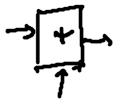
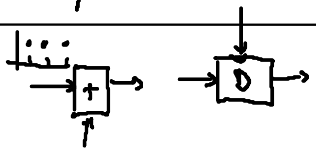
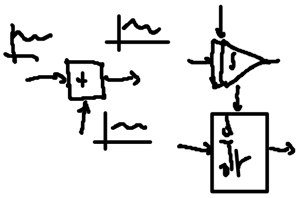
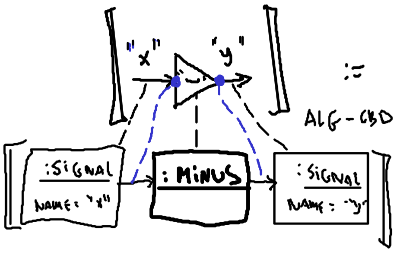
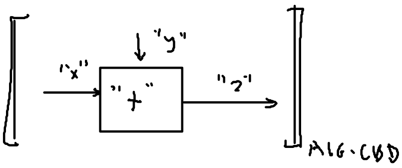
| TIME ↓ | HIERARCHY / FLAT CBD | SYNTAX (FLATTEN) | SEMANTICS: DENOTATIONAL "WHAT" | SEMANTICS: OPERATIONAL "HOW" |
|---|---|---|---|---|
| {NOW} | ALGEBRAIC (ALG-CBD) |  NO LOOPS | ✓ | ✓ |
| | | WITH LOOPS | ✓ | |
| ℕ | DISCRETE-TIME (DT-CBD) |  | | |
| ℝ | CONTINUOUS-TIME (CT-CBD) |  | | |




$:=$ $[\![\text{"y"}]\!] = -[\![\text{"x"}]\!]$ $\quad$ $mvy := -\, mvx$

$mvx, mvy \in \mathbb{R}$

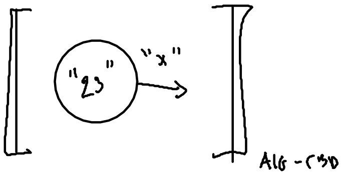
$- : \mathbb{R} \to \mathbb{R}$




$:=$ $[\![\text{"z"}]\!] = [\![\text{"x"}]\!] + [\![\text{"y"}]\!]$

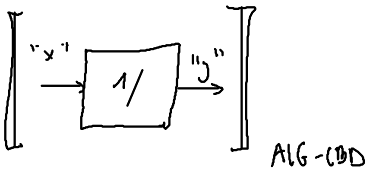
$mvz = mvx + mvy$ $\quad$ $mvx, mvy, mvz \in \mathbb{R}$

$+ : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$



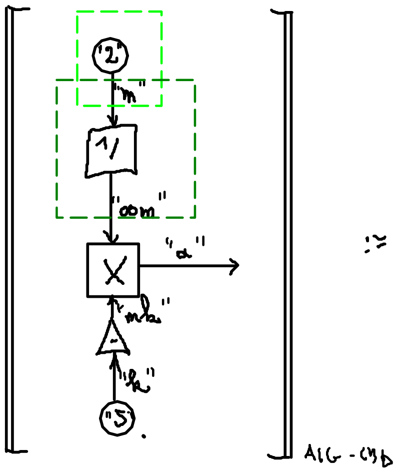

$:=$ $[\![\text{"x"}]\!] = [\![\text{"23"}]\!]$

$mvx = 23$ $\quad$ $mvx \in \mathbb{R}$




$:=$ $[\![\text{"y"}]\!] = 1/\, [\![\text{"x"}]\!]$ $\quad$ $mvx, mvy \in$

$mvy = 1/\, mvx$ $\quad$ $mvx \in \mathbb{R} \setminus \{0\},\ mvy \in \mathbb{R}$ $\quad$ $\mathbb{R} \cup \{\text{EXCEPT}\}$

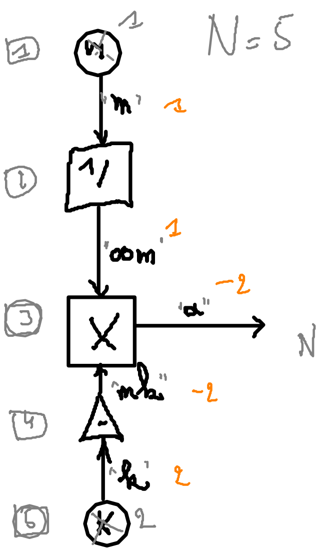



$:=$

$mvm, mvoom, mvm, mva, mvmk, mvk \in \mathbb{R}$

SUCH THAT $\quad$ $mvm \neq 0$

$$\left\{\begin{array}{l} mvm = 2 \\ mvoom = 1/mvm \\ mva = mvoom \times mvmk \\ mvmk = -mvk \\ mvk = 5 \end{array}\right. = (2, 0.5, 2.5, -5, 5) \in \mathbb{R}^5$$




$N = 5$

| | vm | voom | va | vmk | vk |
|---|---|---|---|---|---|
| | UK | UK | UK | UK | UK |
| 1 | 1 | UK | UK | UK | UK |
| 2 | 1 | 1 | UK | UK | UK |
| 3 | 1 | 1 | UK | UK | UK |
| 4 | 1 | 1 | UK | UK | UK |
| 5 | 1 | 1 | UK | UK | 2 |
| 1 | 1 | 1 | UK | UK | 2 |
| 2 | 1 | 1 | UK | UK | 2 |
| N 3 | 1 | 1 | UK | UK | 2 |
| 4 | 1 | 1 | UK | −2 | 2 |
| 5 | 1 | 1 | UK | −2 | 2 |
| 1 | 1 | 1 | UK | −2 | 2 |
| 2 | 1 | 1 | UK | −2 | 2 |
| N 3 | 1 | 1 | −2 | −2 | 2 |
| 4 | | | | | |
| 5 | | | | | |

1 2 ··· 5

"SCHEDULE"

1 5 2 4 3

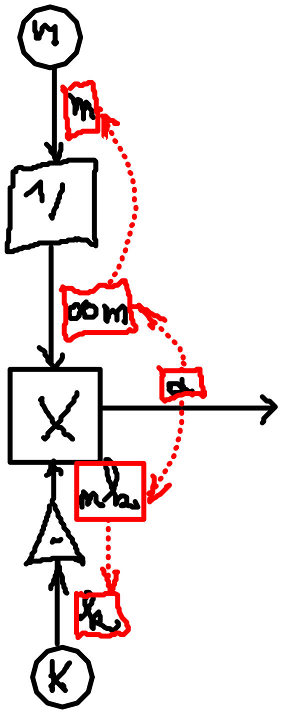
// (1 2) // (5 4) 3

WORST-CASE

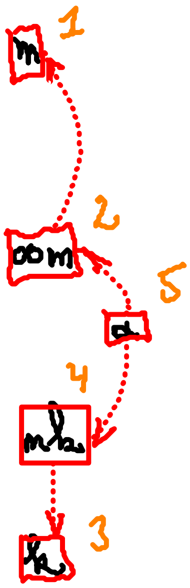
\# ITER $\Theta(N)$ $\Theta(1)$

\# COMP $\Theta(N^2)$ $\Theta(N)$

DISTRIBUTED

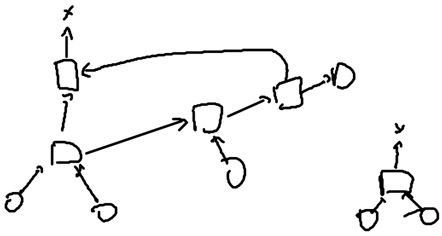



DEPENDENCY GRAPH




schedule = [m, oom, k, mk, a]

1 2 3 4 5



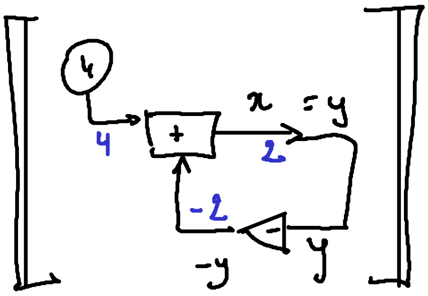
operational semantics $\Theta(N)$

```
depGraph = buildDepGraph(CBD)
schedule = topologicalSort(depGraph)

for block in schedule:
  block.compute()
```

$\Theta(N)$

$\Theta(N)$




$:=$

$$\begin{cases} x + y = 4 \\ x - y = \phi \end{cases} \qquad \begin{aligned} x &= 4 - y \\ y &= x \end{aligned}$$

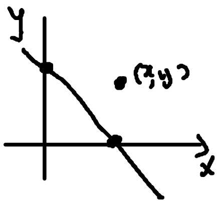
SET LINEAR EQNS.

2 UNKNOWNS

2 EQNS

$$\begin{cases} 2x + 2y = 4 \\ x + y = 2 \end{cases}$$

$$y = 2 - x$$




$$\begin{cases} 1 \times x + 1 \times y = 4 \\ 1 \times x - 1 \times y = \phi \end{cases}$$

$$\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

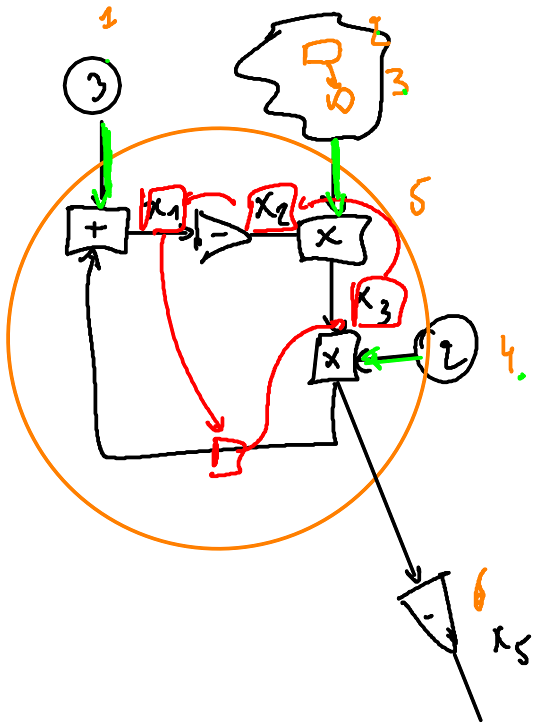
$$\begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = \phi$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\det \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = -2 \neq \phi$$

$$x = \frac{\begin{vmatrix} 4 & 1 \\ \phi & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-4}{-2} = 2$$

$$y = \frac{\begin{vmatrix} 1 & 4 \\ 1 & \phi \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-4}{-2} = 2$$



BLOCKS IN "LINEAR LOOP"

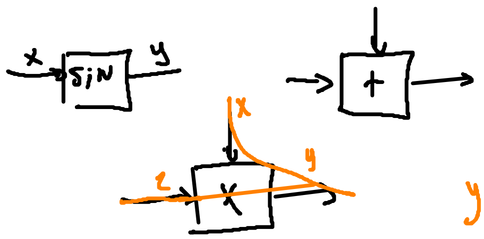
$$\begin{cases} k_{11} x_1 + k_{21} x_2 + \dots + k_{n1} x_n = K_1 \\ k_{12} x_1 + k_{22} x_2 + \dots + k_{n2} x_n = K_2 \\ \vdots \\ k_{1n} x_1 + k_{2n} x_2 + \dots + k_{nn} x_n = K_n \end{cases}$$

$$\underbrace{\begin{bmatrix} k_{11} & \dots & \\ \vdots & & \vdots \\ & \dots & k_{nn} \end{bmatrix}}_{K} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \underbrace{\begin{bmatrix} K_1 \\ \vdots \\ K_n \end{bmatrix}}_{K}$$

UNIQUE? $\det(K) \neq 0$

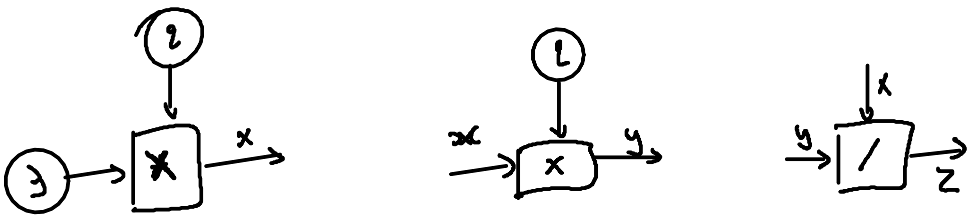
$(x_1, \dots, x_n)$

GAUSSIAN ELIM $O(n^3)$

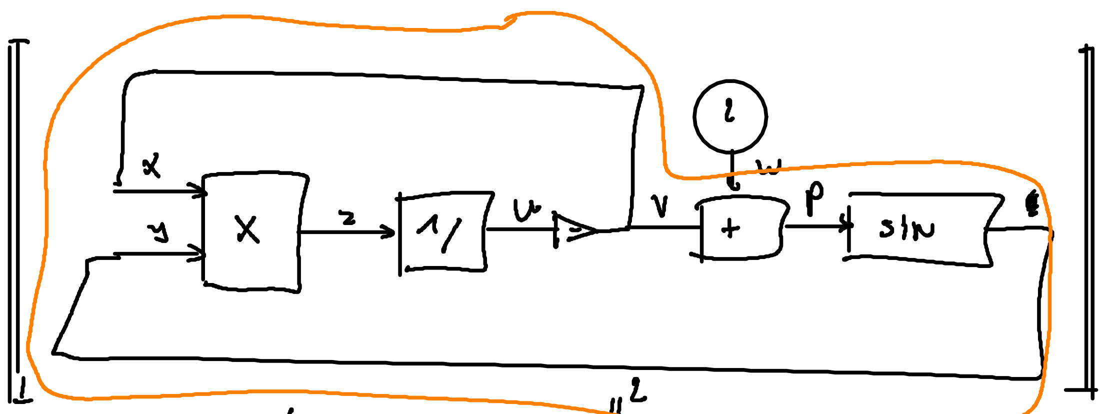


$y = x \times z$

$xy - x \times z = 0$



$z = x/y$



$(x, y, z, w, v, w, p) \in \mathbb{R}$

UNIQUE?

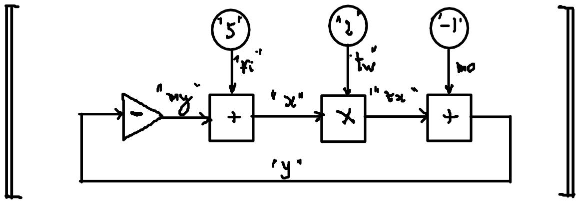
COMPLEXITY? $O(\dots)$

$$\begin{cases} x \times y - z = 0 & \varepsilon_1 \\ z - 1/w = 0 & \varepsilon_2 \\ \vdots & \varepsilon_3 \\ \vdots & \end{cases} \rightarrow 0$$

$(x_0, y_0, z_0, w_0, v_0, w_0, p_0)$

$(x_1, y_1, z_1, w_1, v_1, w_1, p_1)$

"algebraic loop"



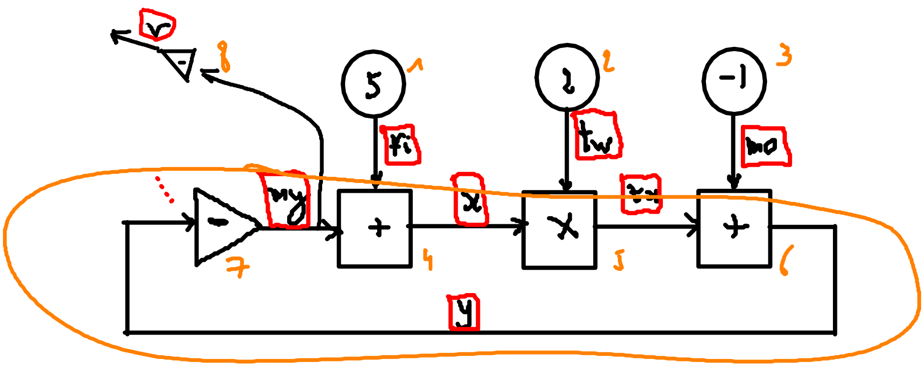

:=

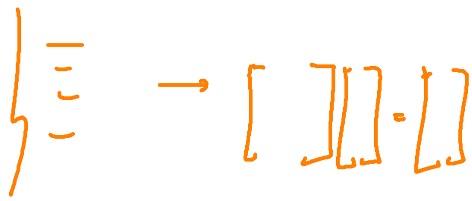
ALG-CBD

$\mathrm{mvfi}, \mathrm{mvx}, \mathrm{mvtw}, \mathrm{mvtx}, \mathrm{mvmo}, \mathrm{mvy}, \mathrm{mvny} \in \mathbb{R}$

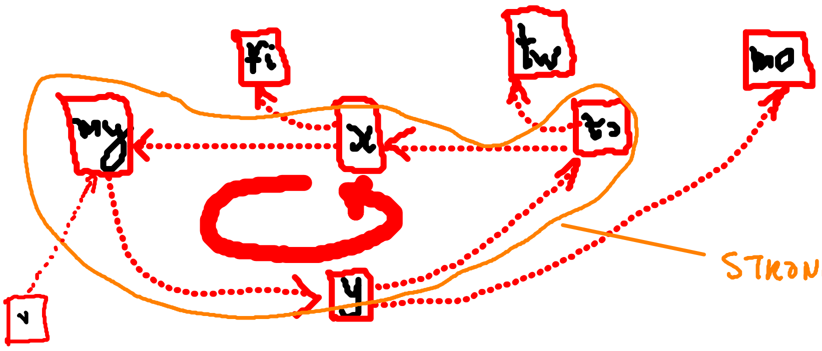
such that

$$\begin{cases} \mathrm{mvfi} = 5 \\ \mathrm{mvx} = \mathrm{mvfi} + \mathrm{mvny} \\ \mathrm{mvtw} = 2 \\ \mathrm{mvtx} = \mathrm{mvtw} \times \mathrm{mvx} \\ \mathrm{mvmo} = -1 \\ \mathrm{mvy} = \mathrm{mvmo} + \mathrm{mvx} \\ \mathrm{mvny} = -\mathrm{mvy} \end{cases} = (5, 2, 2, 4, -1, 3, -3) \in \mathbb{R}^7$$



1 2 3 $\longleftrightarrow$ {4,5,6,7} 8




STRONG COMPONENT

[{1},{2},{3}, {4,5,6,7}, {8}]

M

TARJAN 1974

$\Theta(N+E)$

operational semantics

```
schedule = topologicalSortAndLoopDetect(depGraph(CBD))
```

$\Theta(N+E)$

```
for genBlock in schedule:
  genBlock.compute()
```

$\Theta(N)$ ?

SOME SETS OF COUPLED EQNS (SIZE M)

SOLVER COMPLEXITY $\leq \Theta(M^3)$

USUALLY $M \ll N$