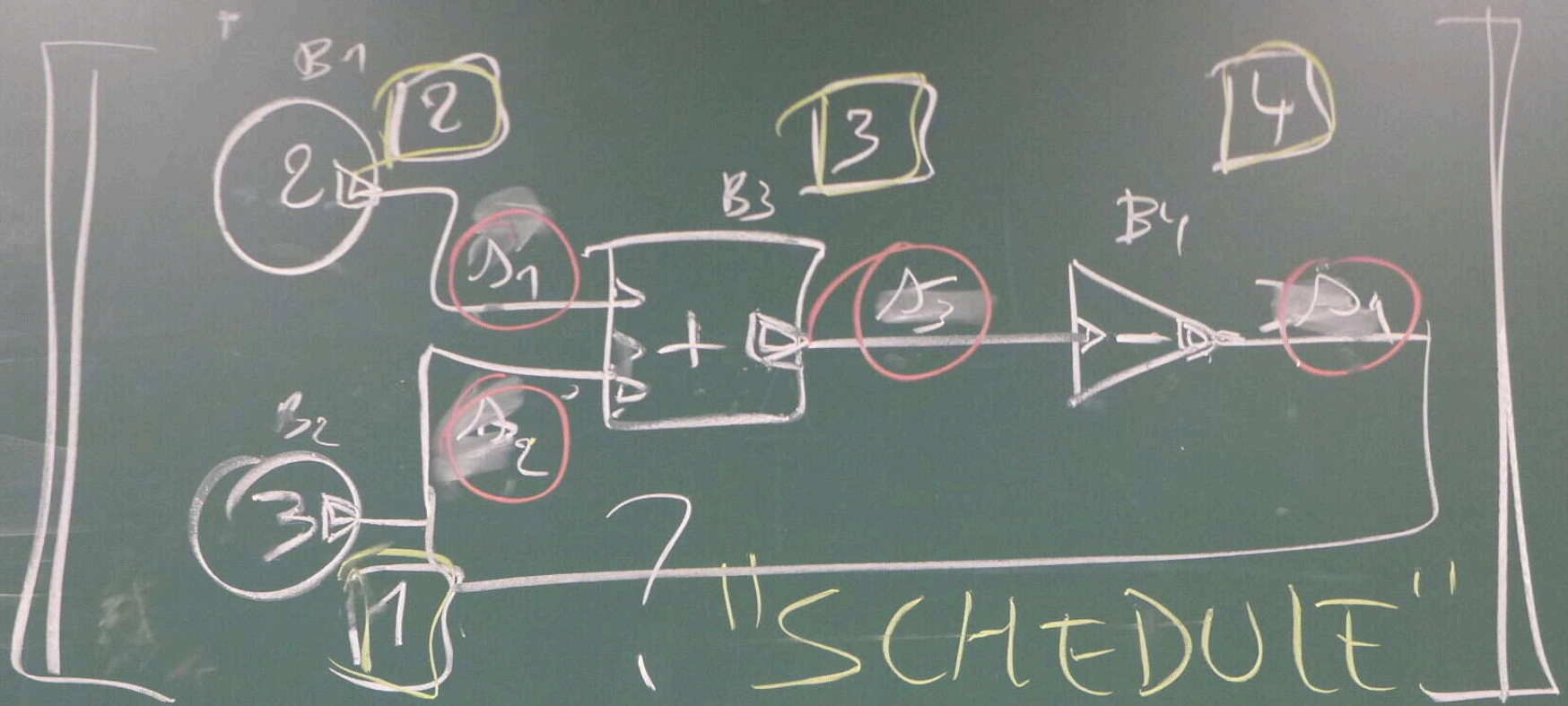
SEMANTICS

	TIME	SYNTAX	DENOT. ≈ OPER
AG	{NOW}		
DT	CN		
CT	CR		


B1 NAME
SNAME
B2 NAME
"1"
OPNAME
"2"
INPNAME
=


B1 NAME
SNAME
B2 NAME
"S"
"B1"
OPNAME
"B2"
INPNAME

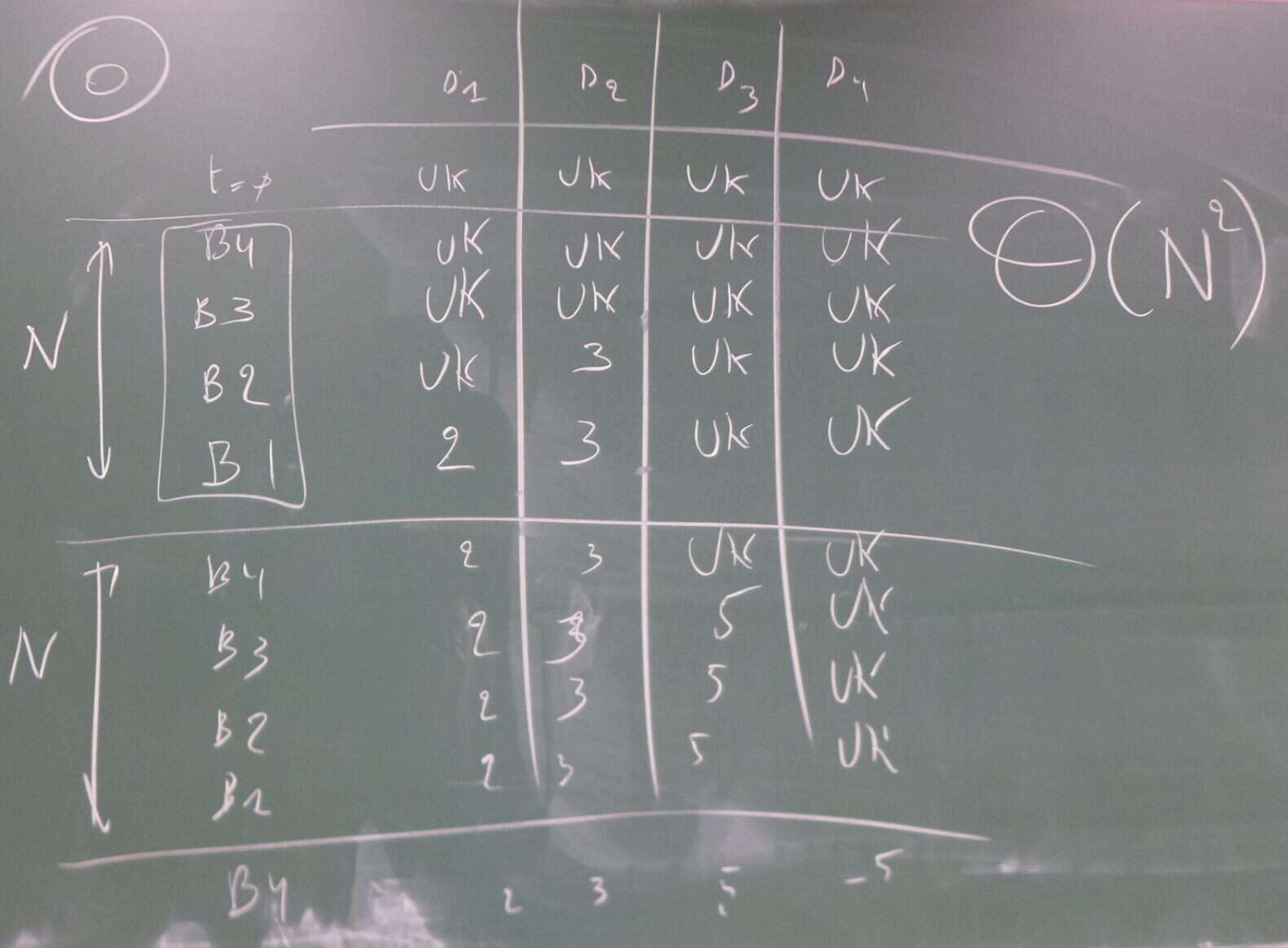

$\hat{OP}_1$
$\hat{OP}_2$
SNAME
"λ"
VAR([B1NAME, OPNAME])
b1_out ∈ B
b1_out = - $\hat{OP}$
VAR([B2NAME, INPNAME])
b2_in ∈ B
b2_in
VAR([B2NAME, INPNAME]) = VAR([B1NAME, OPNAME])
b2_in
b1_out

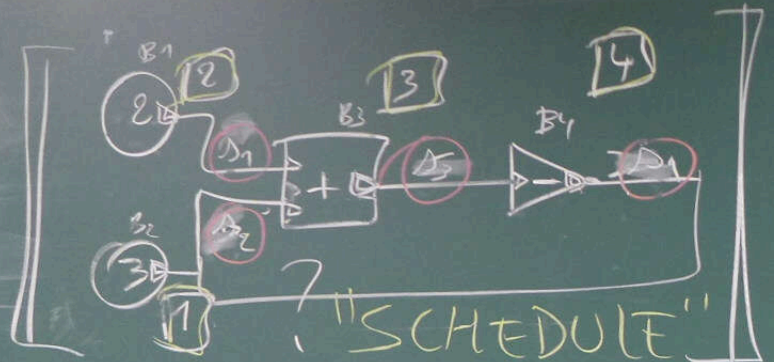

B1
2
2
3
B3
4
B4
D1
D2
D3
D4
B2
3
1
"SCHEDULE"

$$= \begin{cases} \lambda_1 = 2 \\ \lambda_2 = 3 \\ \lambda_3 = \lambda_1 + \lambda_2 \\ \lambda_4 = -\lambda_3 \end{cases}$$

$(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \in \mathbb{R}^4$

$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{R}$


D1 D2 D3 D4
t=p
UK UK UK UK
B4 UK UK UK UK
B3 UK UK UK UK
B2 UK 3 UK UK
B1 2 3 UK UK
N
O(N²)
B4 2 3 UK UK
B3 2 3 5 UK
B2 2 3 5 UK
B1 2 3 5 UK
N
B4 2 3 5

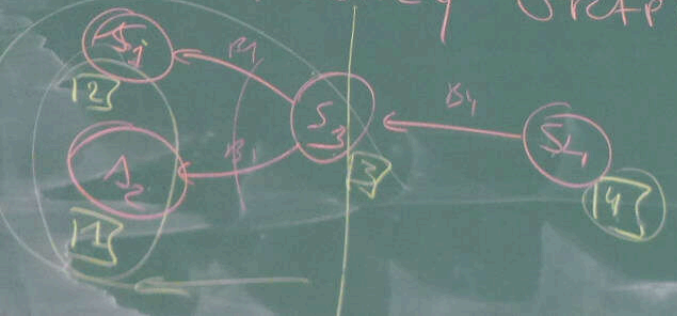


$$= \begin{cases} s_1 = 2 \\ s_2 = 3 \\ s_3 = s_1 + s_2 \\ s_4 = -s_3 \end{cases} \qquad (s_1, s_2, s_3, s_4) \in \mathbb{R}^4$$

$s_1, s_2, s_3, s_4 \in \mathbb{R}$

N BLOCKS

DEPENDENCY GRAPH



for $i = 1 \dots N$:

CSP_BLOCK[i].COMPUTE()

$O(N)$

CBD

HIERARCHY

FLAT

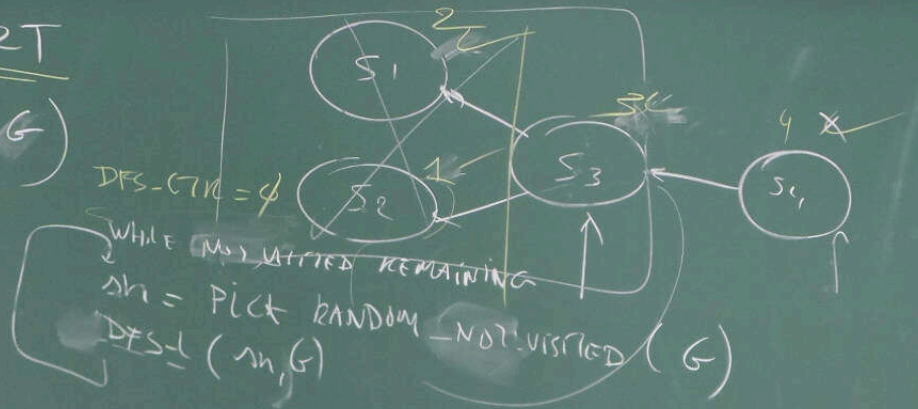
SEMANTICS

	TIME	SYNTAX	DENOT. ≈ OPER
ALG	{NOW}		
DT	CN		
CT	CR		

I TOPSORT

DFS (s_n, G)

$O(N)$



DFS_L (S_n, G).

mark S_n as visited

$\forall$ CHILDREN C of S_n in G:

if not visited C:

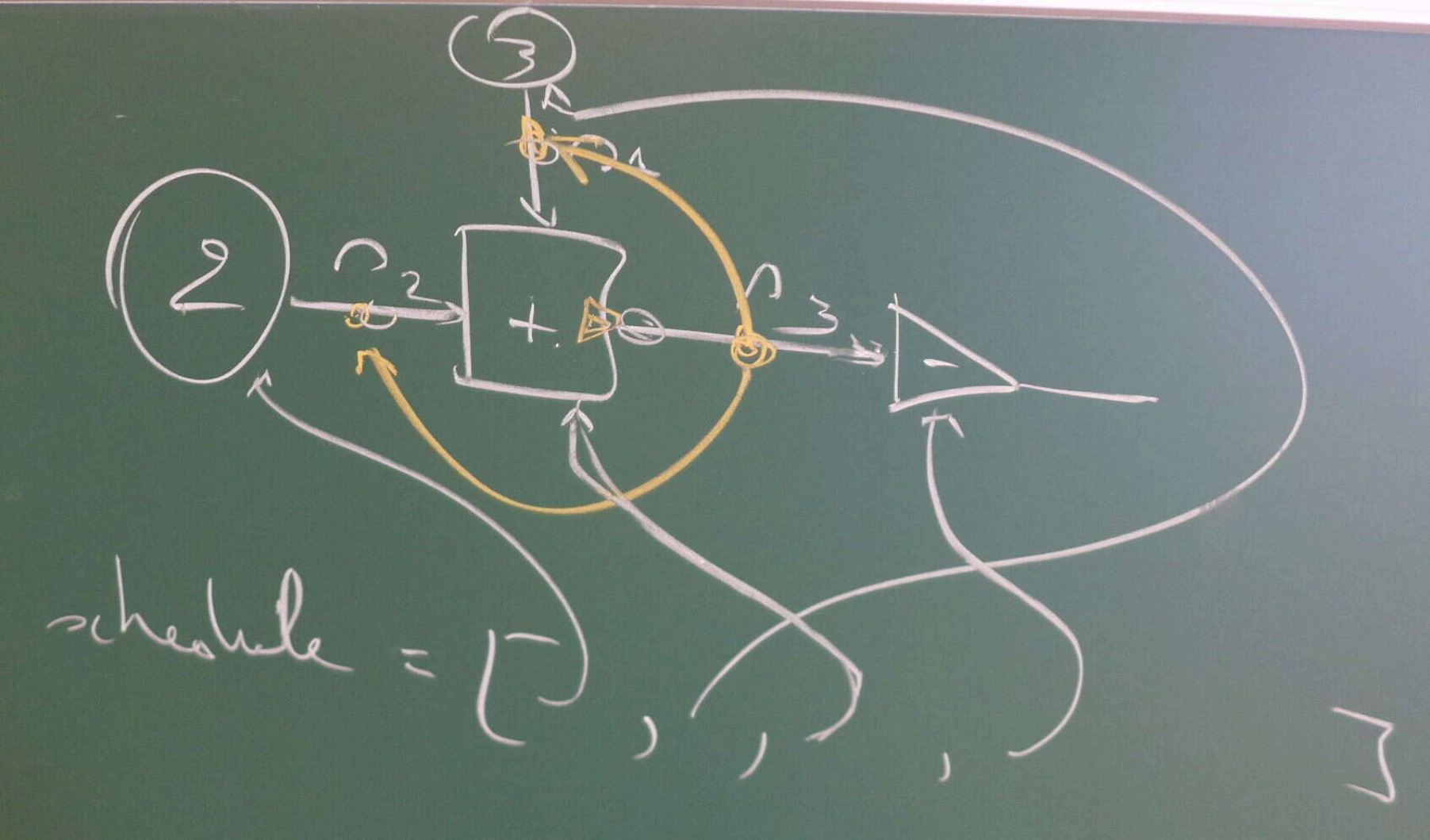
DFS_L (C, G)

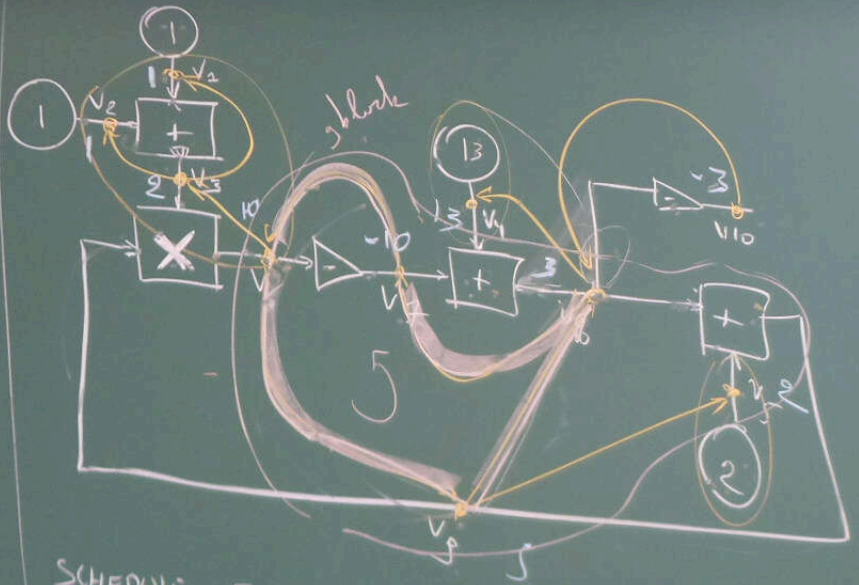
← DFS_CTR ++

← LABEL S_n WITH DFS_CTR

AIG-CBD $T = \{Now\}$

```
depGraph = buildDepGraph(cbd)
schedule = topoDetectSort(depGraph)
for gblock in schedule:
    gblock.compute()
```


3
2
+
-
schedule = (



"STRONG COMPONENT" TARJAN 1972

"DEPENDENCY CYCLE" $\Theta(N+E)$

"ALGEBRAIC LOOP" ⟵

FIXED-POINT ITERATION

SCHEDULE = $[V_1, V_2, V_3, V_4, V_5, \{V_6, V_7, V_8, V_9\}, V_{10}]$

$[\![\text{Cbd}]\!] =$

DENOTATIONAL ALG - CSD

$V_1 = 1$

$V_2 = 1$

$V_3 = 2$

$V_4 = 13$

$V_5 = 2$

$V_6 = 2V_5$

$V_7 = -V_6 = -2V_5$

$V_8 = V_7 + V_4 = 13 - 2V_5$

$V_9 = 13 - 2V_5 + 2 = 15 - 2V_5$

$V_{10} = -V_8 = 2V_5 - 13$

SATISFY

$V_1, \ldots, V_{10} \in \mathbb{R}$

$(V_1, V_2, \ldots, V_{10}) \in \mathbb{R}^{10}$

$(1, 1, 2, 13, 2, 10, -10, 3, 5, -3)$

$V_4 = 13$

$V_5 = 2$

$V_6 = 10$

$V_7 = -10$

$V_8 = 3$

$V_9 = 5$

$V_{10} = -3$

$$\begin{cases} 1x + 1y = 4 \\ 1x - 1y = 0 \end{cases}$$

$$A\,\vec{x} = \vec{B}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

SOLUTION

1. CRAMER

$$x = \frac{\begin{vmatrix} 4 & 1 \\ 0 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}}$$

$$= 2, \quad = 2$$

$$\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2$$

$\Rightarrow$ UNIQUE SOLUTION

2. OPTIMIZATION / SHOOTING
   WORKS FOR NON-LINEAR

3. GAUSS - ELIMINATION

$$\begin{cases} \sin(x) + y^2 - 4 = 0 & \varepsilon_x \\ x^2 - \cos^2(x/y) + 3 = 0 & \varepsilon_y \end{cases}$$

DEFORMATIONAL ALG - CKD

$(x_0, y_0) \rightarrow (\varepsilon_{x_0}, \varepsilon_{y_0})$

$(x_1, y_1) \rightarrow (\varepsilon_{x_1}, \varepsilon_{y_1})$

$\vdots$

$(x_*, y_*) \rightarrow (\varepsilon_{x_*}, \varepsilon_{y_*})$

i = 0

D7 - CBD OPERATIONAL SEMANTICS

WHILE NOT END_COND(i, CBD)

depGraph = buildDepGraph(CBD)

schedule = topoDataSort(depGraph)

for gblock in schedule:

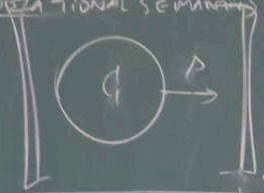
gblock.compute()

i++

SCHEDULE = [∫, ∫, ∫, { ∫, ∫, ∫ }, ∫, ∫, { ∫, ∫, ∫ }, ∫, ∫]

DT - CBD $T = \mathbb{N}$

DENOTATIONAL SEMANTICS



$\forall i \in \mathbb{N}$

$s(i) = c$ ...

DENOT OF $s : \mathbb{N} \longrightarrow \mathbb{R}$

$i \longrightarrow s(i)$

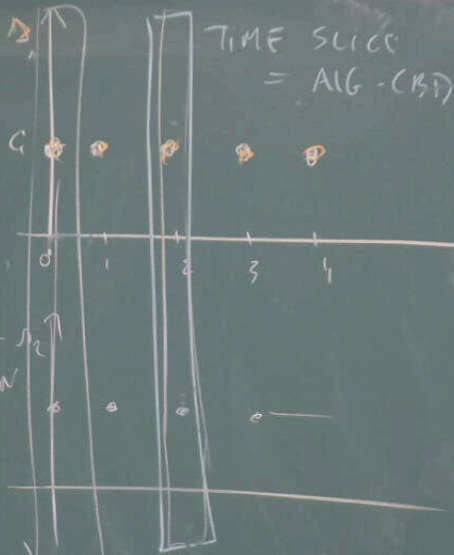
$s_3(i) = s_1(i) + s_2(i), \forall i \in \mathbb{N}$

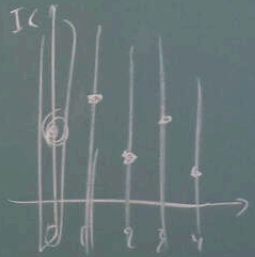
DENOT OF

$y(i) = x(i-1), \forall i \in \mathbb{N}$

$y(0) = IC$

DENOT OF

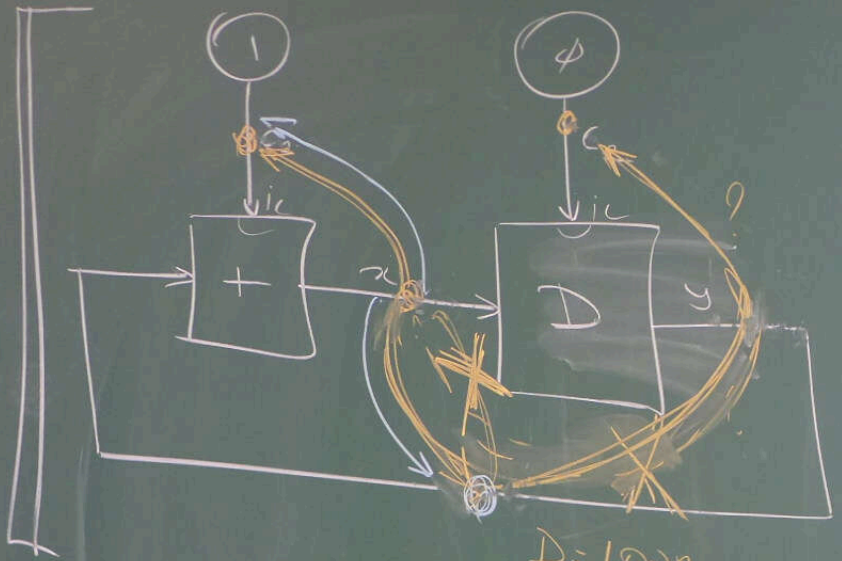




```
i = 0
WHILE NOT END_COND(i, CBD):
    if i < 2:
        DEP_GRAPH = build Dep Graph(CBD)
        SCHEDULE = top Detect Sort(DEP_GRAPH)
    FOR GBLOCK IN SCHEDULE:
        GBLOCK.COMPUTE()
    i++
```

OPSEM

D7-(BD)



$$
\begin{cases}
c_1(i) = 1 & \forall i \in \mathbb{N} \\
c_2(i) = \varphi & \forall i \in \mathbb{N} \\
x(i) = c_1(i) + y(i) & \forall i \in \mathbb{N} \\
y(0) = c_2(0) & i = \varphi \\
y(i) = x(i-1) & i \in \mathbb{N} \setminus \{\varphi\}
\end{cases}
$$

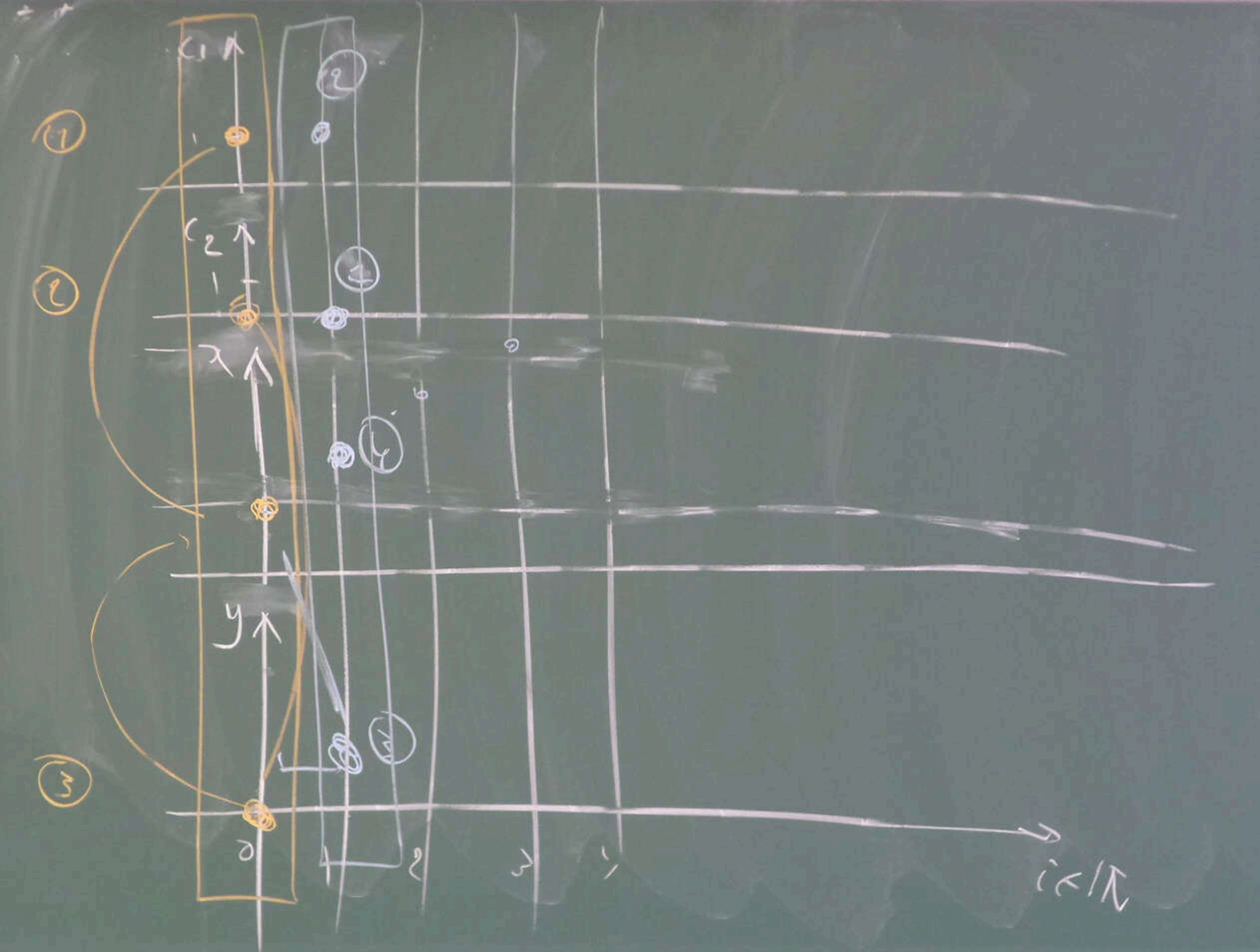
$$
= \begin{cases} x(i) = 1 + y(i) \\ y(i) = x(i-1) \end{cases}
\quad
\begin{cases} x(i) = x(i-1) + 1 & i > 0 \\ y(0) = \varphi & i = 0 \end{cases}
$$

DENOTATIONAL

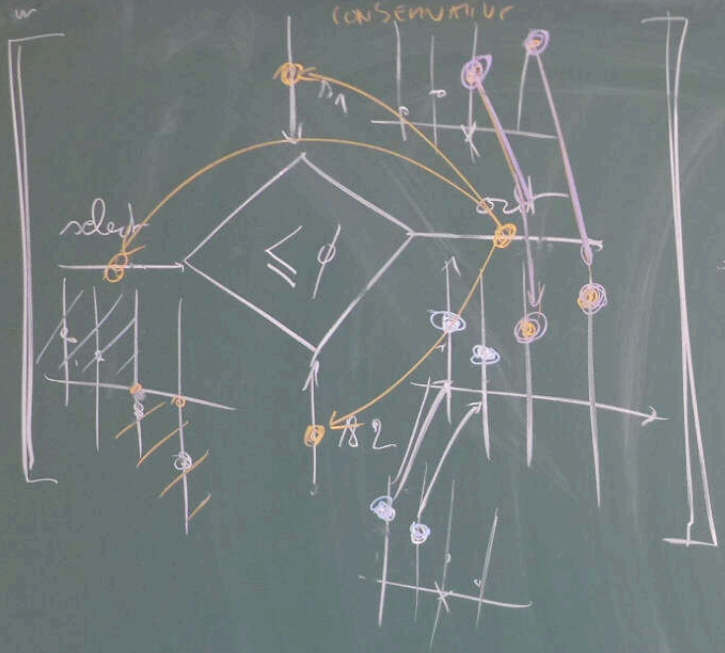
"DIFFERENCE EQN."

$$x_i = x_{i-1} + c$$

$$x_0 = -c$$



1
2
3
c_1
c_2
y
0
1
2
3
$i\varepsilon/\hbar$



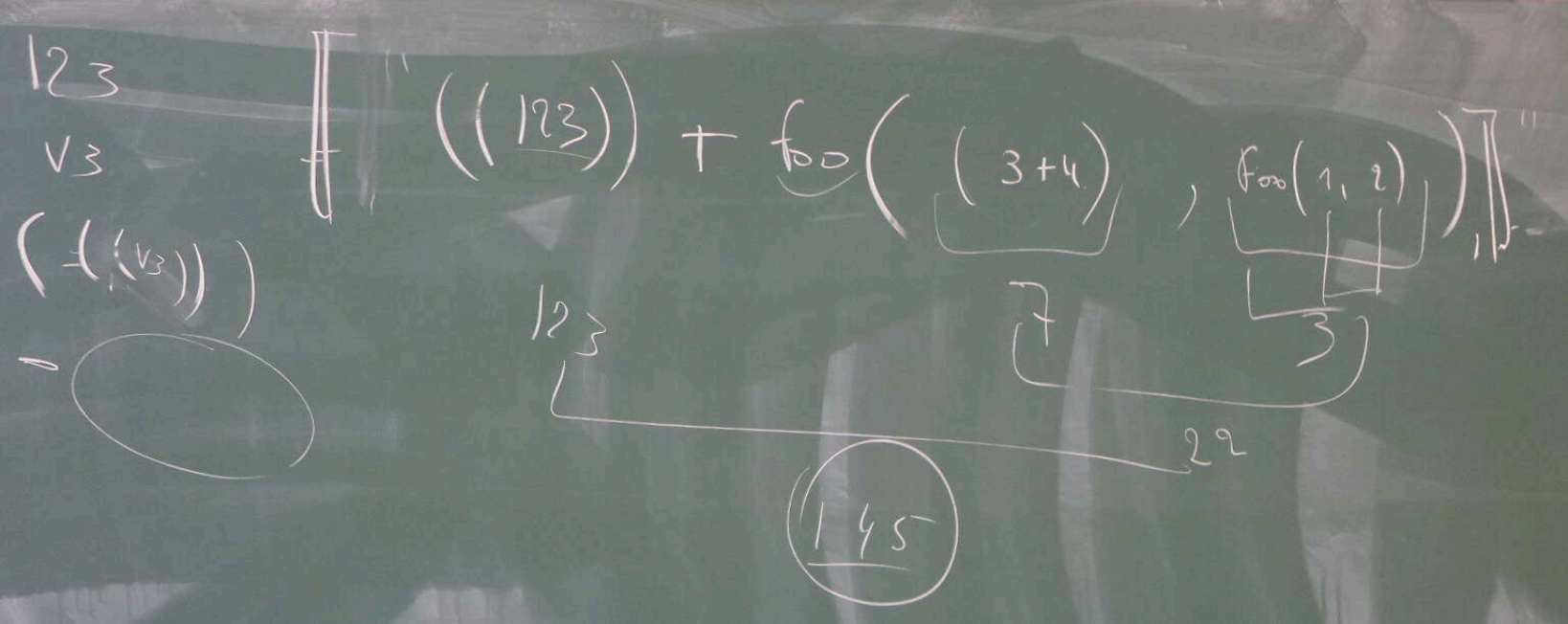
$=$

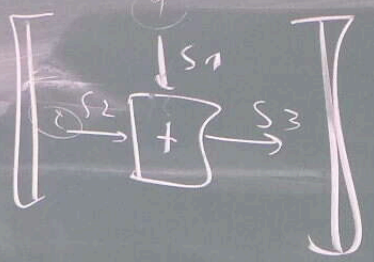
XOR
BASED ON
SELECT

DENOTED

$\forall i \in \mathbb{N}$

$$out(i) = \begin{cases} P_1(i) & \text{if } select(i) \leq \phi \\ P_2(i) & \text{if } select(i) > \phi \end{cases}$$


123
V3
(+((V3)))
-
[[((123)) + foo((3+4) , foo(1, 2))]]
123
7
3
22
145



$= \left[\!\left[\begin{array}{l} n_1, n_2, n_3 \in \mathbb{R} \\ n_3 = n_1 + n_2 \end{array} \right]\!\right]$ Alg

UNIQUE

$= (n_1, n_2, n_3) \in \mathbb{R}^3$

$f : \mathbb{N} \longrightarrow \mathbb{N}$

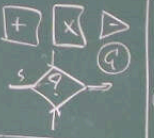
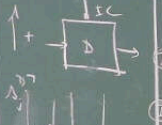
$n \longrightarrow n * (n+1)$ Div 2

SEMANTICS

HIERARCHICAL

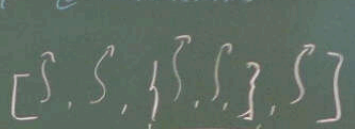
FLAT

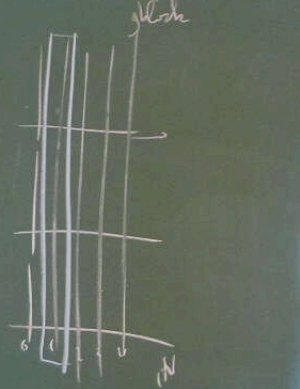
FLATTENING

CBD	T	BLOCKS	DENOTATIONAL	OPERATIONAL
ALG	NOW		① $\lambda_j^{ALG} \in \mathbb{R}$, ALG EQNS OVER Λ^{ALG} ② $(\lambda_1^{ALG}, \ldots, \lambda_{NUMSIGN}^{ALG}) \in \mathbb{R}^{NS}$ UNIQUE	DG = depGraph(CBD) SCHEDULE = topSort(DG) FOR gblock IN SCHEDULE: gblock.compute()
DT DISCRETE-TIME AKA SDF SYNCHRONOUS DATAFLOW EXACT	$\mathbb{N}$		① $\lambda_j^{DT} \in \mathbb{N} \to \mathbb{R}$, DIFFERENCE EQNS OVER Λ^{DT} ② $(\lambda_1^{DT}, \ldots, \lambda_{NUMSIGN}^{DT}) \in (\mathbb{N} \to \mathbb{R})^{NS}$ UNIQUE	i = 0 WHILE NOT END_COND(i, CBD): if i < 2
CT CONTINUOUS-TIME	$\mathbb{R}$		① $\lambda_j^{CT} \in \mathbb{R} \to \mathbb{R}$, DIFFERENTIAL/INTEGRAL EQNS OVER Λ^{CT} ② $(\lambda_1^{CT}, \ldots, \lambda_{NUMSIGN}^{CT}) \in (\mathbb{R} \to \mathbb{R})^{NS}$ UNIQUE	APPROX ~ ERROR

APPROX

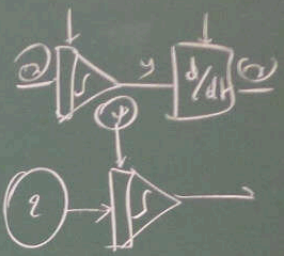
APPROX





$$\left[\left[\; G \;\right]\right] = \left[\left[\; \right]\right]_{CT}$$

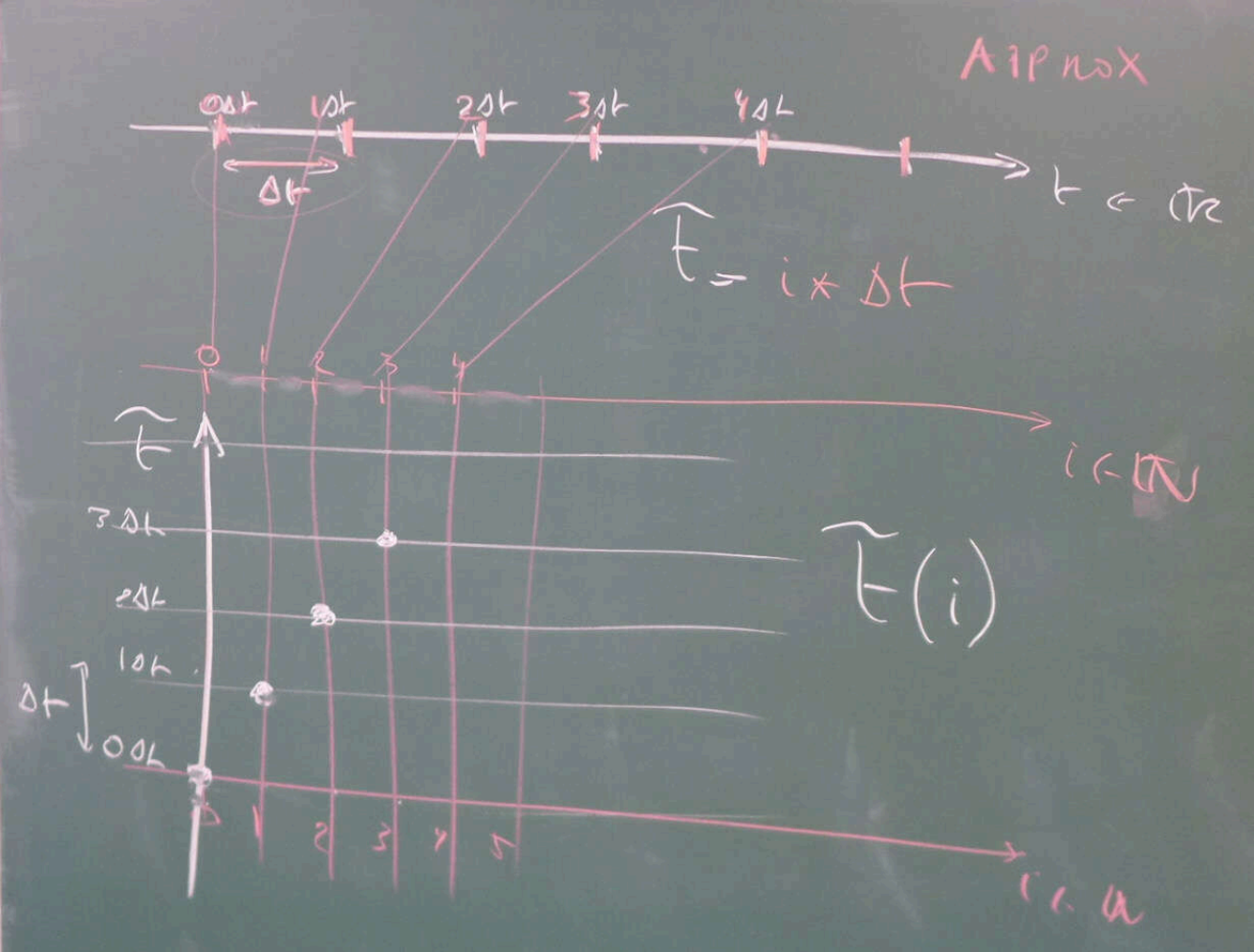
$$\Lambda : \mathbb{R} \longrightarrow \mathbb{R}$$
$$t \longrightarrow \Lambda(t) = \phi$$

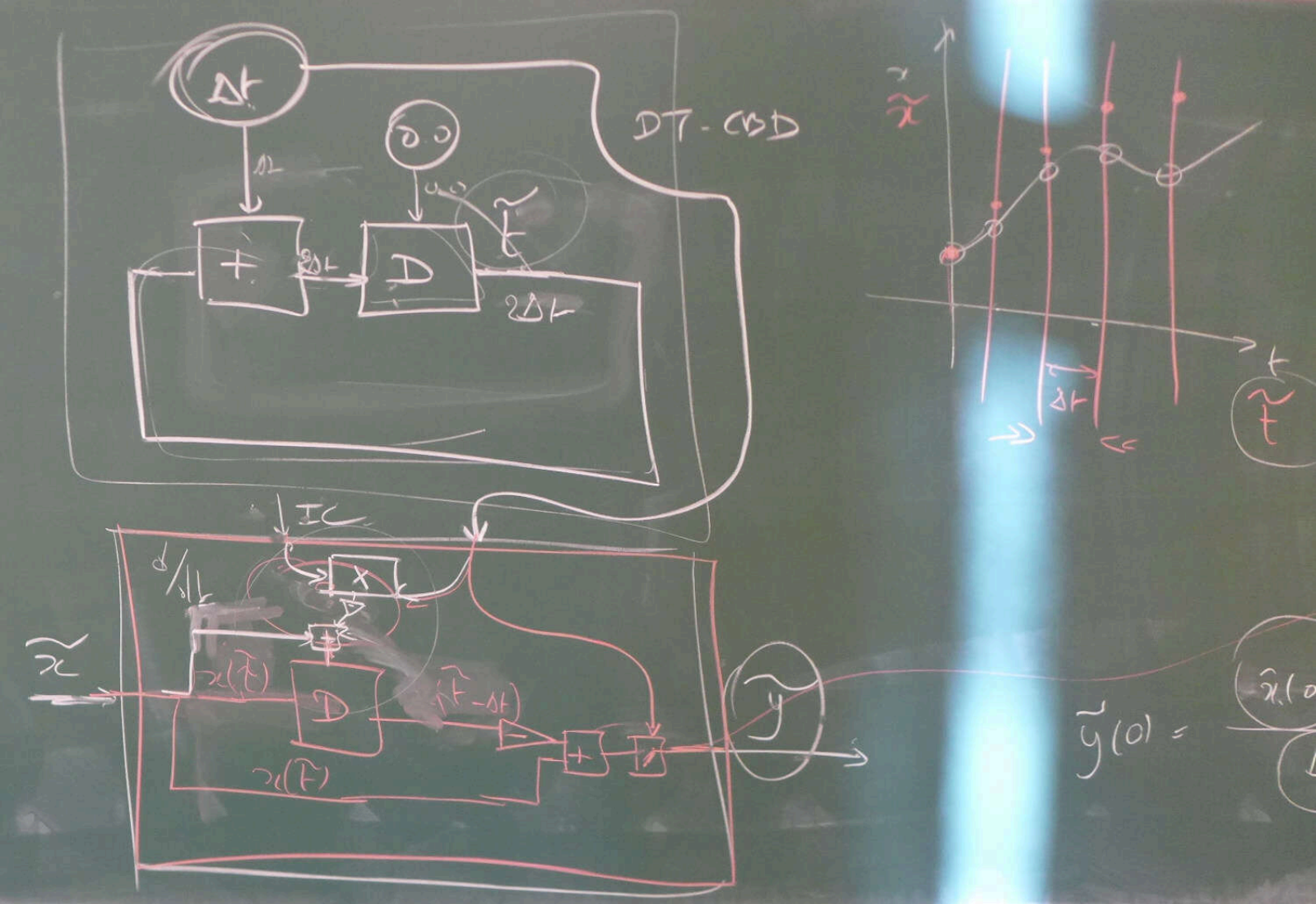


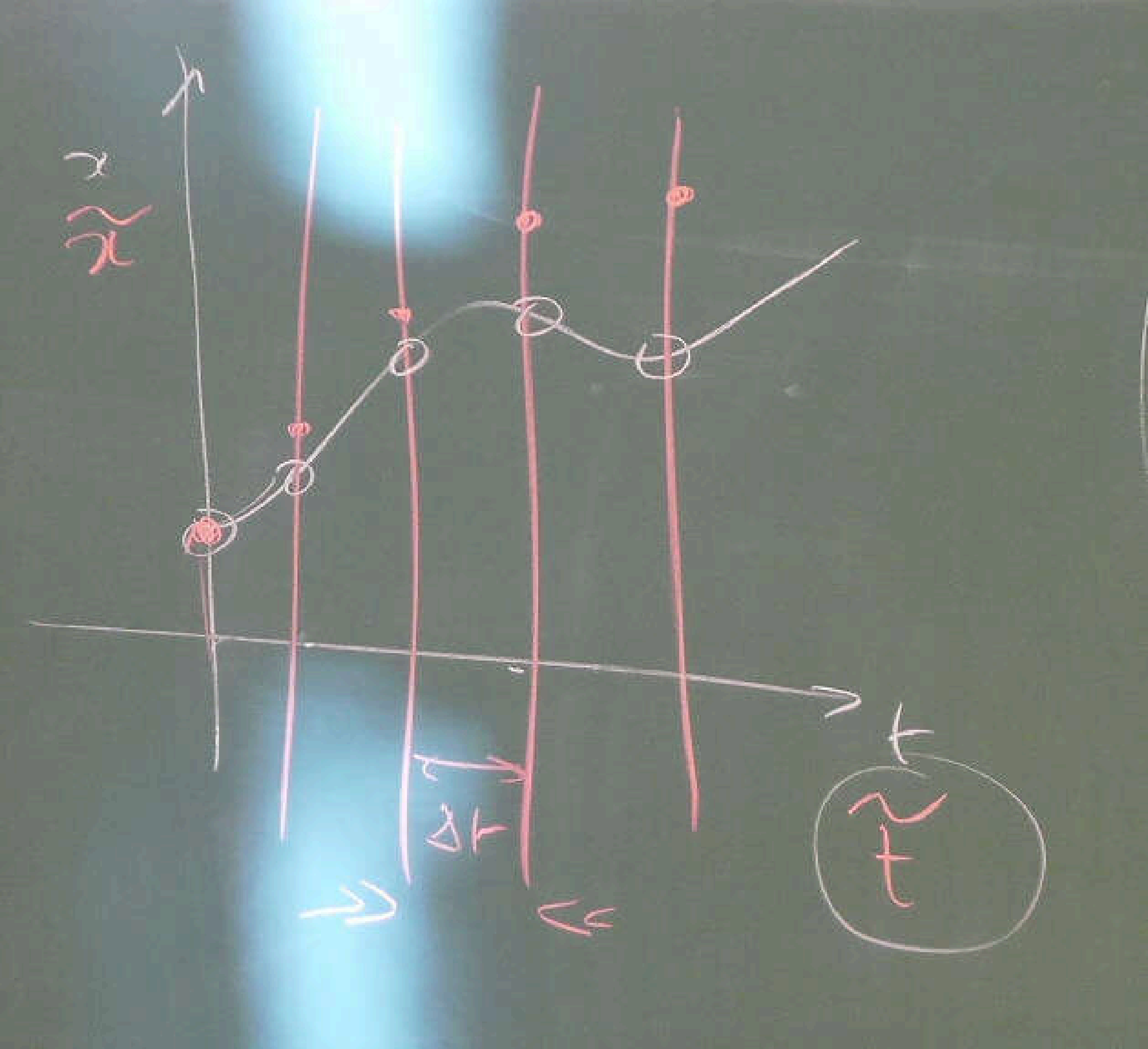
$$= \begin{cases} y(t) = \dfrac{d}{dt}x(t) & t > \phi \quad \in \mathbb{R} \\ y(0) = IC(0) & t = \phi \end{cases}$$

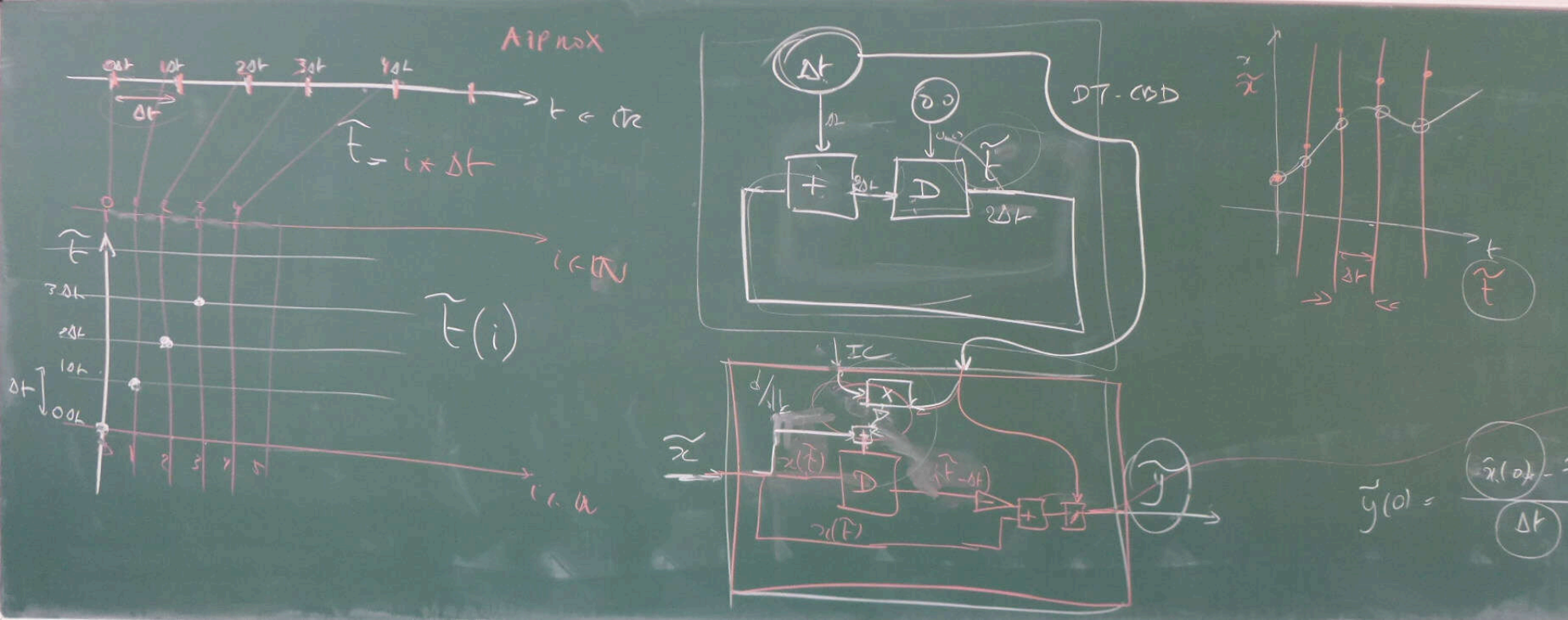
$x(t) = \underbrace{A}_{\in}t + B$

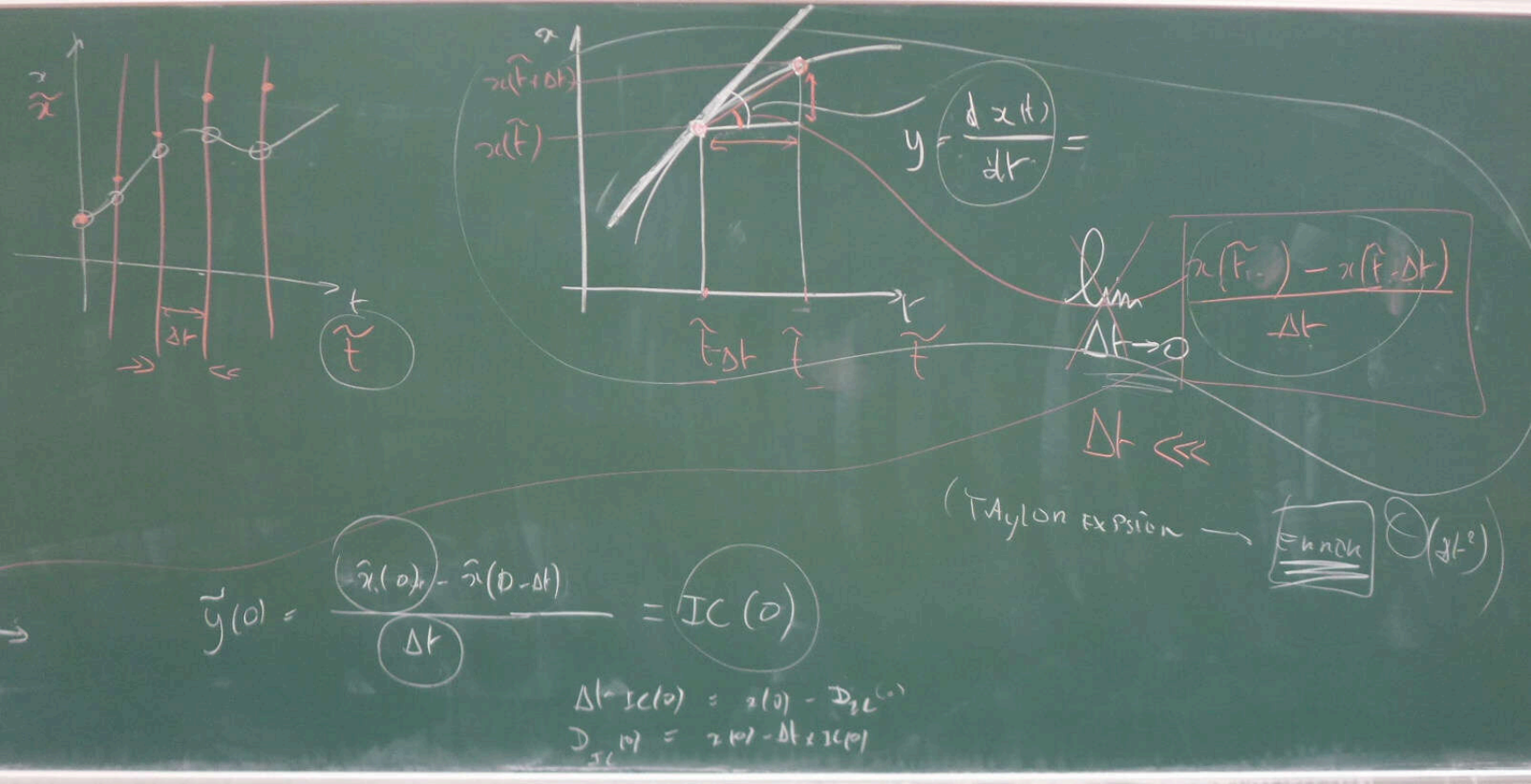
$$= \begin{cases} y(t) = \displaystyle\int_0^t x(\tau)\, d\tau & t > \phi \\ y(0) = IC(\phi) & t = \phi \end{cases}$$

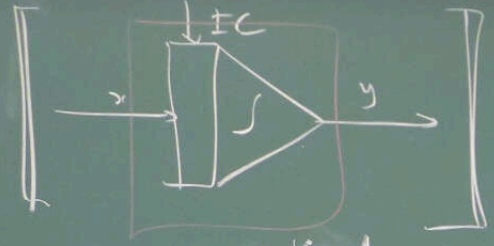

Aipnox
0Δt
1Δt
2Δt
3Δt
4Δt
Δt
t = (t
t = i * Δt
0
1
2
3
4
i = (N
t(i)
3Δt
2Δt
1Δt
0Δt
Δt
0
1
2
3
4
5
i = N


Δt
DT - CBD
+
D
2Δt
IC
x
D
+
Δt
Δt
t


x
$\tilde{x}$
t
Δt
$\tilde{t}$


Algorithme
0Δt
1Δt
2Δt
3Δt
4Δt
Δt
t ∈ ℝ
t̃ = i * Δt
i ∈ ℕ
t̃(i)
3Δt
2Δt
1Δt
0Δt
Δt
i ∈ ℕ
DT-CSD
Δt
+
D
2Δt
t̃
IC
x̃
x(t̃)
x(t̃-Δt)
ỹ
x̃
t
Δt
t̃
ỹ(0) =
x̃(0)
Δt


$\tilde{x}$
t
Δt
$\tilde{t}$
x
$x(\hat{t}+\Delta t)$
$x(\hat{t})$
$\hat{t}+\Delta t$
$\hat{t}$
$\tilde{t}$
$y = \frac{dx(t)}{dt} =$
$\lim_{\Delta t \to 0} \frac{x(\tilde{t}_+) - x(\tilde{t}-\Delta t)}{\Delta t}$
$\Delta t \ll$
(Taylon expsion → Ennoh $\mathcal{O}(\Delta t^2)$)
$\tilde{y}(0) = \frac{\tilde{x}(0_+) - \tilde{x}(0-\Delta t)}{\Delta t} = IC(0)$
$\Delta t \cdot IC(0) = x(0) - D_{IC}(-)$
$D_{IC}(t) = x(t) - \Delta t \cdot IC(t)$



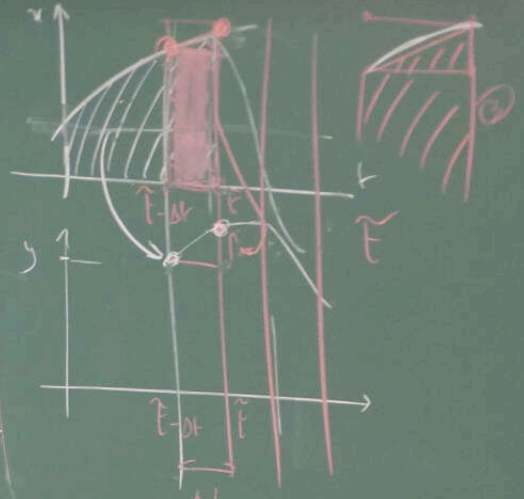
$$= \begin{cases} y(0) = y_0 \\ y(t) = \int_0^t x(\tau)\,d\tau \end{cases}$$

$$\frac{dx}{dt} = 0$$

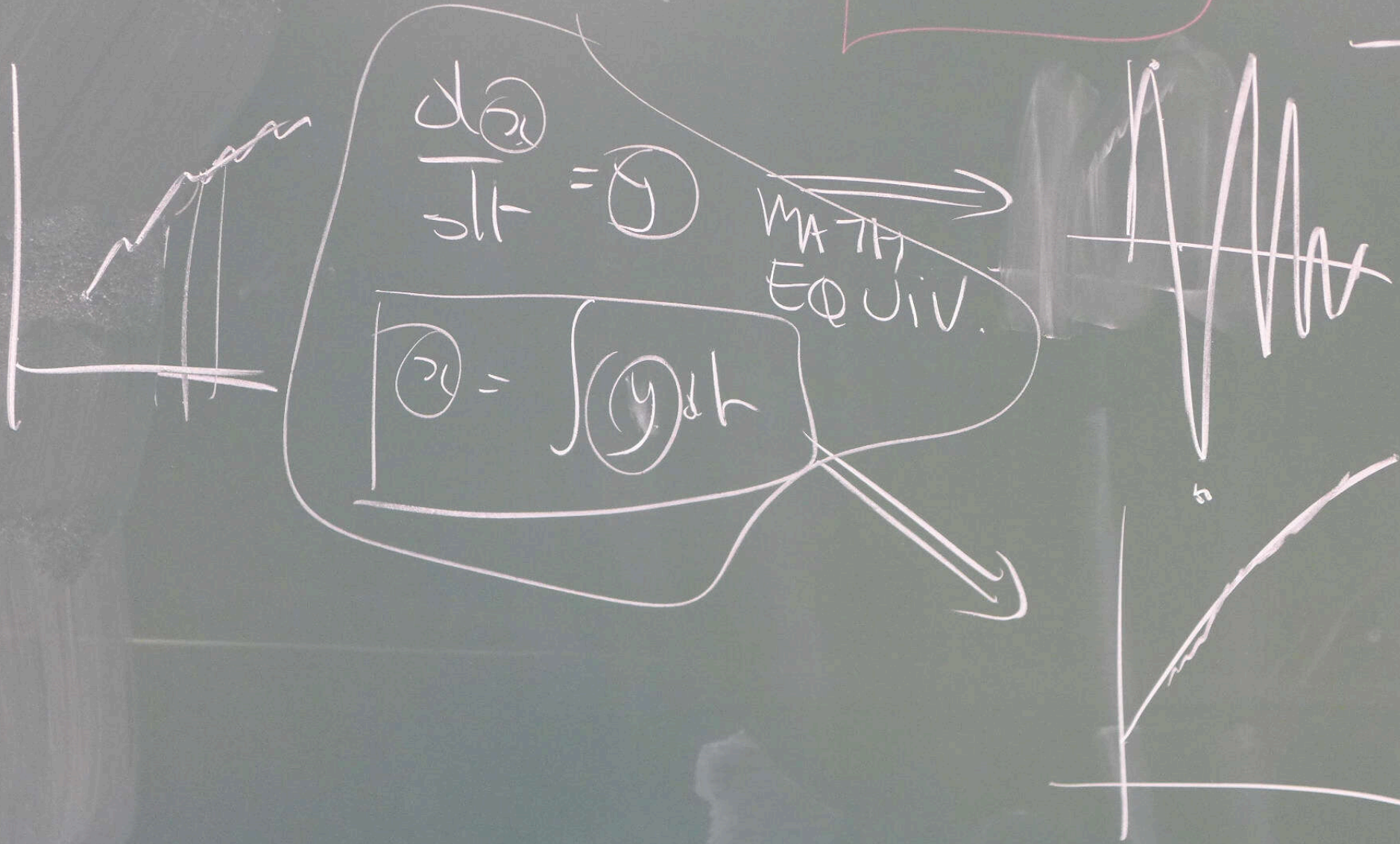
EQUIV.

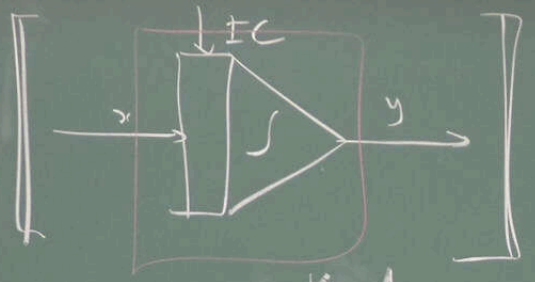
$$x = \int 0\,dt$$

$$\tilde{y}(t) = \tilde{y}(t-\Delta t) + \int_{t-\Delta t}^{t} x(\tau)\,d\tau$$

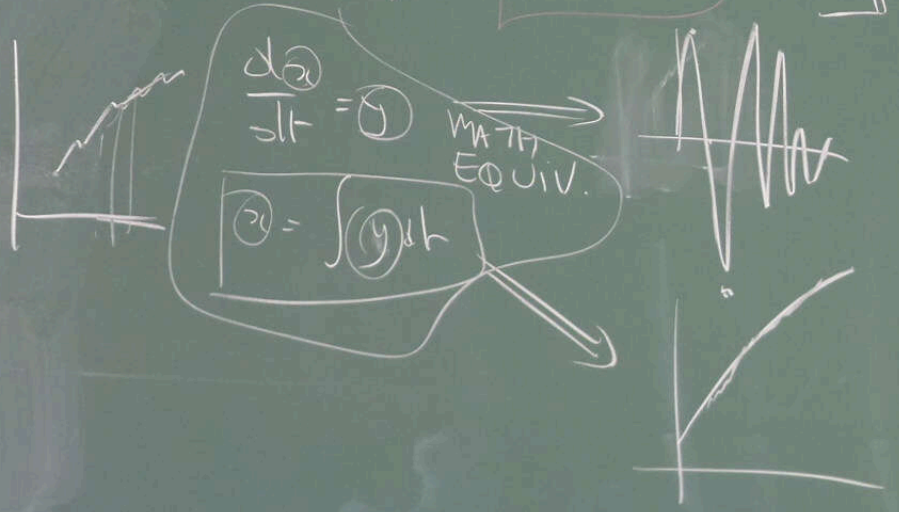


1. $\Delta t \times x(t-\Delta t)$
2. $\Delta t \times x(t)$
3. $\Delta t \times \left(x(t) + x(t-\Delta t) \right)/2$

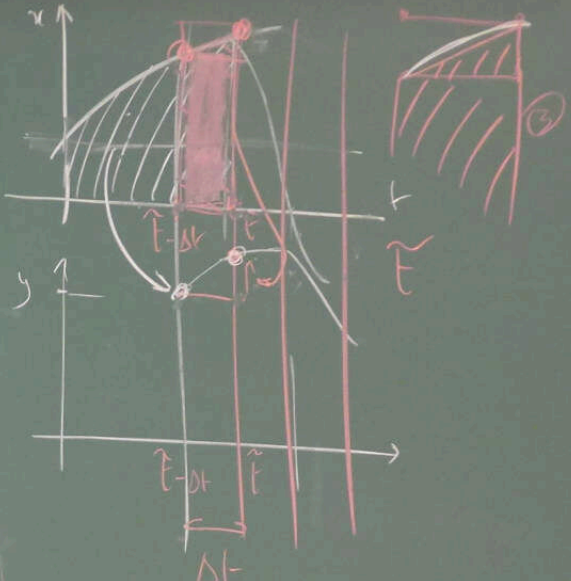

MATH
EQUIV.



$$= \begin{cases} y(0) = y_0 \\ y(t) = \int_0^t x(\tau)\,d\tau \end{cases}$$



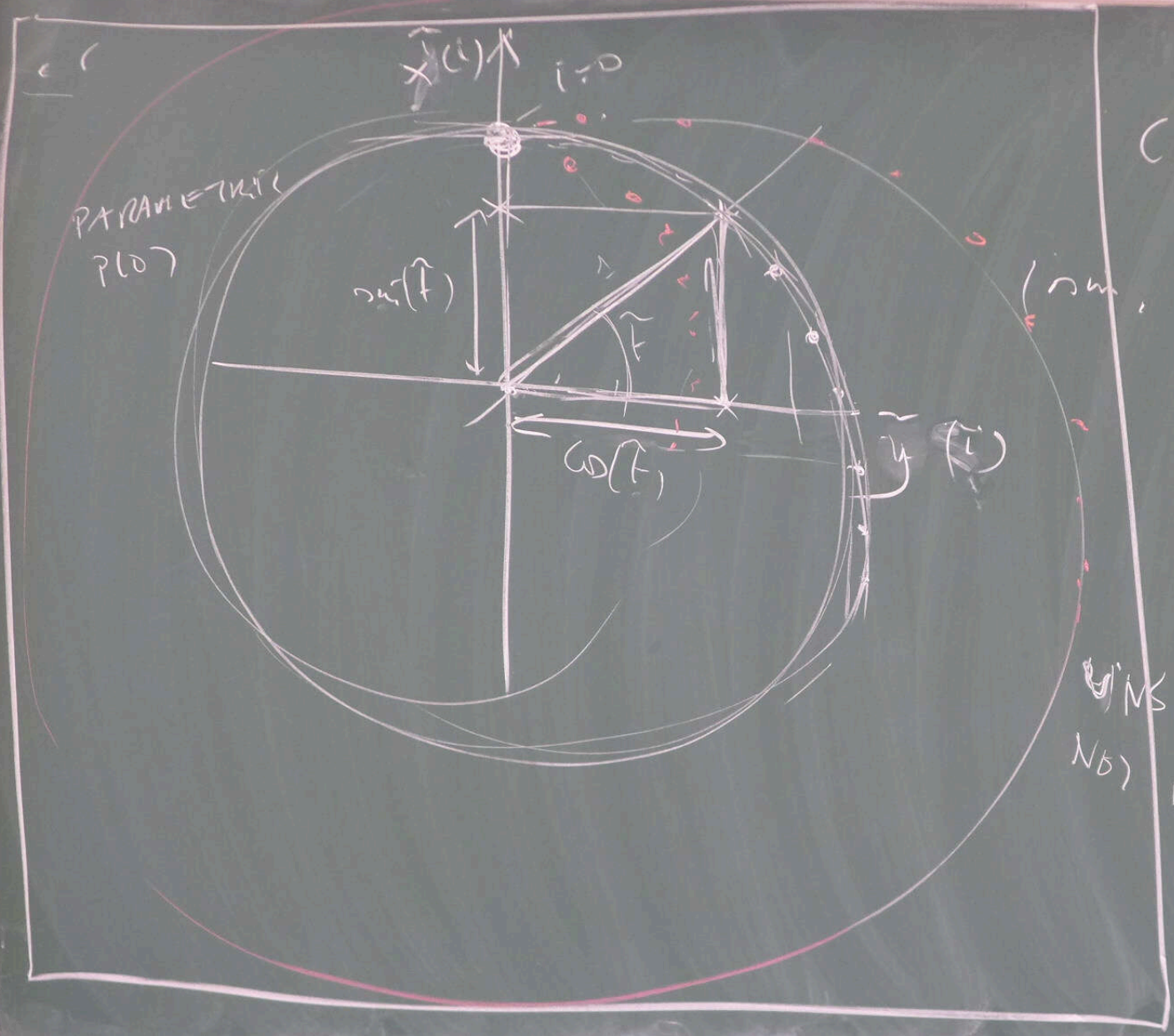
$$\tilde{y}(t) = \tilde{y}(\tilde{t}-\Delta t) + \int_{\tilde{t}-\Delta t}^{\tilde{t}} x(\tau)\,d\tau$$



① $\Delta t \times x(\tilde{t}-\Delta t)$

② $\Delta t \times x(\tilde{t})$

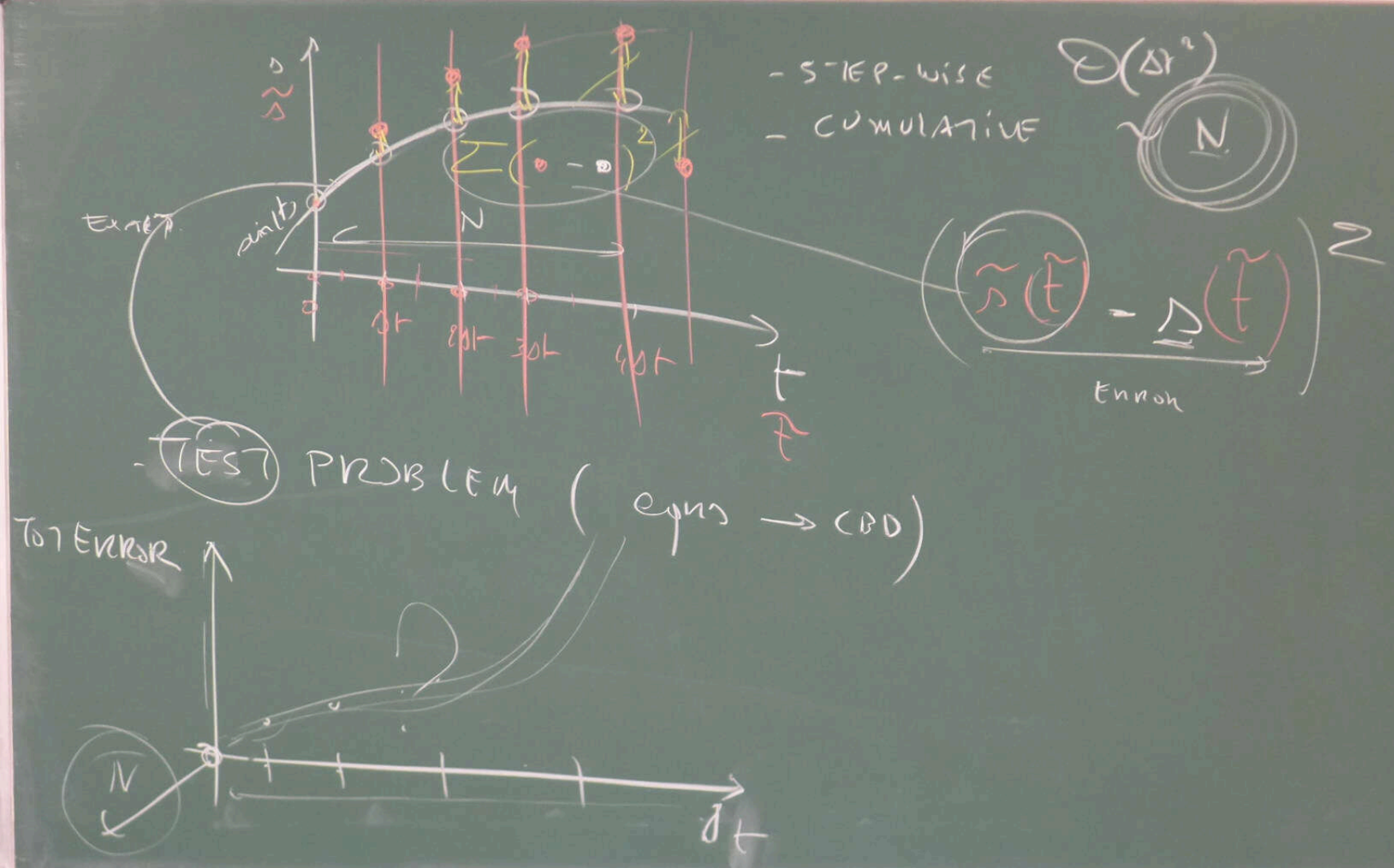
③ $\Delta t \times \left(x(\tilde{t}-\Delta t) + x(\tilde{t}) \right)/2$

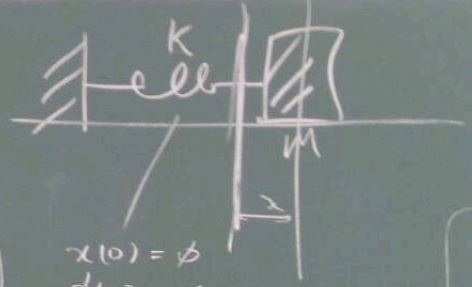


CIRCLE TEST

VISIBLE

NOT IN BOUNDED AREA


- STEP-WISE
- CUMULATIVE
$\mathcal{O}(\Delta t^2)$
N
$\tilde{\Delta}$
Δ
Exact
$\sum (\tilde{\Delta} - \Delta)^2$
N
Δt
$2\Delta t$
$3\Delta t$
$4\Delta t$
t
$\tilde{t}$
$\left(\tilde{\Delta}(\tilde{t}) - \Delta(\tilde{t})\right)^2$
Error
- TEST PROBLEM (eqns → CBD)
TOT ERROR
N
Δt



$$F = \frac{d(mv)}{dt} = \frac{dm}{dt}v + m\frac{dv}{dt}$$

$$\begin{cases} x(0) = 0 \\ \frac{dx}{dt}(0) = 1 \\ m\frac{d^2x}{dt^2} = -kx \end{cases}$$

$$\frac{d}{dt}\left[\frac{dx}{dt} = v\right]$$

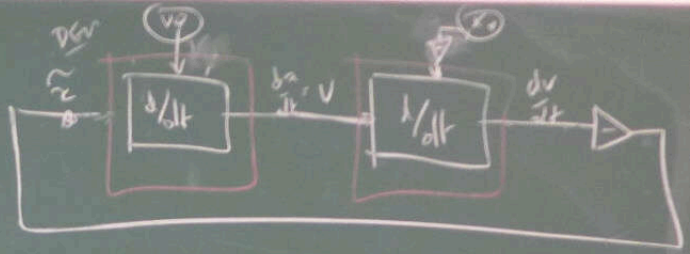
$$\begin{cases} \frac{dv}{dt} = -\frac{k}{m}x & v(0) = v_0 \\ \frac{dx}{dt} = v & x(0) = x_0 \end{cases}$$

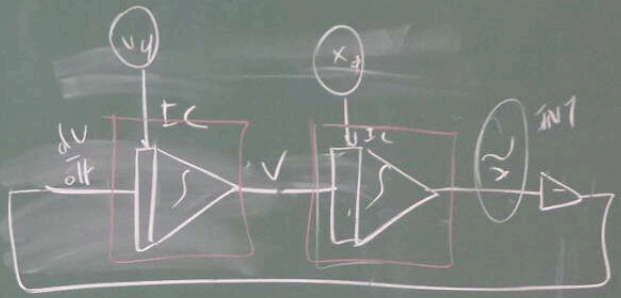
ODE

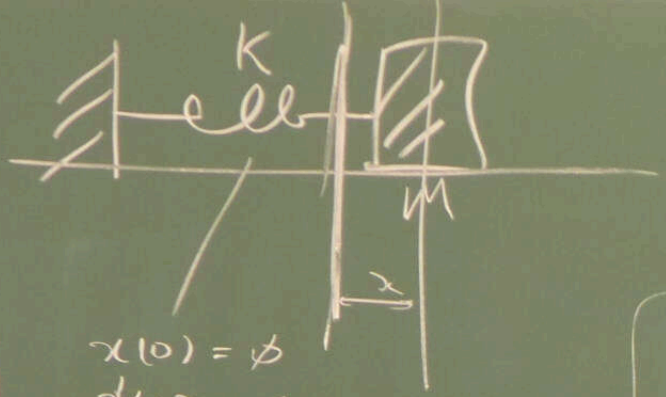
$$x(t) = \underset{1}{A}\sin(t) + \underset{0}{B}\cos(t) \qquad x(0) = B$$

$$\frac{dx}{dt} = A\cos(t) - B\sin(t) \qquad \frac{dx}{dt}(0) = A$$

$$\frac{dv}{dt} = -(A\sin(t) + B\cos(t))$$







$$F = \frac{d(m\,v)}{dt} = \frac{dm}{dt}\,v + m\,\frac{dv}{dt}$$

$$\begin{cases} x(0) = 0 \\ \frac{dx}{dt}(0) = 1 \end{cases}$$

$$\frac{d^2x}{dt^2} = -kx$$

$$\begin{cases} \frac{dv}{dt} = -\frac{k}{m}x & v(0) = v_0 \\ \frac{dx}{dt} = v & x(0) = x_0 \end{cases}$$

ODE

$$x(t) = A\sin(t) + B\cos(t) \qquad B = x(0) = x_0$$

$$\frac{dx}{dt} = A\cos(t) - B\sin(t) \qquad A = \frac{dx}{dt}(0) = v_0$$

$$\frac{dv}{dt} = -\left(A\sin(t) + B\cos(t)\right) = -x$$

DC
V
dV/dt
dV/dt = V
dV/dt

V
dV/dt
V
LNA

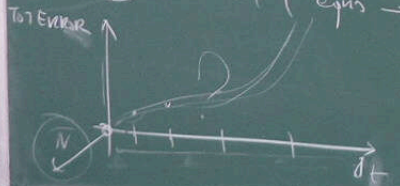
$\varepsilon(\Delta t^2)$

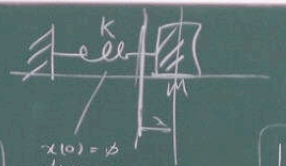
N

$\left(\tilde{x}(t) - \Delta(t)\right)^2$

Error

TEST PROBLEM (eqns → COD)





$$\frac{d^2x}{dt^2} = -k x$$

$$\frac{d}{dt}\left[\frac{dx}{dt}\right]$$

$$F = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

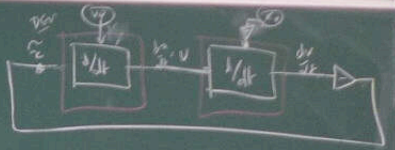
$\frac{dv}{dt} =$	$-\frac{k}{m}x$	$v(0) = v_0$
$\frac{dx}{dt} =$	v	$x(0) = x_0$

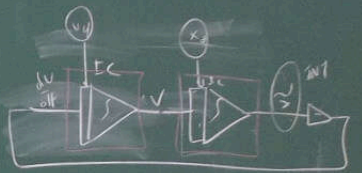
ODE

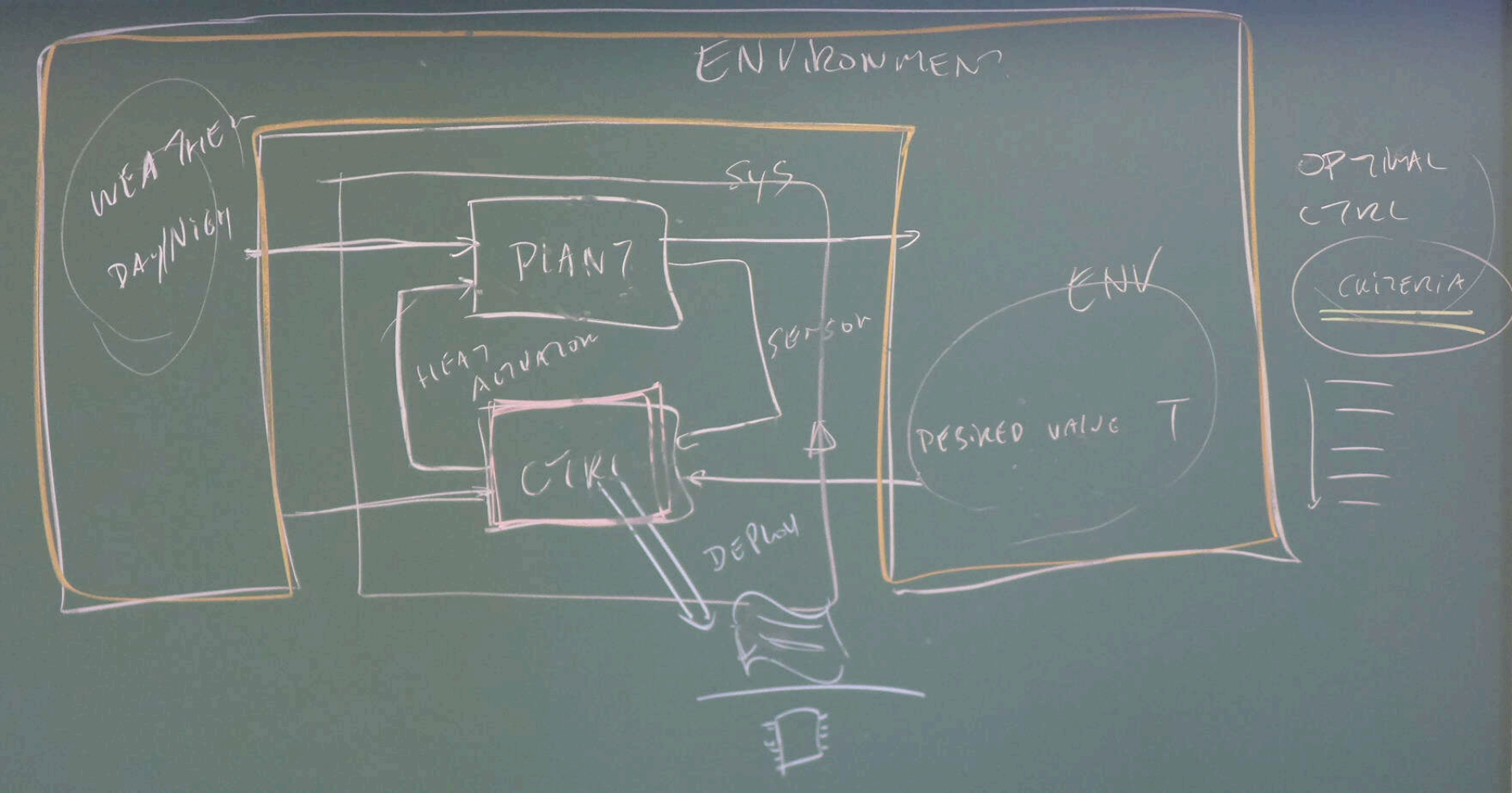
$x(t) = A\sin(t) + B\cos(t)$ $\quad \psi = x(0) = B$

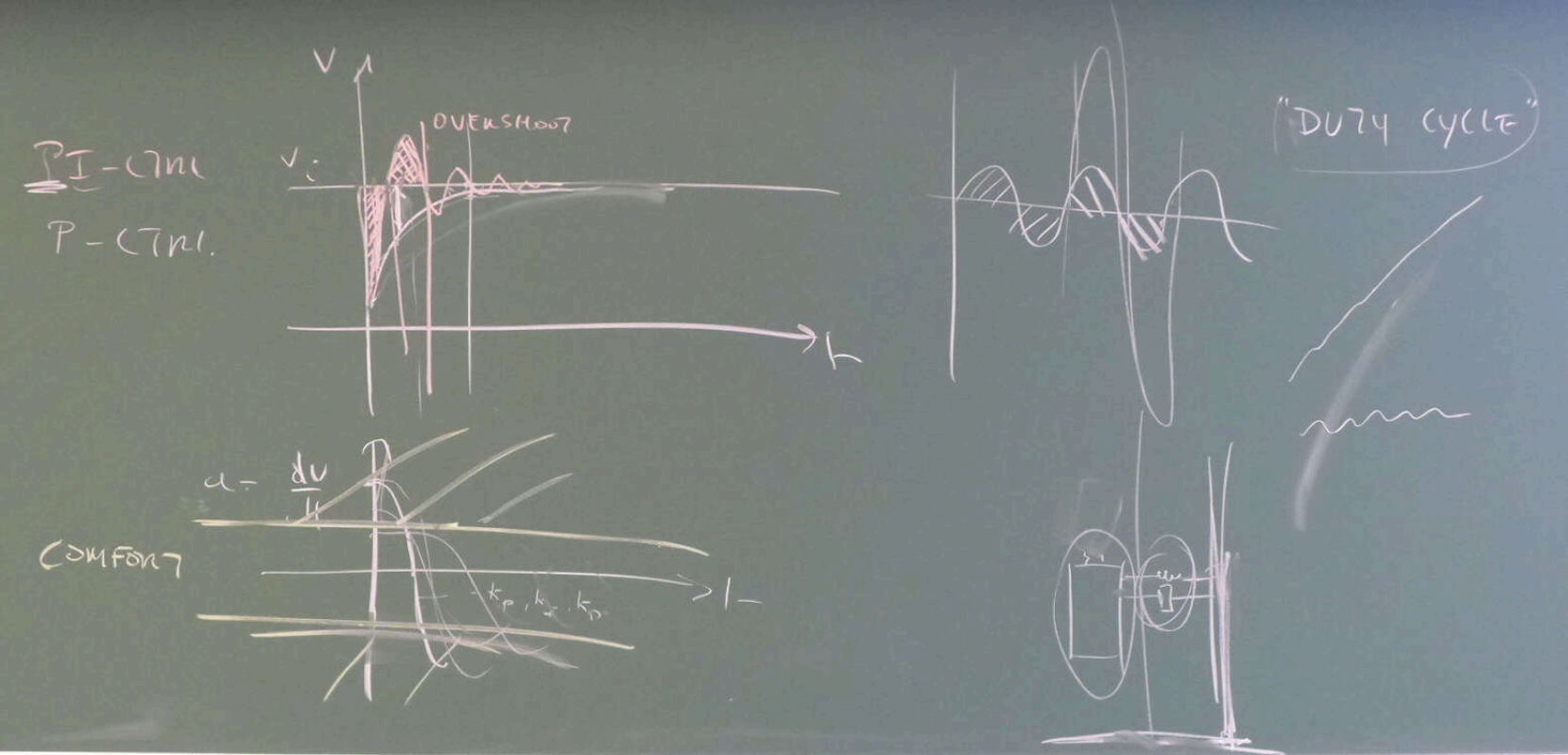
$\frac{dx}{dt} = A\cos(t) - B\sin(t)$ $\quad \left(= \frac{dx}{dt}(v) = A\right)$

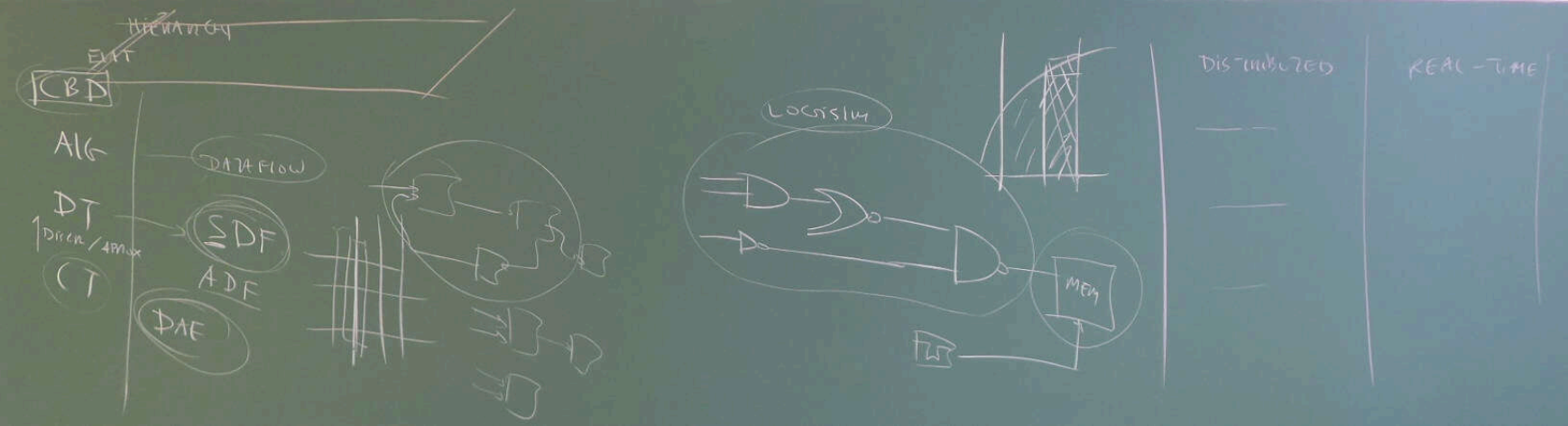
$\frac{dv}{dt} = -\left(A\sin(t) + B\cos(t)\right)$






ENVIRONMENT
WEATHER
DAY/NIGHT
SYS
PLANT
HEAT ACTUATOR
SENSOR
CTRL
DESIRED VALUE T
ENV
DEPLOY
OPTIMAL CTRL
CRITERIA

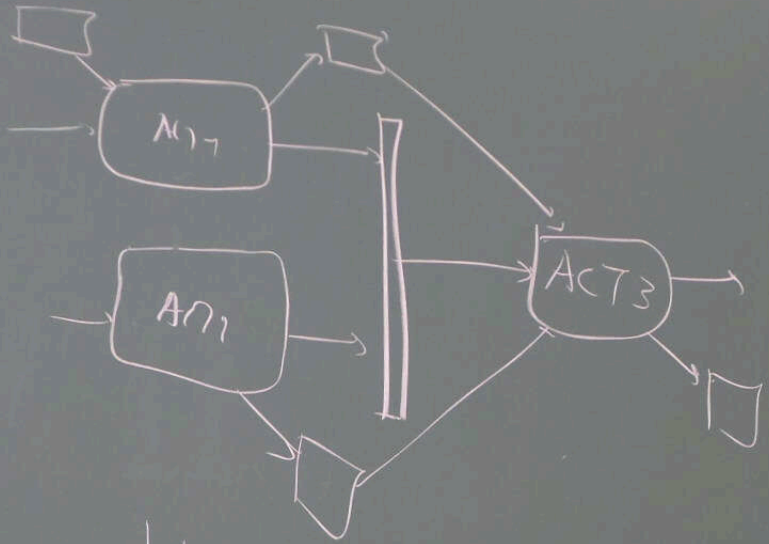

PI-CTRL
P-CTRL.
V
OVERSHOOT
v_i
t
a = dv/dt
COMFORT
k_p, k_i, k_D
t
"DUTY CYCLE"


HIERARCHY
EXIT
CBD
ALG
DATAFLOW
DT
DIRECT / APPROX
CT
SDF
ADF
DAE
LOGISIM
MEM
DISTRIBUTED
REAL-TIME

BPMN

YAWL

UML ACTIVD



DESCRIBE

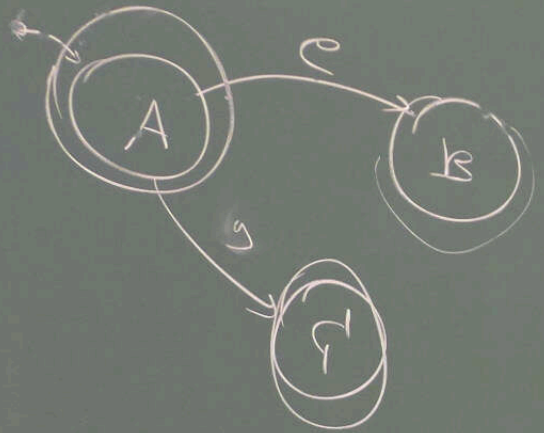
PRESCRIBE

PRESCRIBE

PN

A-SYNCH DATAFLOW

WORKFLOW



$E = \{e, g\}$

Q

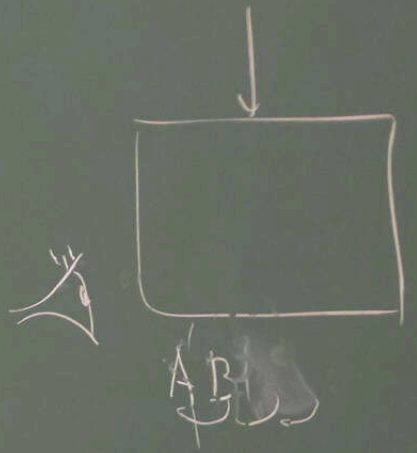
$S - X = \{A, B, C\}$

$\delta_f : X \times E \longrightarrow X$

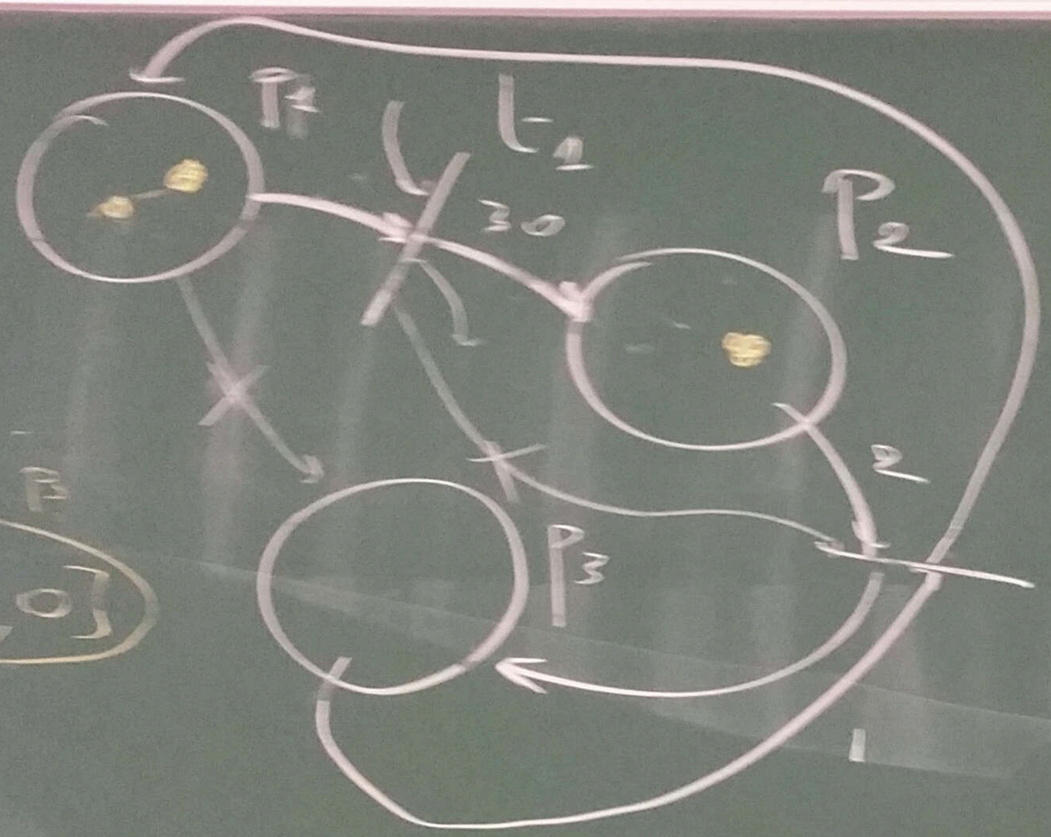
$(A, e) \longrightarrow B$

$(A, g) \longrightarrow C$

$\in E^*$ "ljje"



$X = N^3$

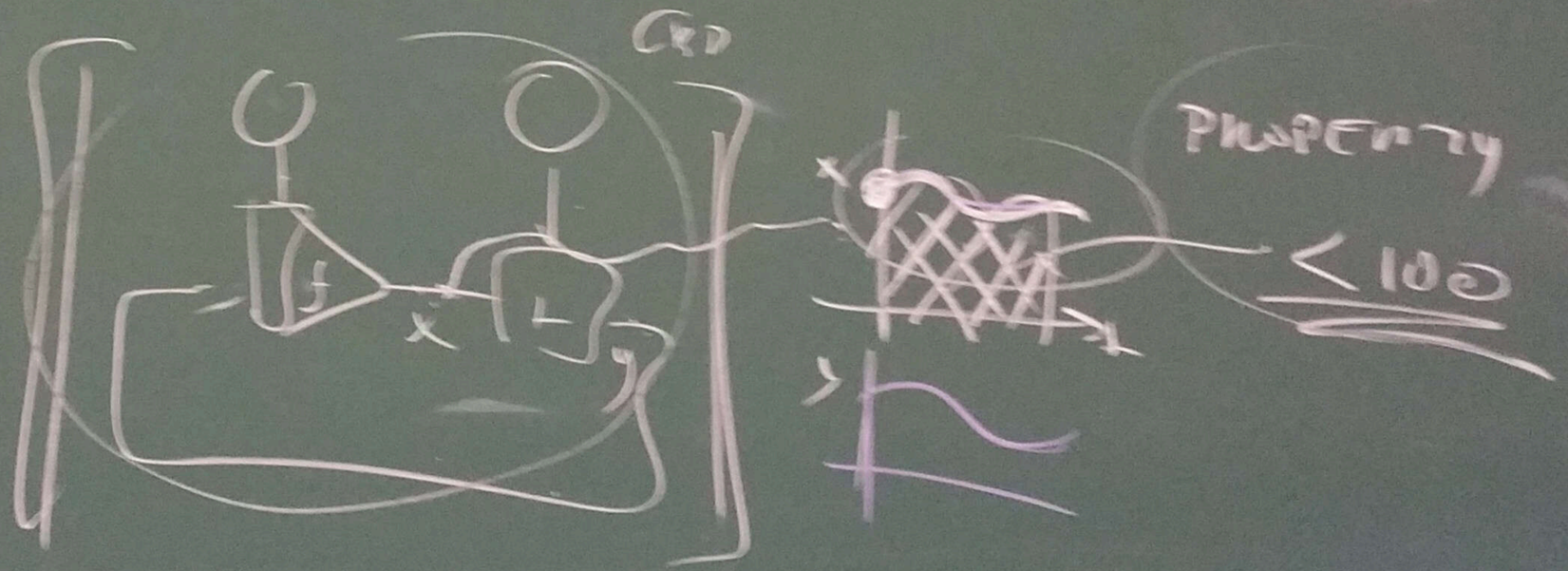


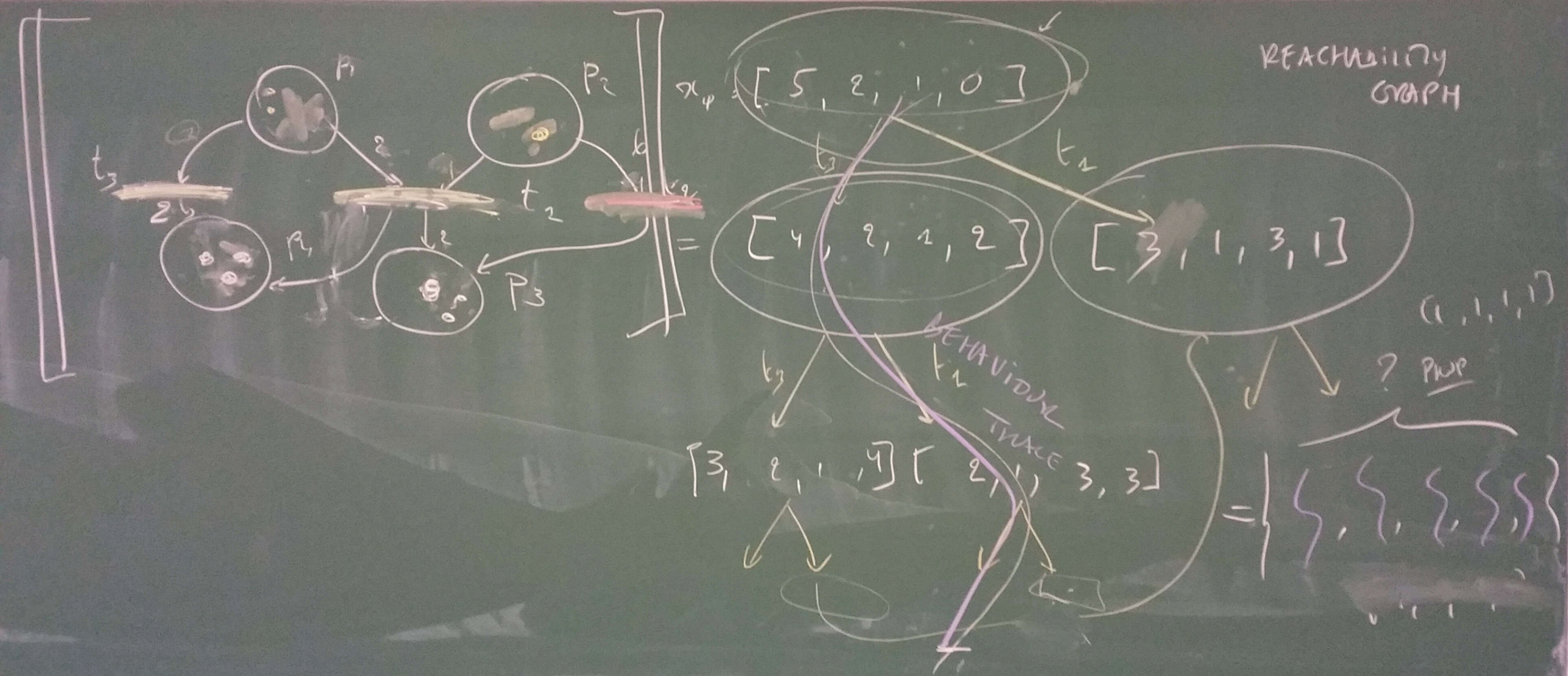
$P = \{P_1, P_2, P_3\}$

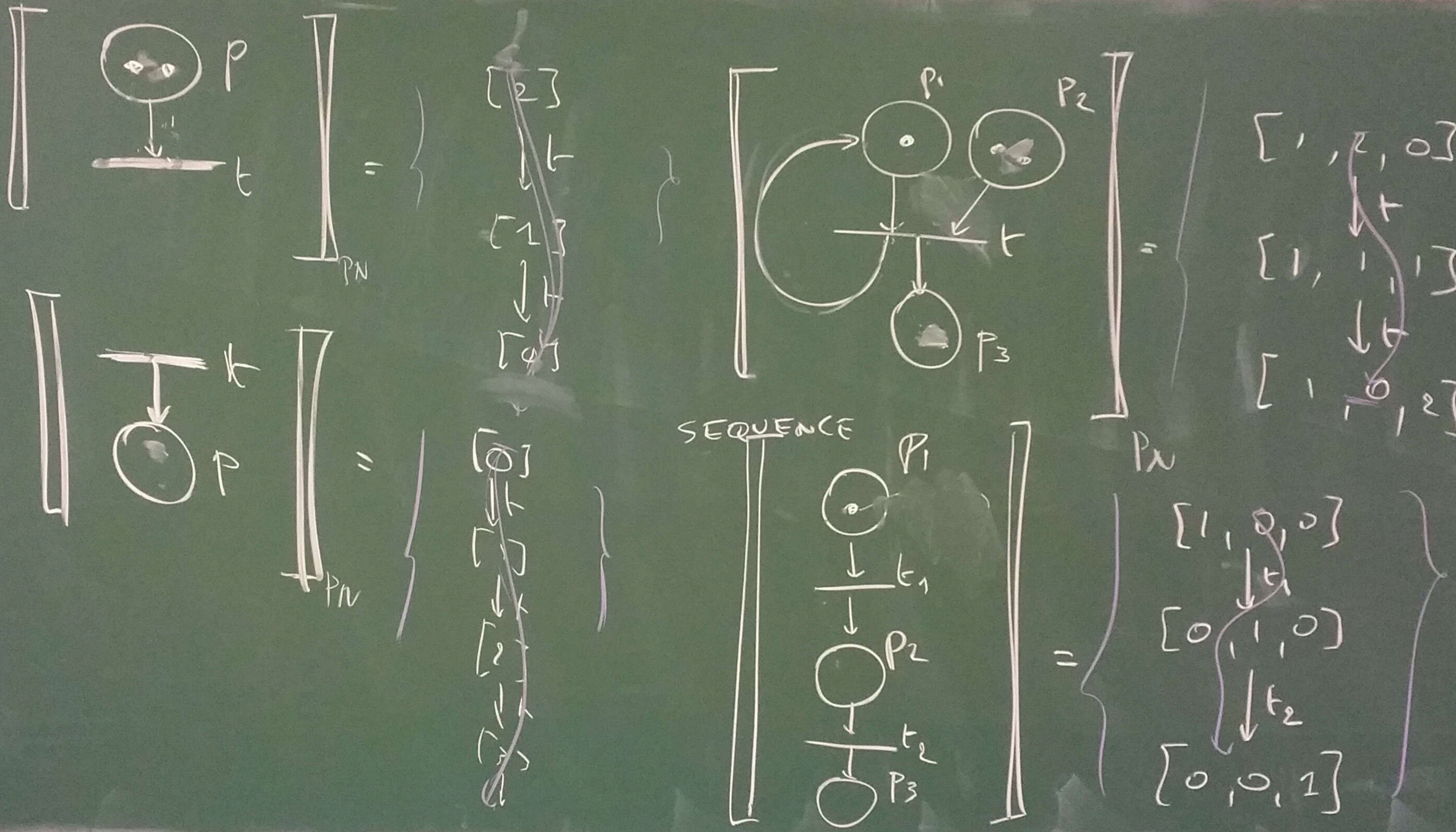
$T = \{L_1, L_2\}$

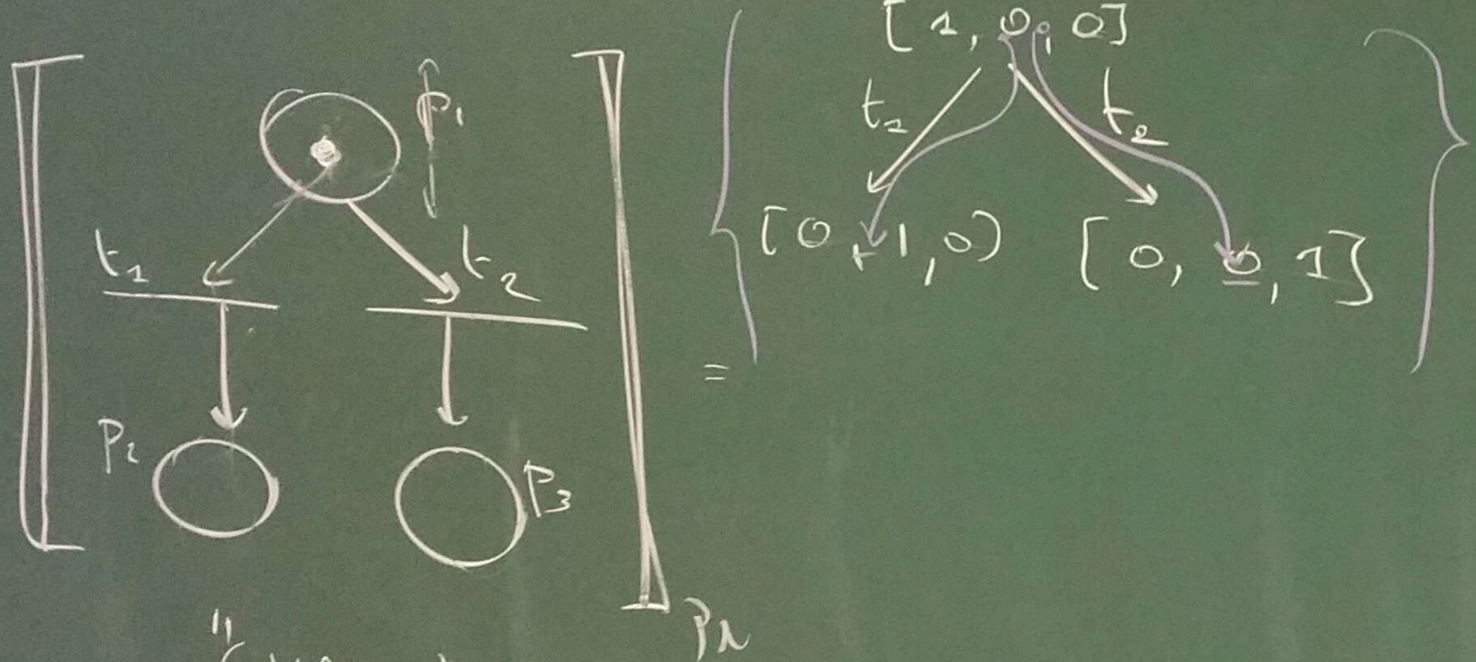
P_1 P_2 P_3

$\{2, 2, 0\}$




P1
P2
P3
P4
t1
t2
t3
t4
x_0 = [5, 2, 1, 0]
[4, 2, 1, 2]
[3, 1, 3, 1]
[3, 2, 1, 4]
[2, 1, 3, 3]
REACHABILITY GRAPH
BEHAVIOUR
TRACE
(1, 1, 1, 1)
? PROP


p
t
PN
[2]
t
[1]
t
[0]
p1
p2
t
p3
[1, 1, 0]
t
[1, 1, 1]
t
[1, 0, 1]
SEQUENCE
p1
t1
p2
t2
p3
[1, 0, 0]
t1
[0, 1, 0]
t2
[0, 0, 1]



"CHOICE"

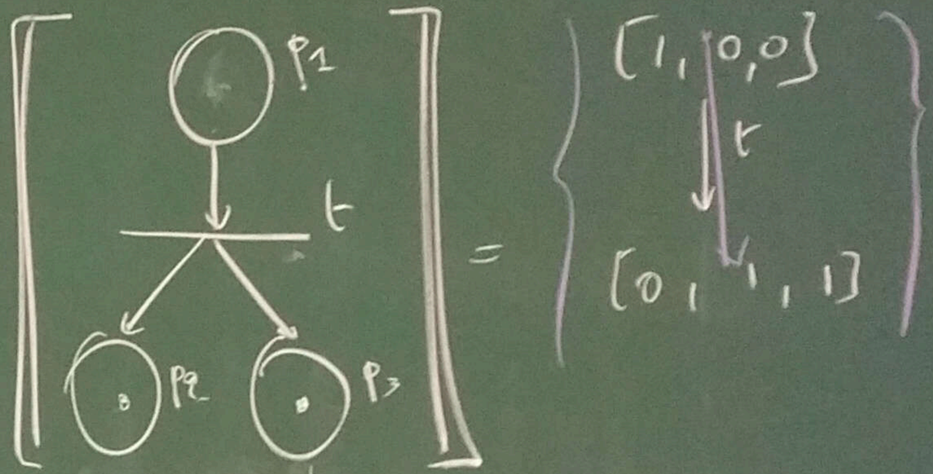
(IF THEN ELSE

XOR)

P/T NETS

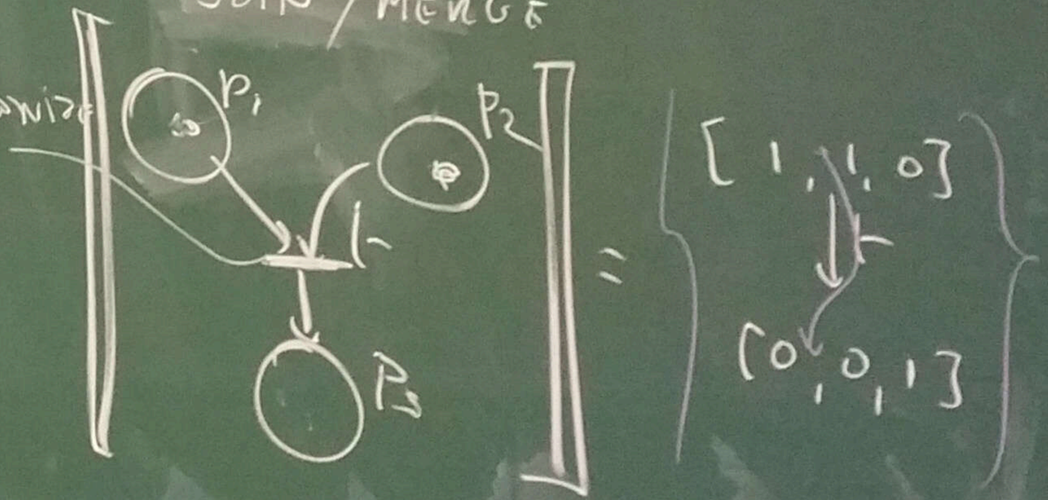
PLACE - TRANSITION

"SPLIT" SEQ → //

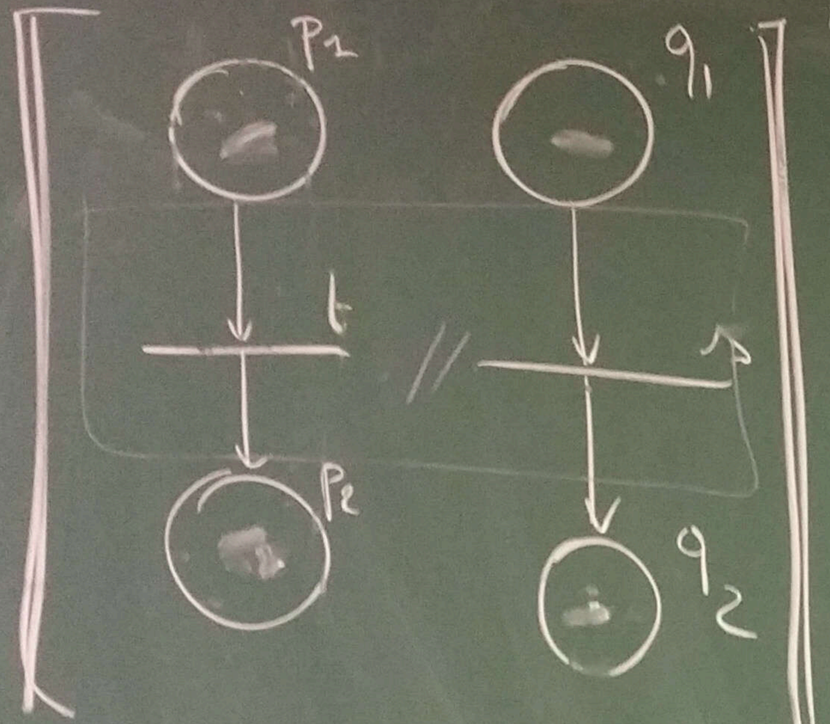


JOIN / MERGE

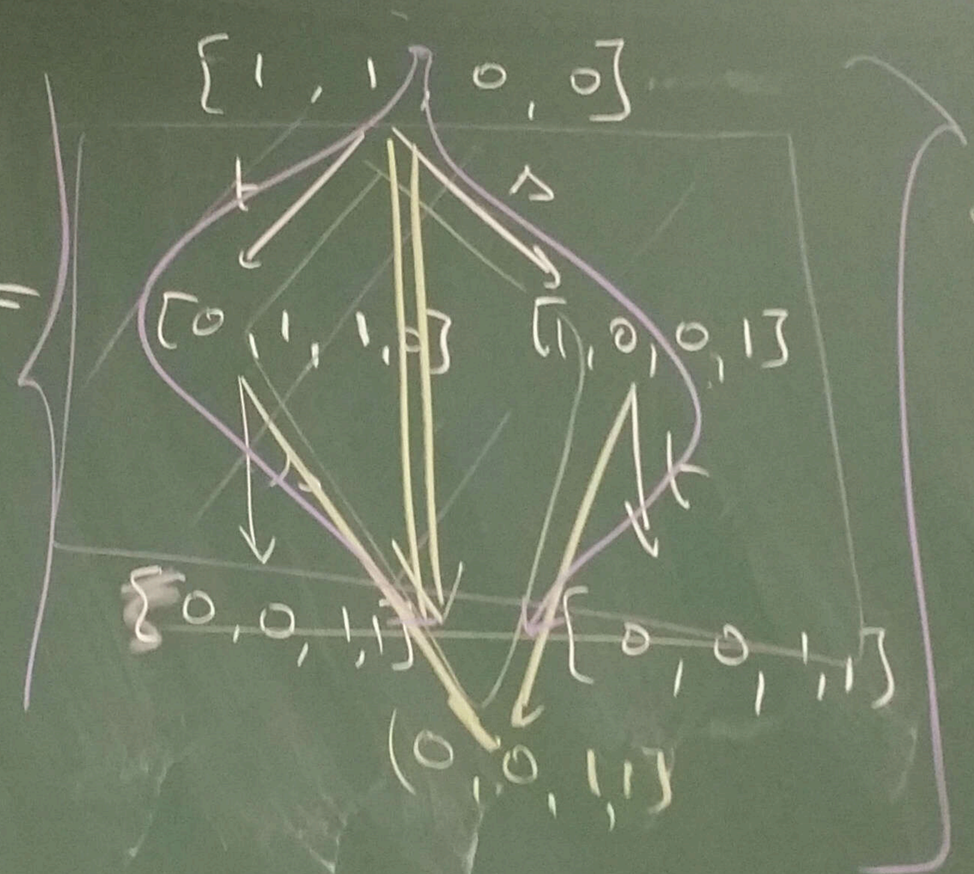
SYNCHRONIZE



CONCURRENCY
"INTERLEAVING"

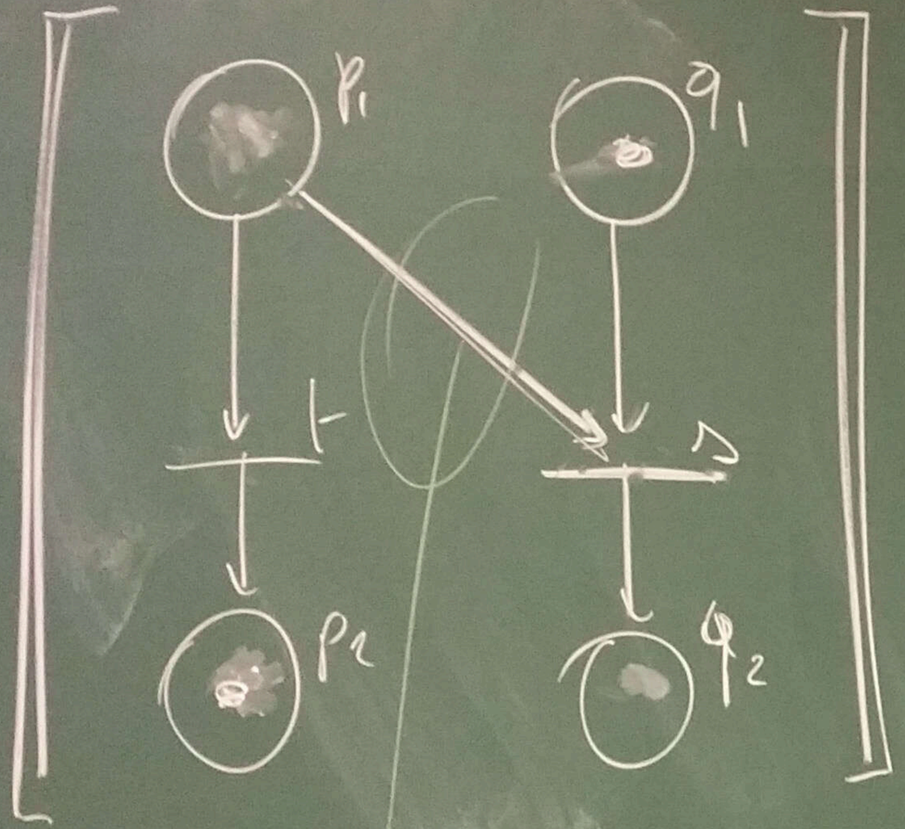


=



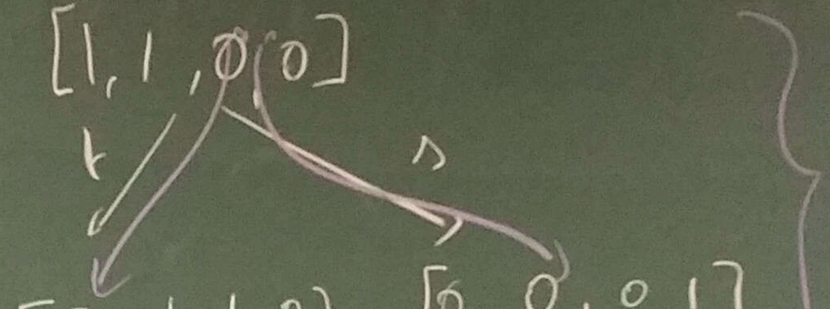
PARALLEL INDEPENDENT

CONFLUENT

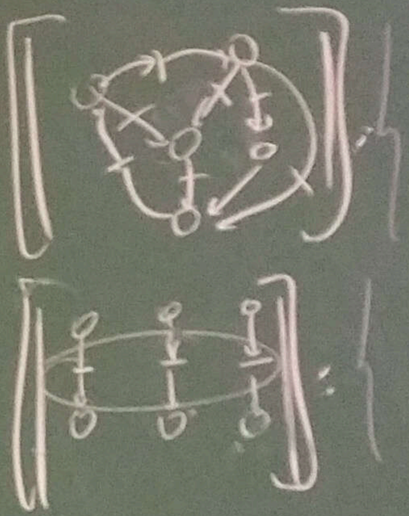


PARAMETER DEPENDENT

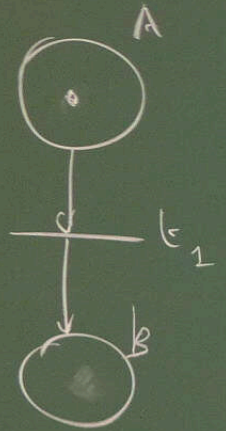
=



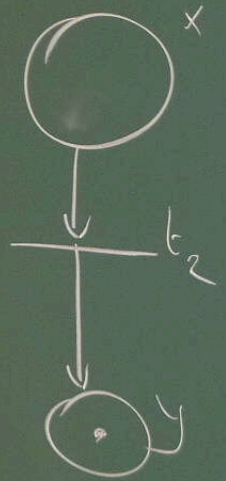
INTUITION

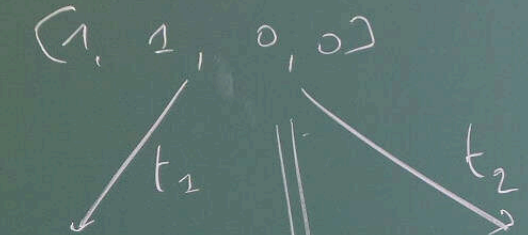


PROC 1



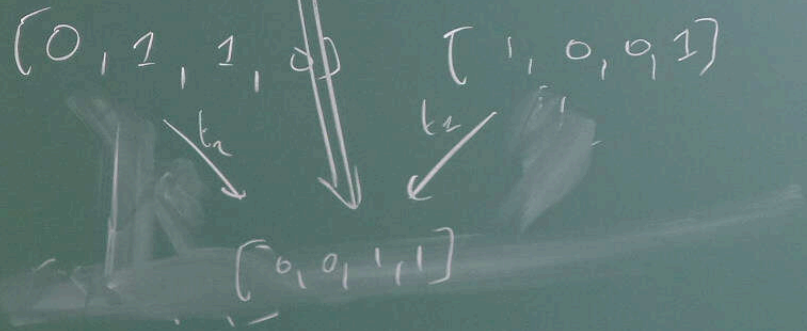
PROC 2





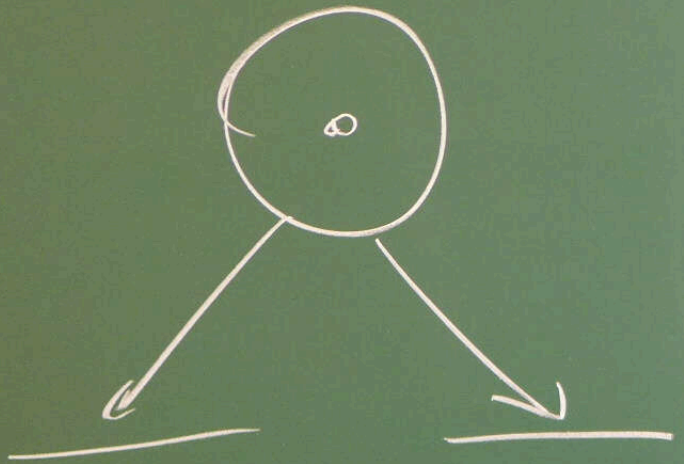
// INDEPENDENCE

CONFLUENCE

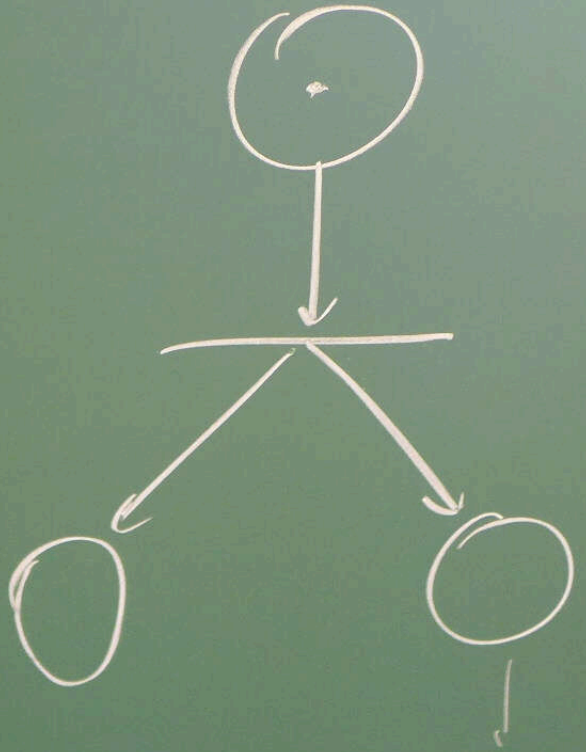


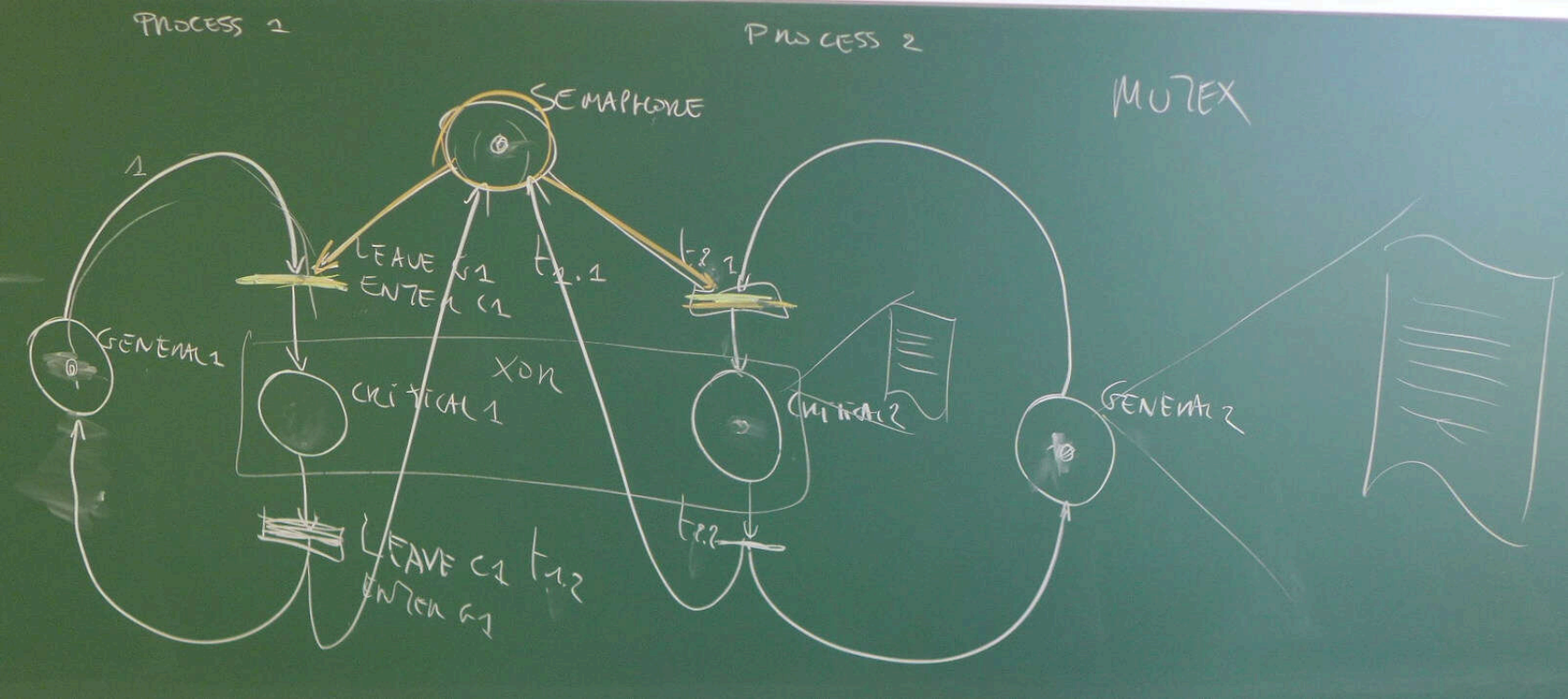
PLACE CONDITION

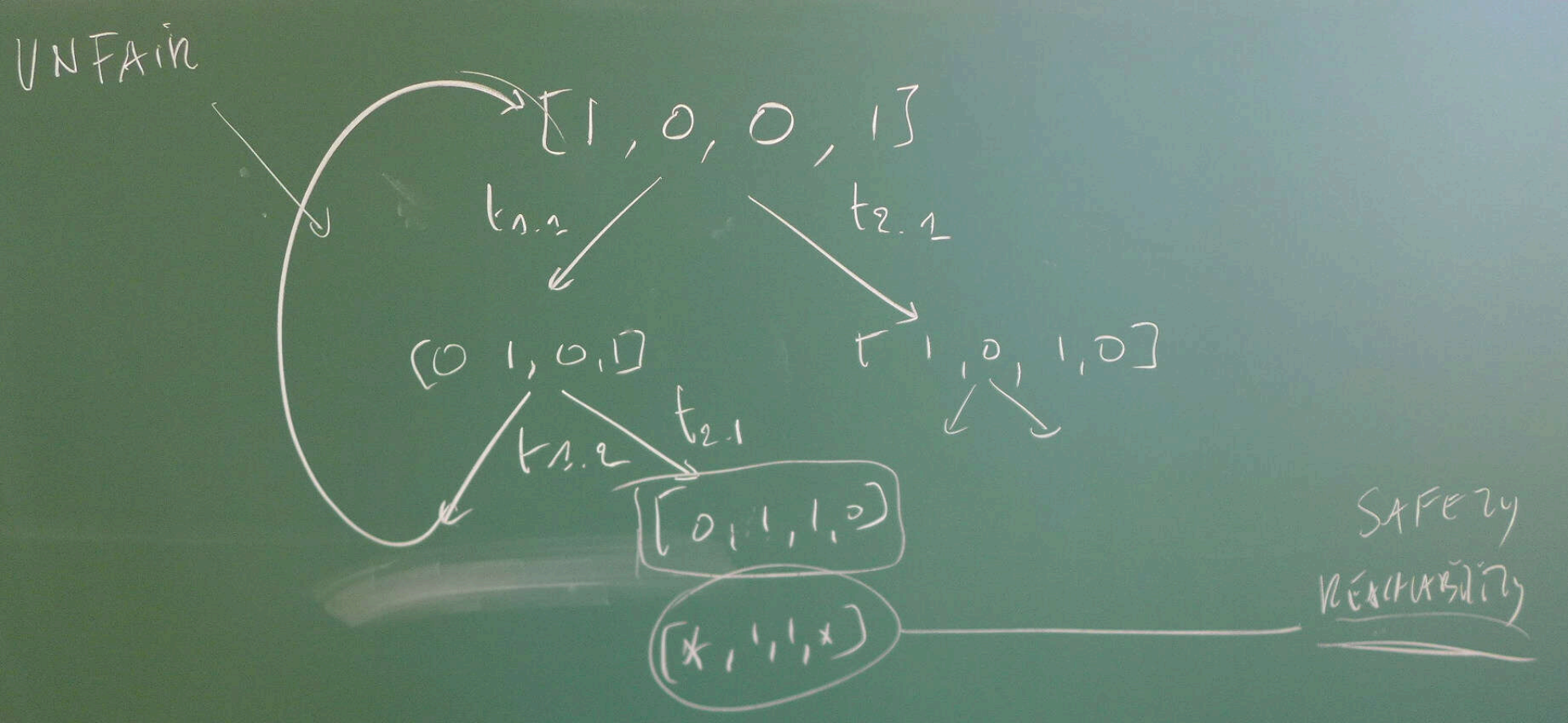
Choice

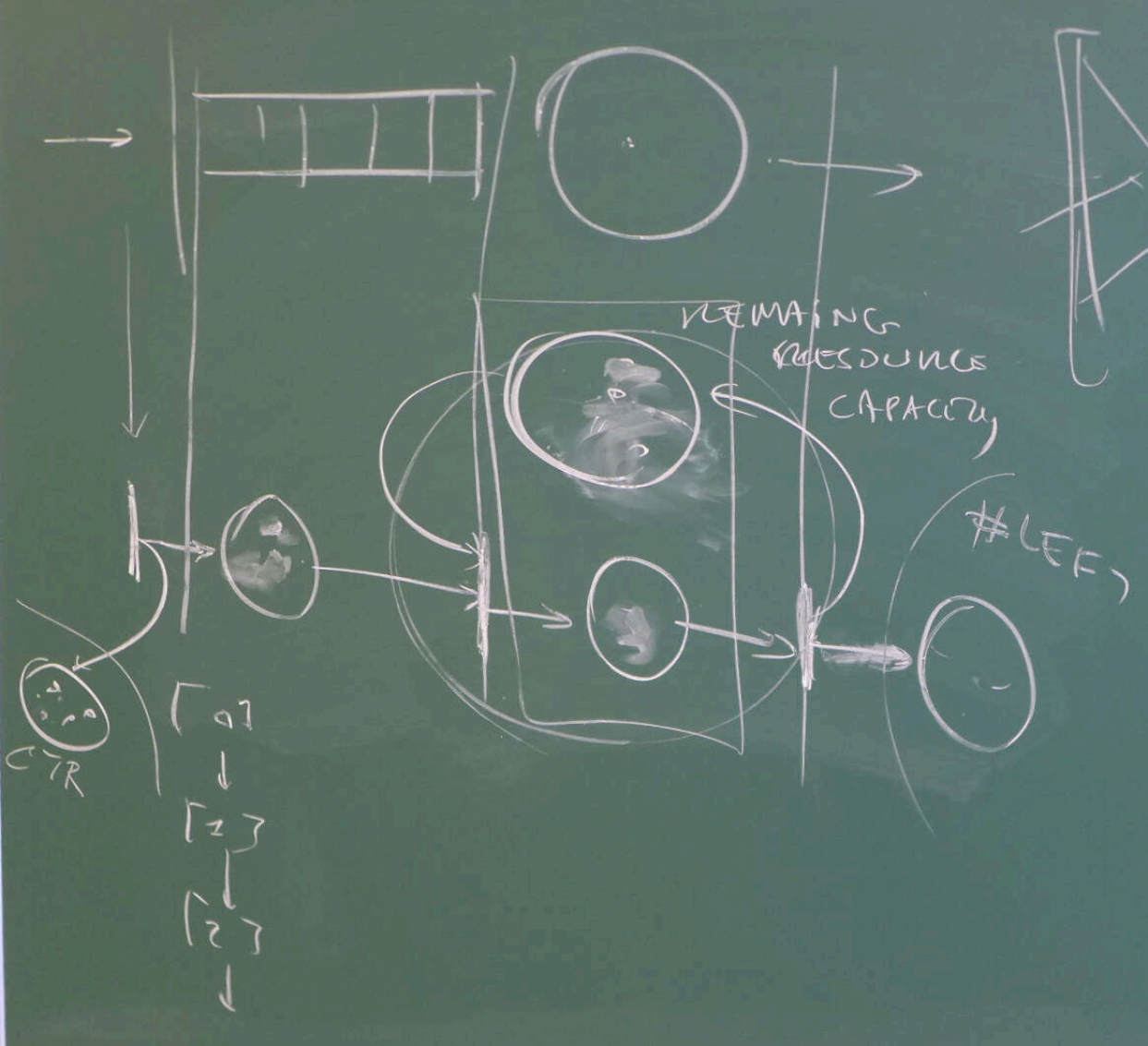


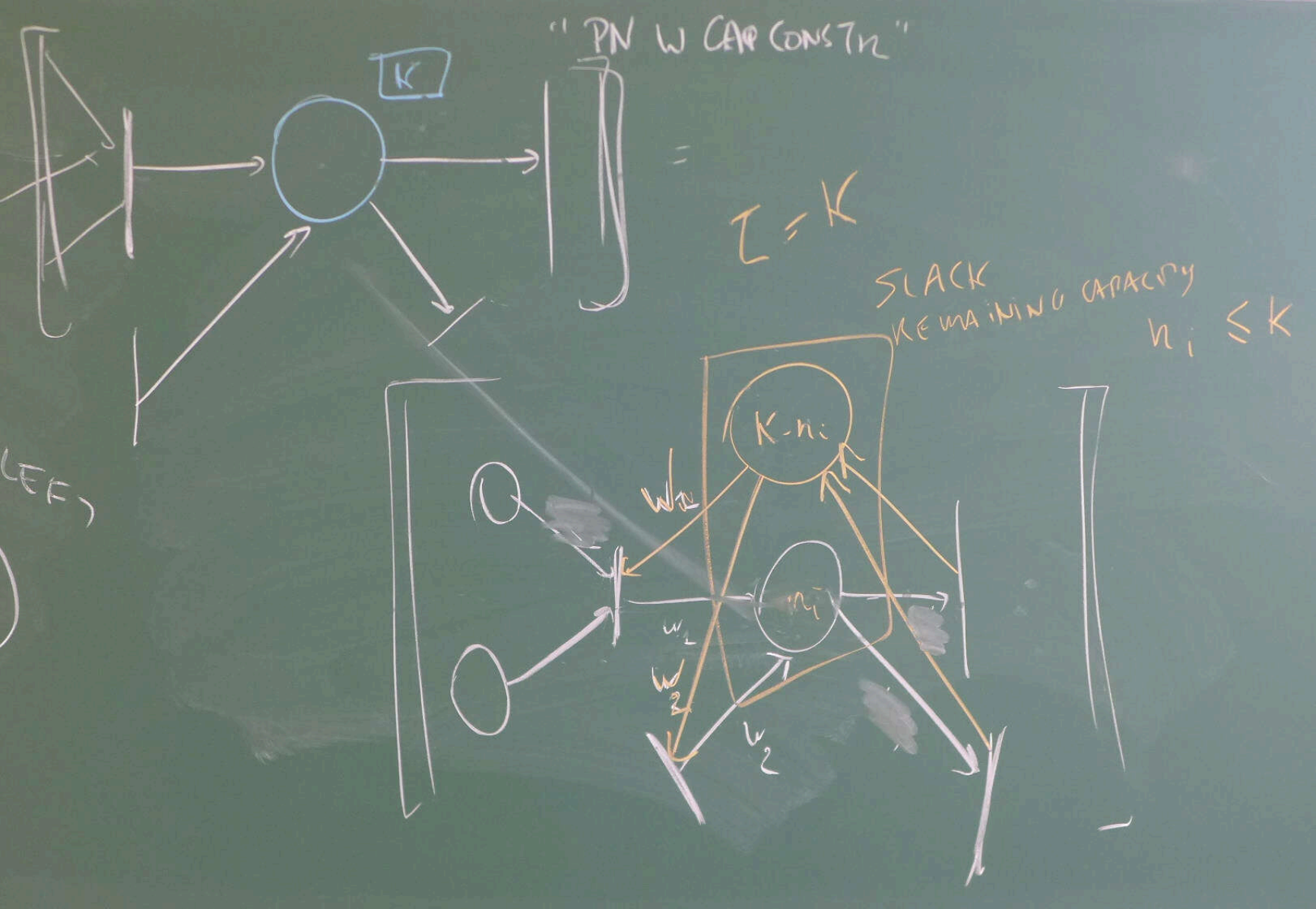
Split




PROCESS 1
PROCESS 2
SEMAPHORE
MUTEX
1
LEAVE C1
ENTER C1
t1.1
t2.1
GENERAL 1
XOR
CRITICAL 1
CRITICAL 2
GENERAL 2
LEAVE C1
ENTER C1
t1.2
t2.2


UNFAIR
[1, 0, 0, 1]
t1.1
t2.1
[0 1, 0, 1]
[1, 0, 1, 0]
t1.2
t2.1
[0, 1, 1, 0]
[*, 1, 1, *]
SAFETY
REACHABILITY


REMAINING
RESOURCE
CAPACITY
#LEFT

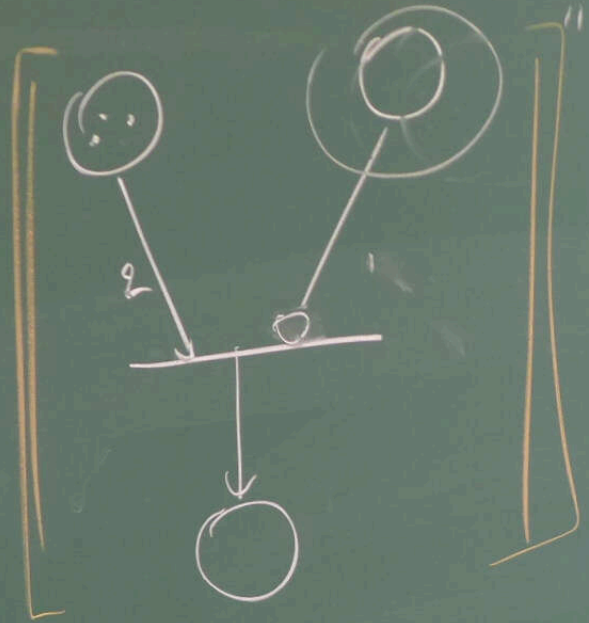

"PN W CAP CONSTR"
K
Σ = K
SLACK
REMAINING CAPACITY
$n_i \leq k$
K - n_i
n_i
w_1
w_2
w_2
w_2

INHIBITOR ARCS

↓

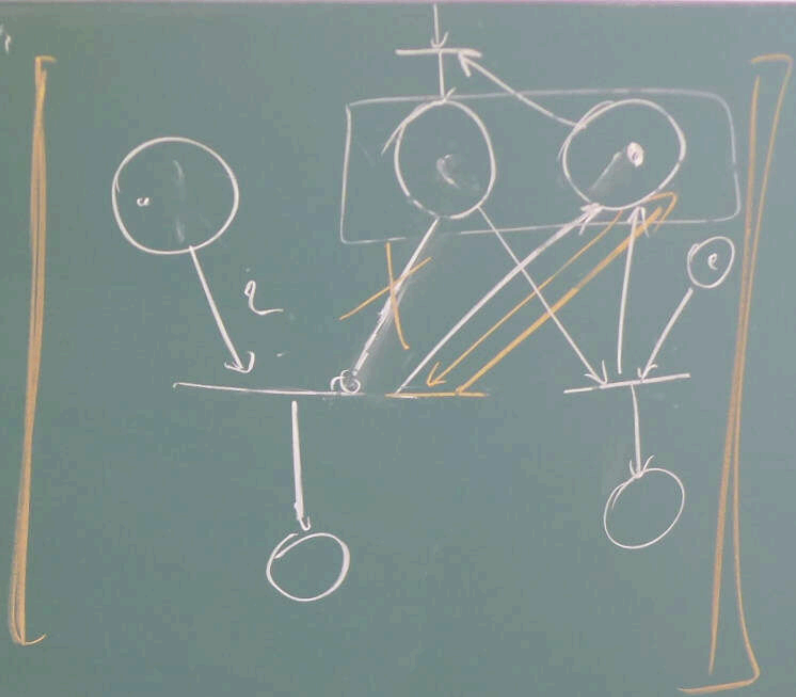
TURING COMPLETE

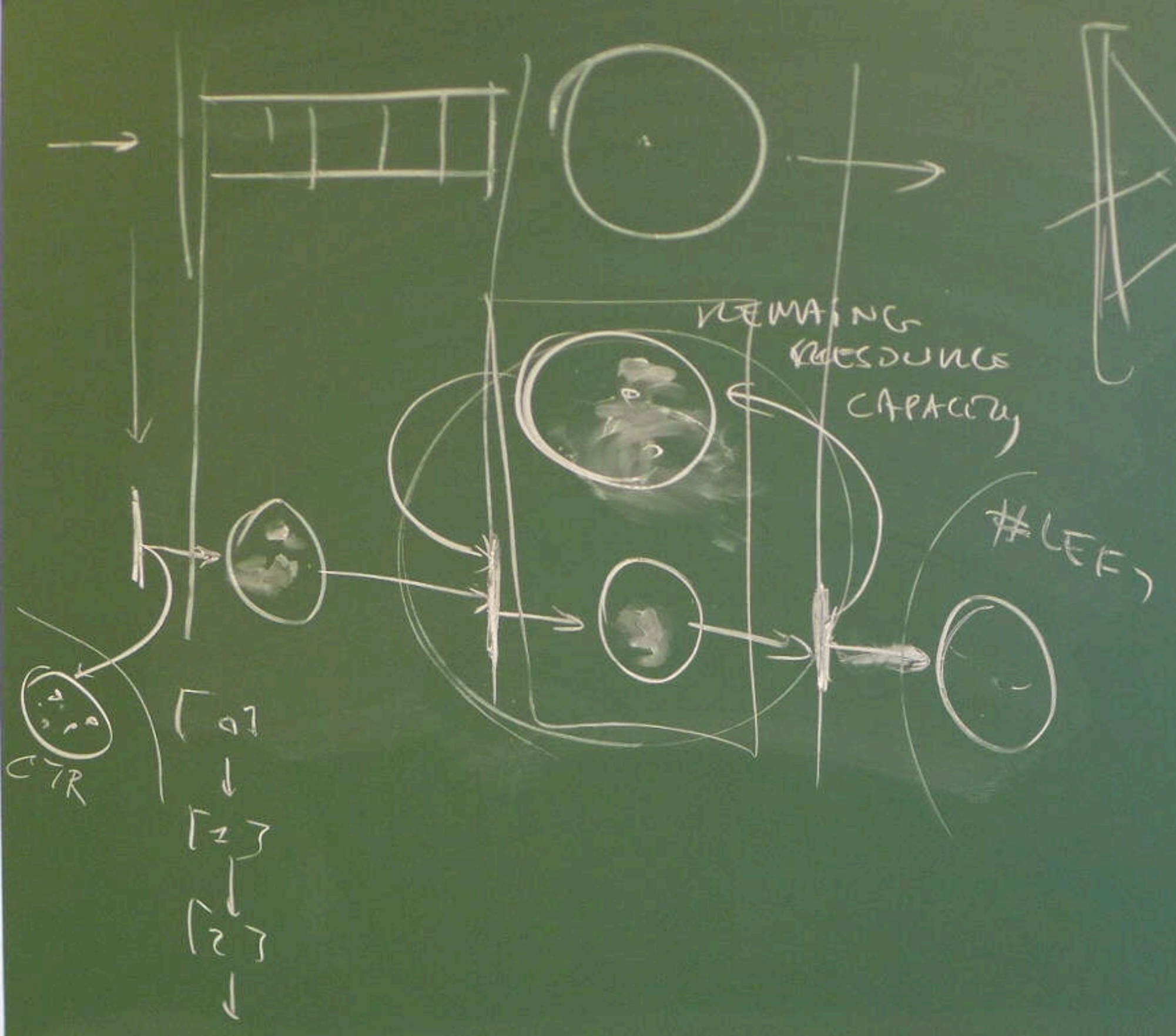
k



"BOUNDED"

=

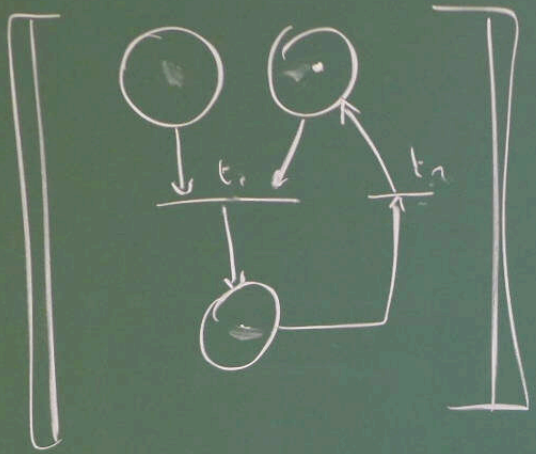


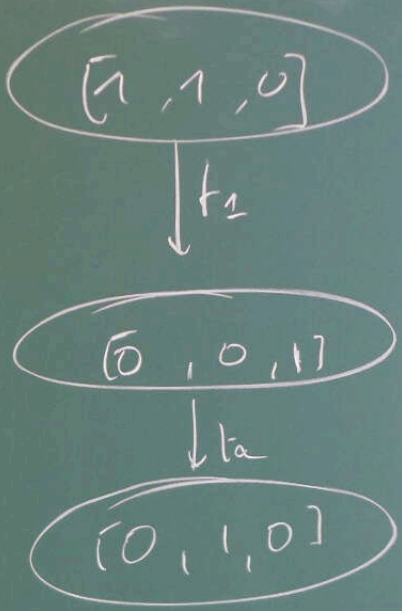

REMAINING
RESOURCE
CAPACITY
#LEFT
CTR
[0]
[1]
[2]

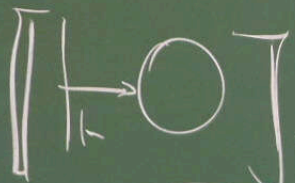
FSA

TIME

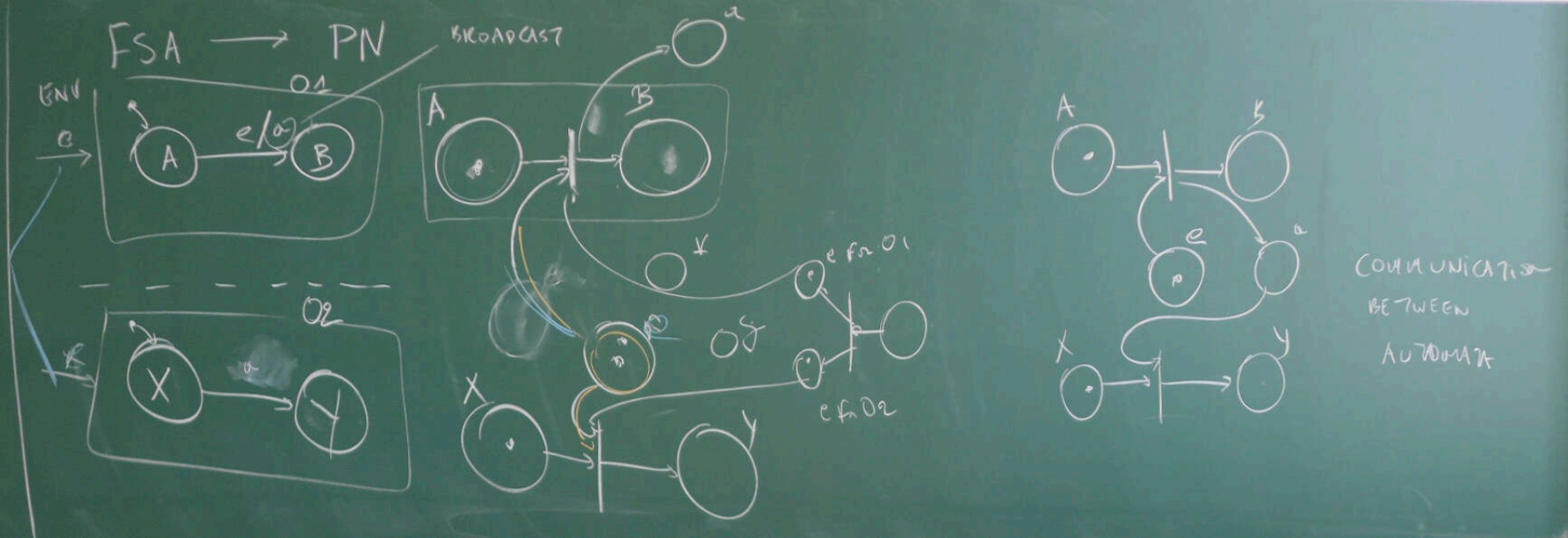
FAIRNESS

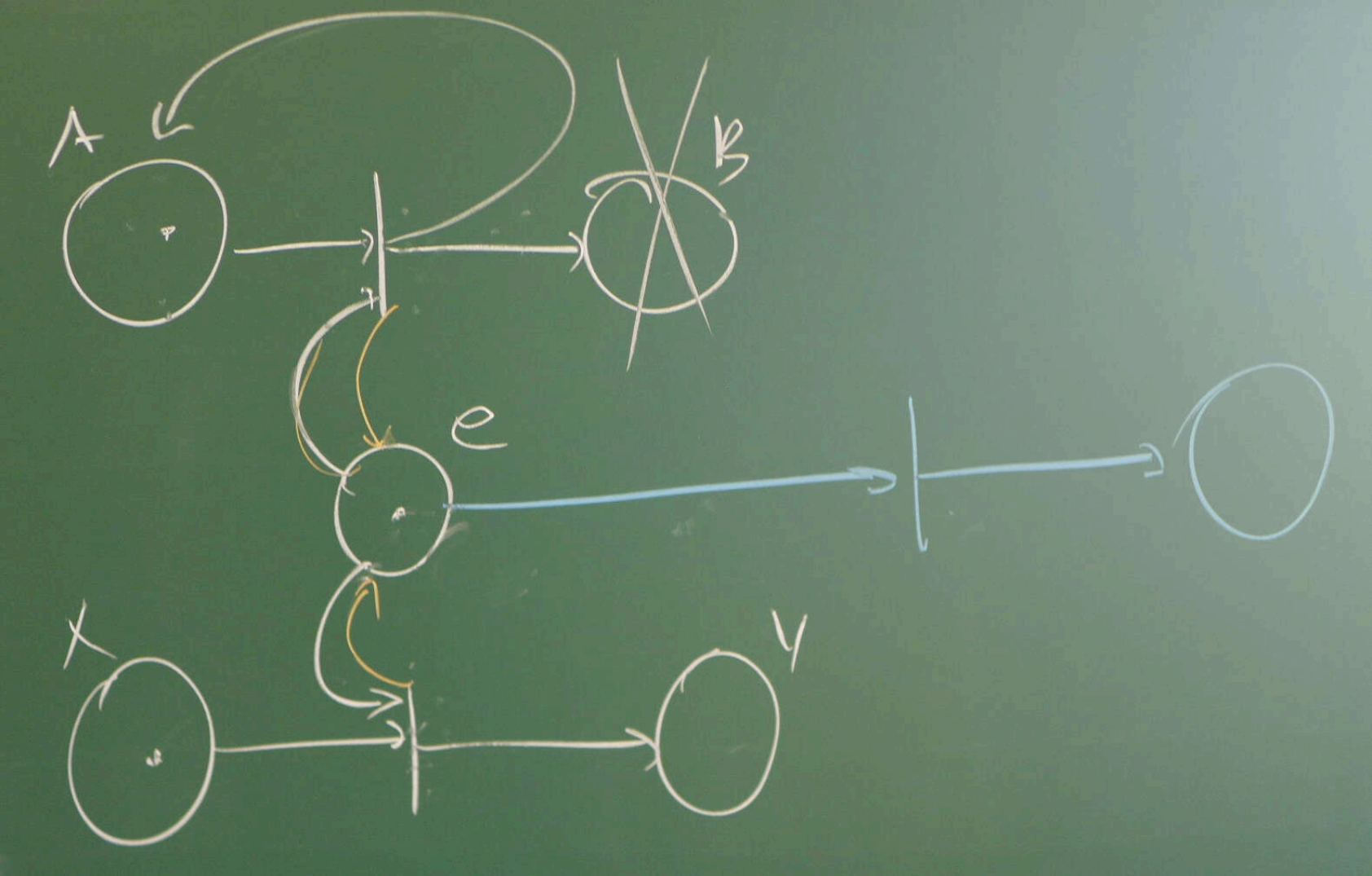


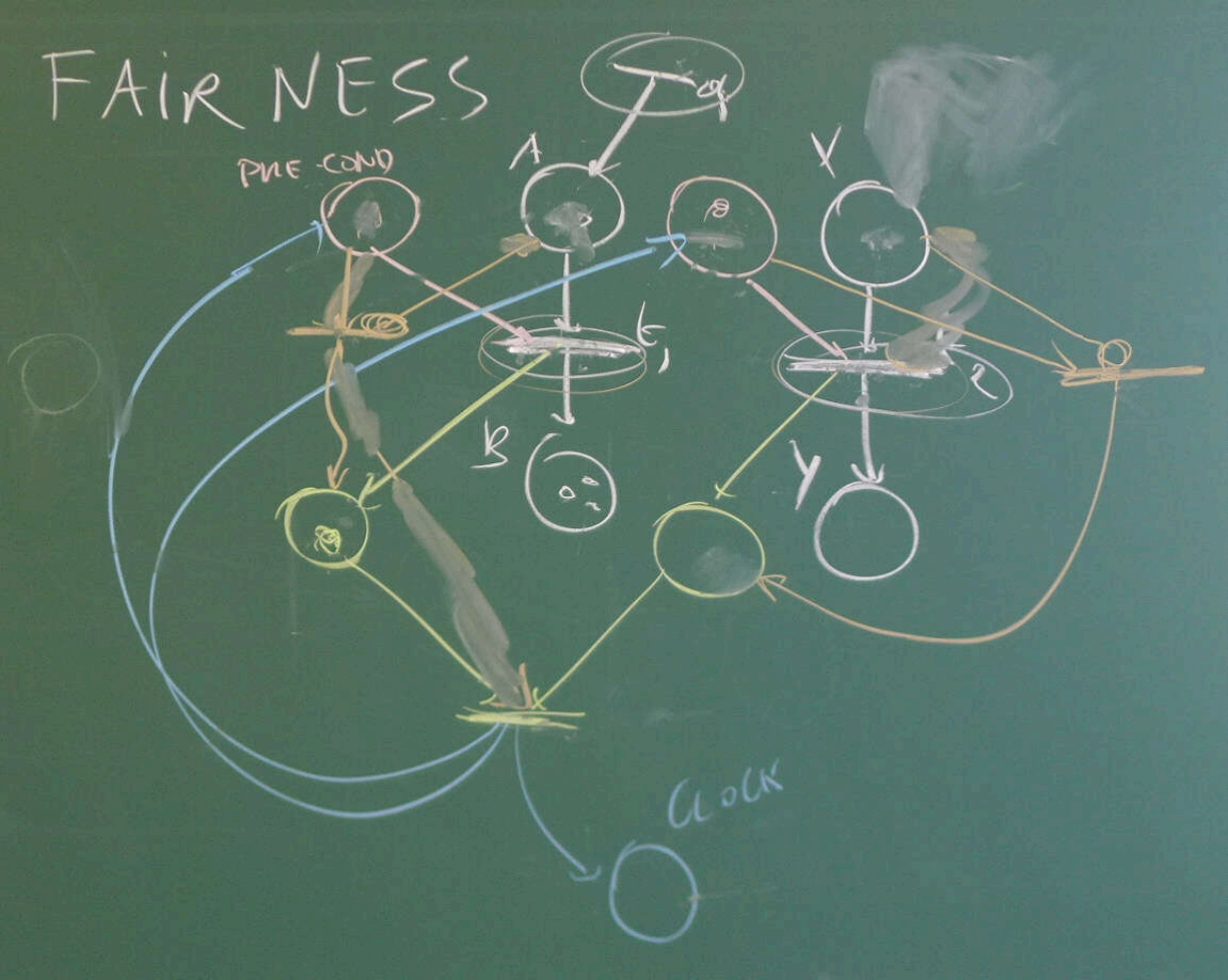




PN $\xrightarrow{[\,]}$ FSA

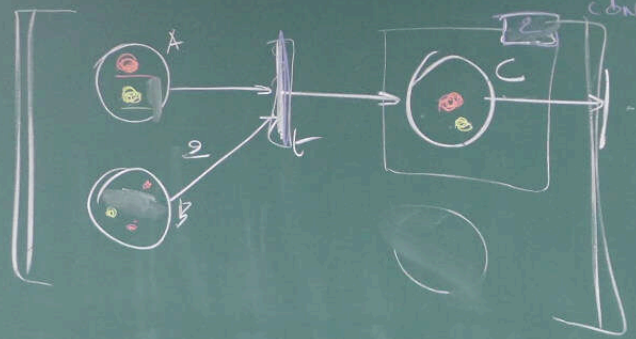

FSA → PN
BROADCAST
ENV
e
O1
A
e/a
B
O2
X
Y
a
A
B
b
c
e from O1
O3
c from O2
X
Y
A
b
e
a
X
Y
COMMUNICATION
BETWEEN
AUTOMATA


A
B
e
X
Y

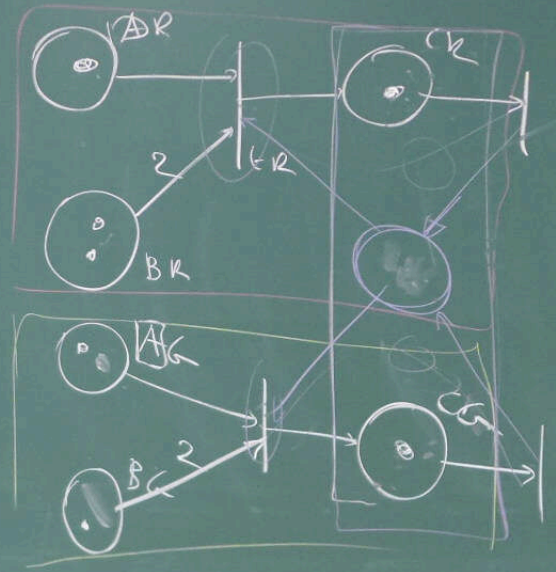

FAIRNESS
PRE-COND
A
B
X
Y
CLOCK

COLOURED PN

① ② CAPACITY CONSTRAINT

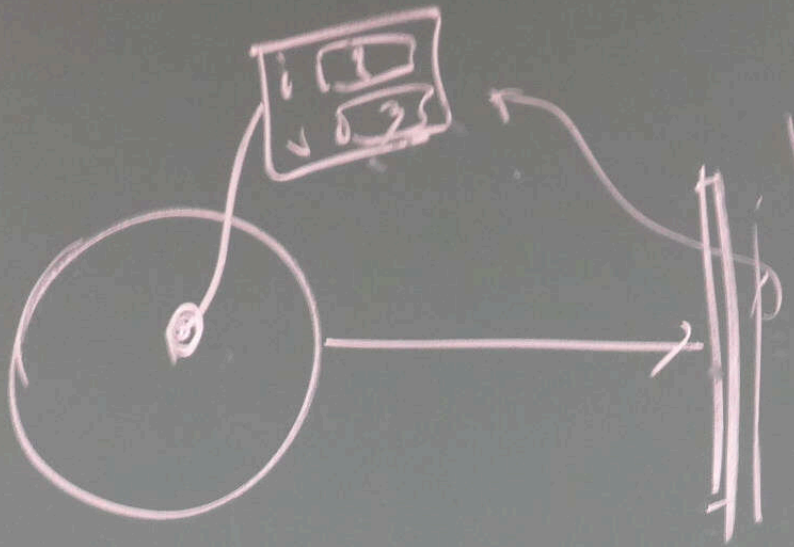


=



① + ② = EXPRESSIVENESS AS P/T NETS

"SYNTACTIC SUGAR"



AIG PN

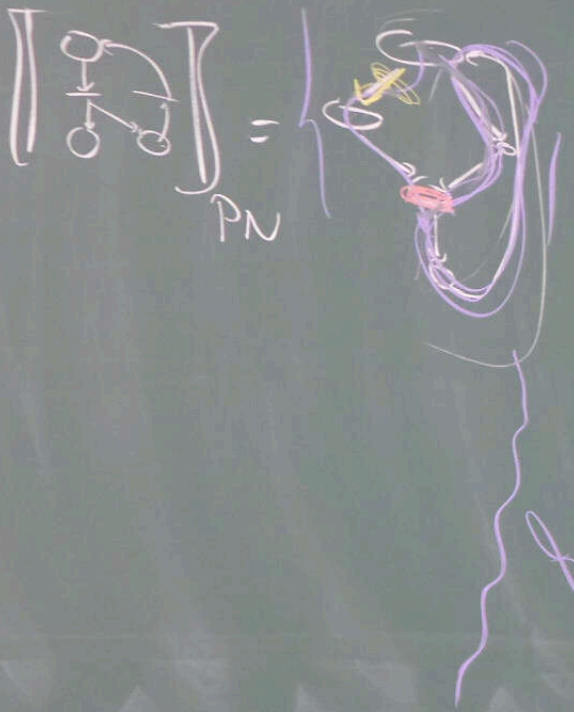
MODEL PN

SATISFACTION

PROPERTIES

LTL, CTL, STL ...

REQUIREMENTS



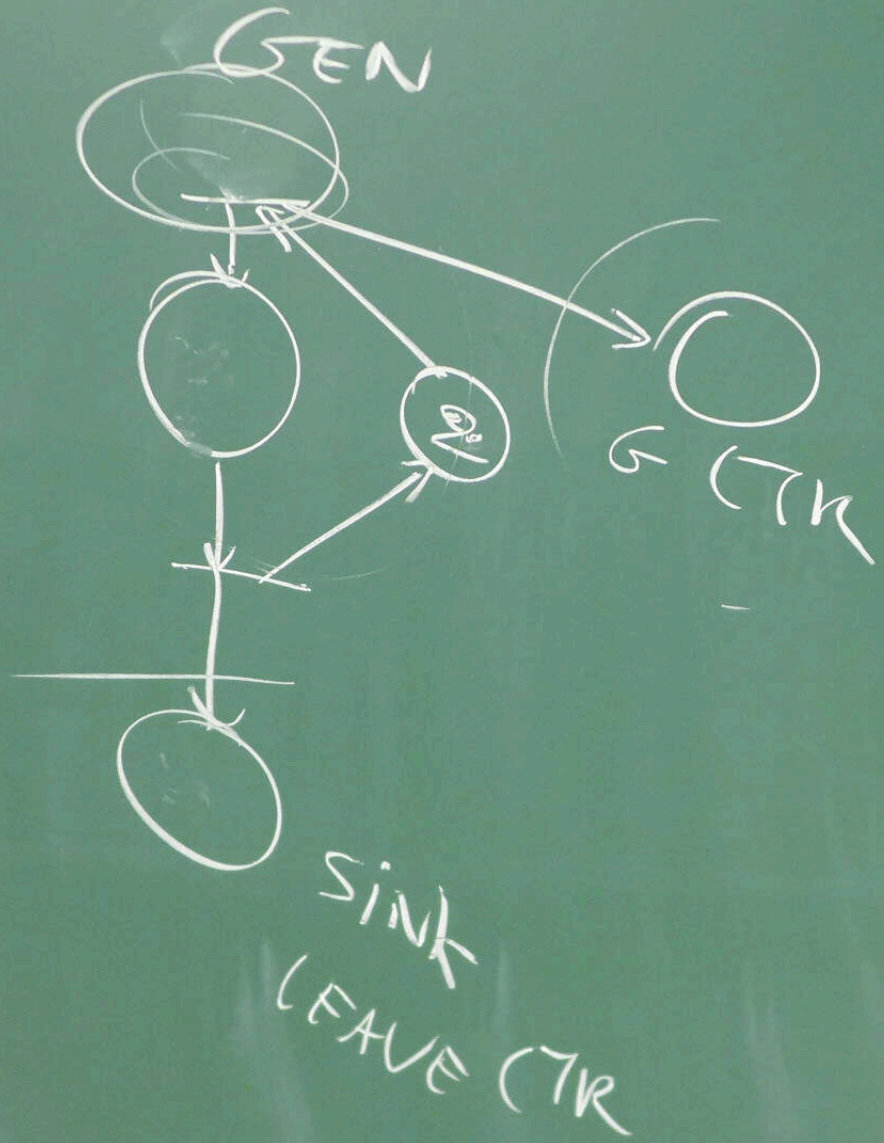
$\models$

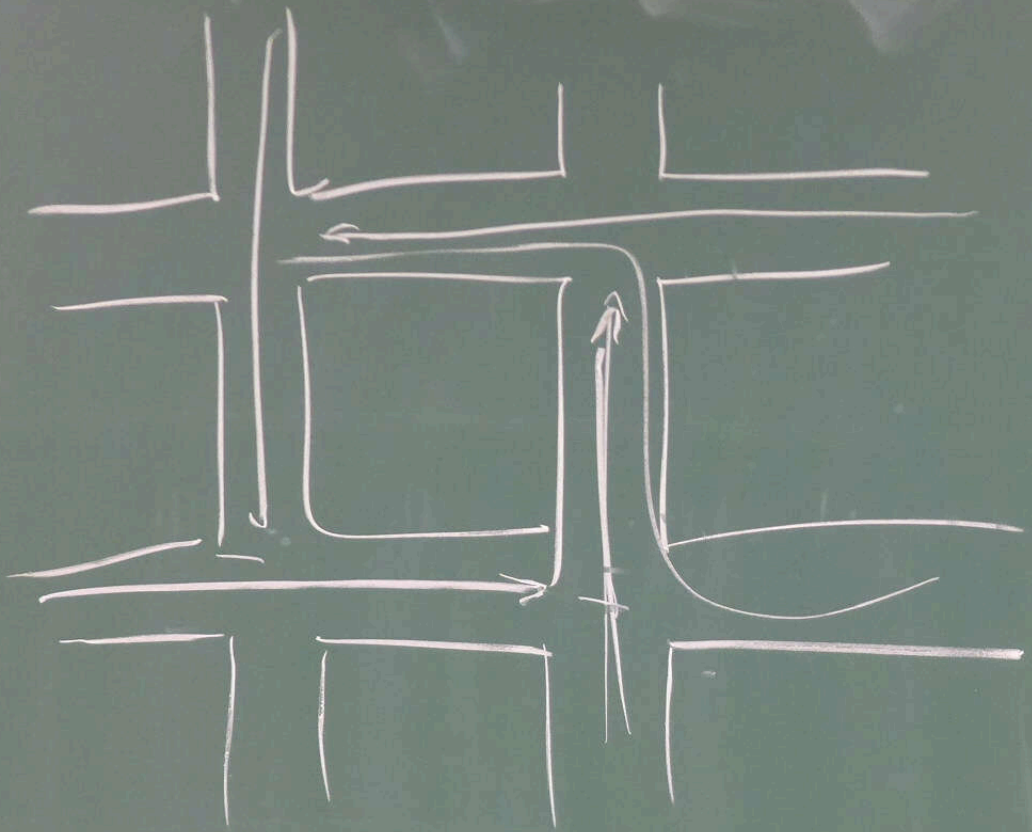
FAIR

NOT REACHABLE

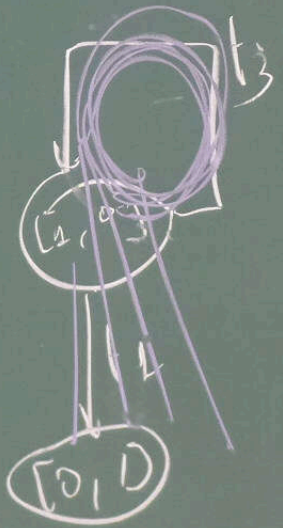
EF .

AF .


GEN
ENTR
SINK
LEAVE CTR

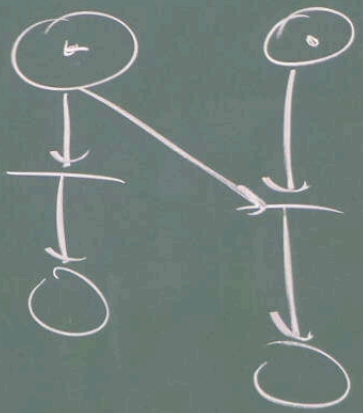


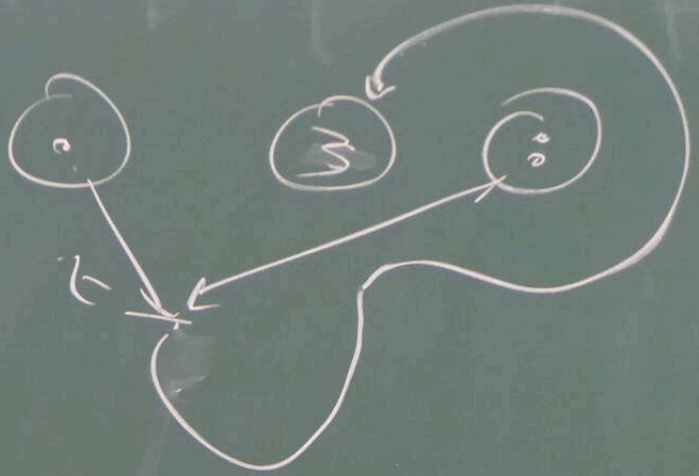
GRIDLOCK

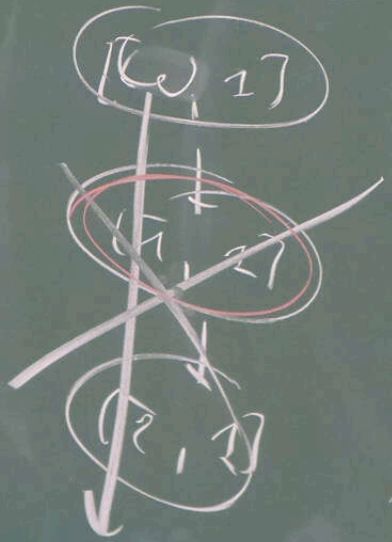


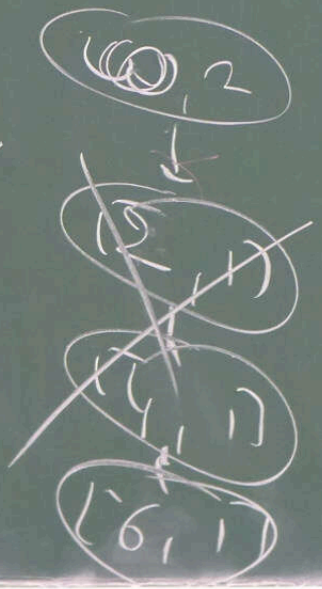
$\bar{y} = [1, 0, 1]$

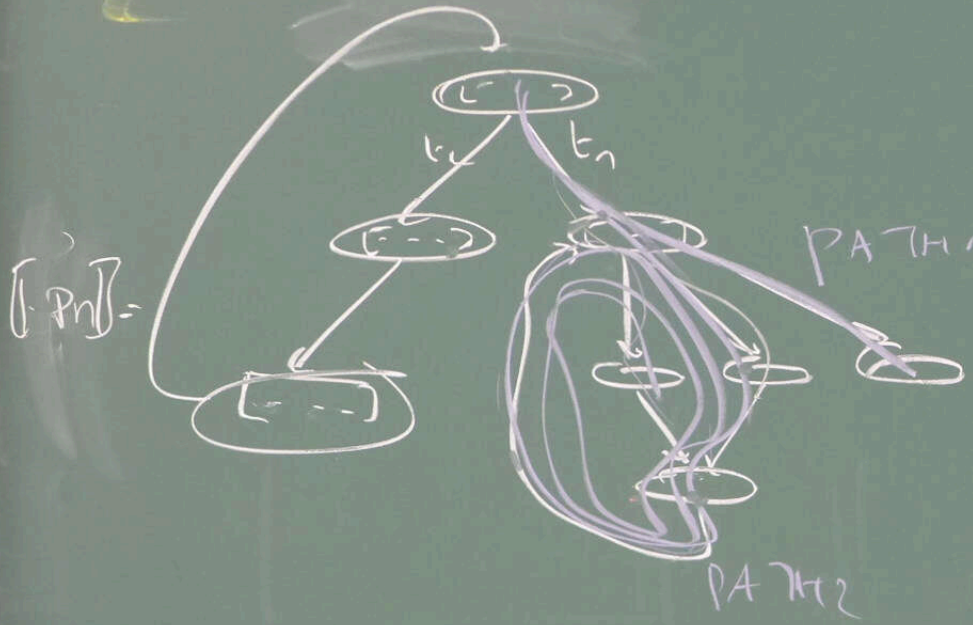
$\bar{x} = [1, 3, 2]$











$= \{ \text{PATH2}, \text{PATH2}, \ldots \}$

$\forall \quad \exists$

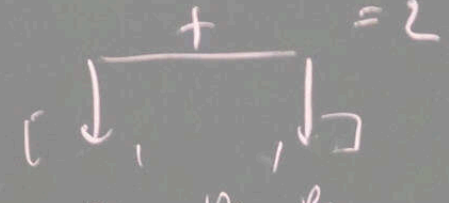
$A \quad E$

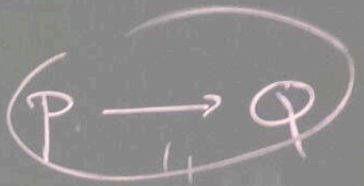
CTL

True
False

p OVER STATES

$\wedge$, $\vee$, $\neg$





$\neg p \vee q$

G p — for All

F p — Eventually

X p — NEXT

p U q

LTL

$FX_p = F_p$