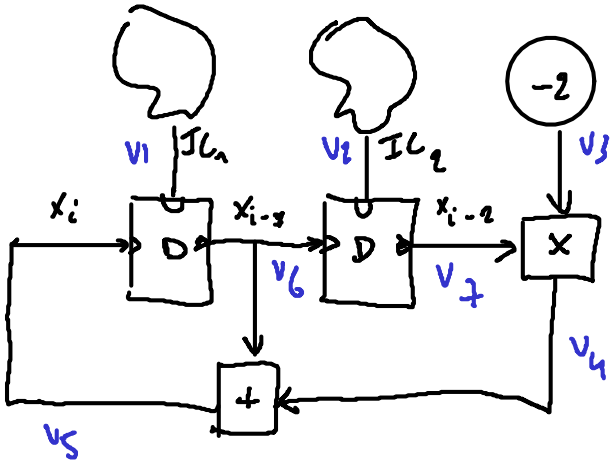
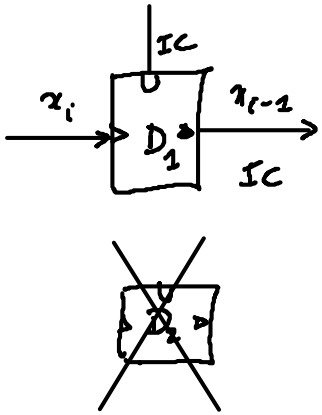
$x_{i+1} = y_i - 2 \times x_{i-1}$

$y_i = y_{i-1} - 2 \times x_{i-2}$

memory over more than 1 time-slice





$V3_i = -2$

$V7_i = V7(i) = V6(i-1), \quad i > \emptyset$

$V7_\emptyset = V7(\emptyset) = V2(\emptyset)$

```
\documentclass{article}
\begin{document}
\[
 \left\{
 \begin{eqnarray}{rcl}
  v_3(i) & = & -2\\
 \end{eqnarray}
 \right.
\]
\end{document}
```

PSEUDO - RNG

PRNG$_1$ SEED (1)

2 4

3 2

STREAM

4 0

5 (5)

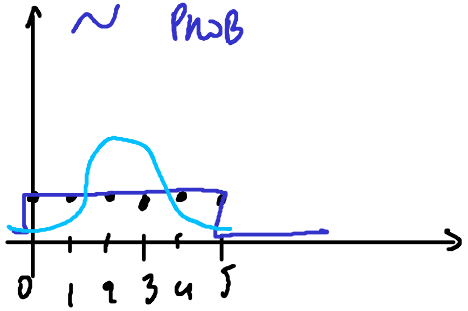
PERIOD

FULL PERIOD

6 3

1

$\mathcal{V}$ # ~ PMB



$\mathcal{U}_{[0,5]}$

6

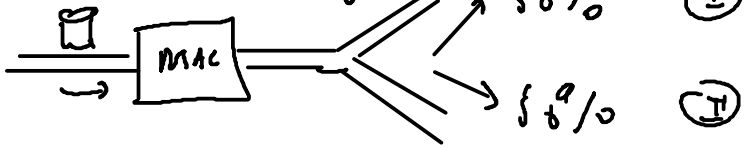
PRNG$_2$ SEED (1)

2

1

5

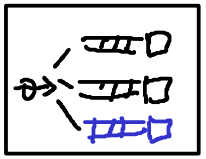
PRNG



I REPEATABILITY

REPRODUCIBILITY (NOT THE SAME MEANS)

II



LINEAR CONGRUENTIAL GENERATOR

A1 LCG

$$x_{i+1} = (A \times x_i + C) \,\%\, M$$

x_0 SEED

DIFFERENCE EQN.

	LCG_1	LCG_2	LCG
A	2	4	1664525
C	0	1	1013904223
M	9	9	2^{32}

$x_i \in [0, 8]$ $\subseteq \mathbb{N}_0$

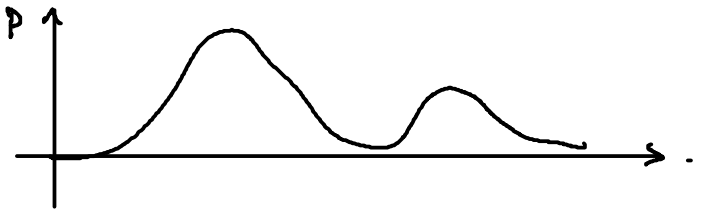
$x_i \in [0, 2^{32}-1]$ $\subseteq \mathbb{N}_0$

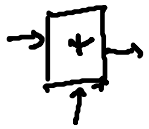
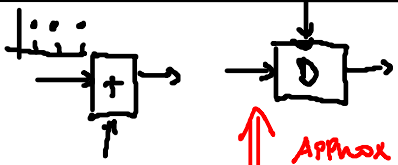
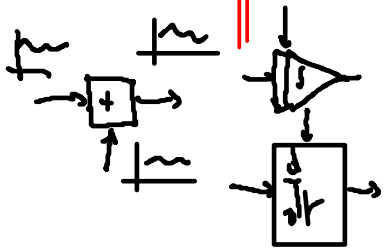
$U_{[0,1[} \subseteq \mathbb{R}^+$

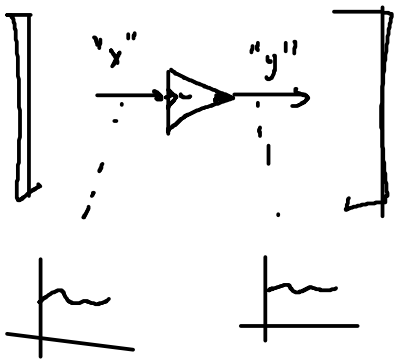
$$\frac{x_i}{2^{32}-1}$$

LCG_1, LCG_2

SEED $0, 1, \ldots, 8$] → PERIOD

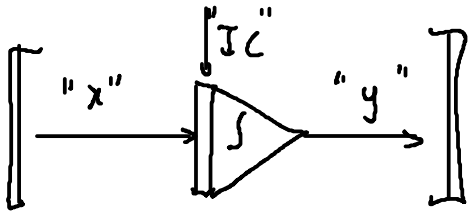


TIME ↓	HIERARCHY → FLAT CBD	SYNTAX →	SEMANTICS → DENOTATIONAL "WHAT"	SEMANTICS → OPERATIONAL "HOW"
{NOW}	ALGEBRAIC (ALG-CBD)	 NO LOOPS / WITH LOOPS	✓ ✓	✓
DN	DISCRETE-TIME (DT-CBD)		✓	✓
CR	CONTINUOUS-TIME (CT-CBD)	 Approx	✓	✗

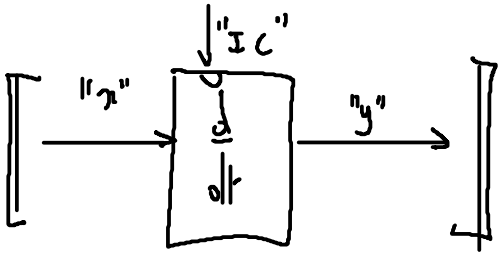


$:=$ $\quad m v_y(t) = -m v_x(t), \quad \forall t \in \mathbb{R}$

$- : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$



$:= \quad m v_y(t) = \underbrace{m v_y(0)}_{IC(0)} + \int_0^t x(\tau)\, d\tau, \quad \forall t \in \mathbb{R}$



$:= \quad m v_y(t) = \frac{d\, m v_x(t)}{dt}, \quad \forall t \in (\text{differentiable})$

$m v_y(0) = IC(0)$

A2

EXPLICIT
$$\begin{cases} x_{i+1} = x_i + \Delta t\, y_i & x_0 = 0 \\ y_{i+1} = y_i - \Delta t\, x_i & y_0 = 1 \end{cases}$$

IMPLICIT
$$\begin{cases} x_{i+1} = x_i + \Delta t\, y_{i+1} & x_0 = 0 \\ y_{i+1} = y_i - \Delta t\, x_{i+1} & y_0 = 1 \end{cases}$$

EXACT
$$\begin{cases} x_i^* = \sin(i\,\Delta t) \\ y_i^* = \cos(i\,\Delta t) \end{cases}$$

$$x_i^{\text{ERR\_E}} = \left| x_i^* - x_i^{\text{EXPL}} \right|$$

$$x_i^{\text{ERR\_I}} = \left| x_i^* - x_i^{\text{IMPL}} \right|$$

$\Theta(N)$

RUNTIME $\sim$ #COMPUTATIONS

$\wedge$

$\Theta(N^3)$

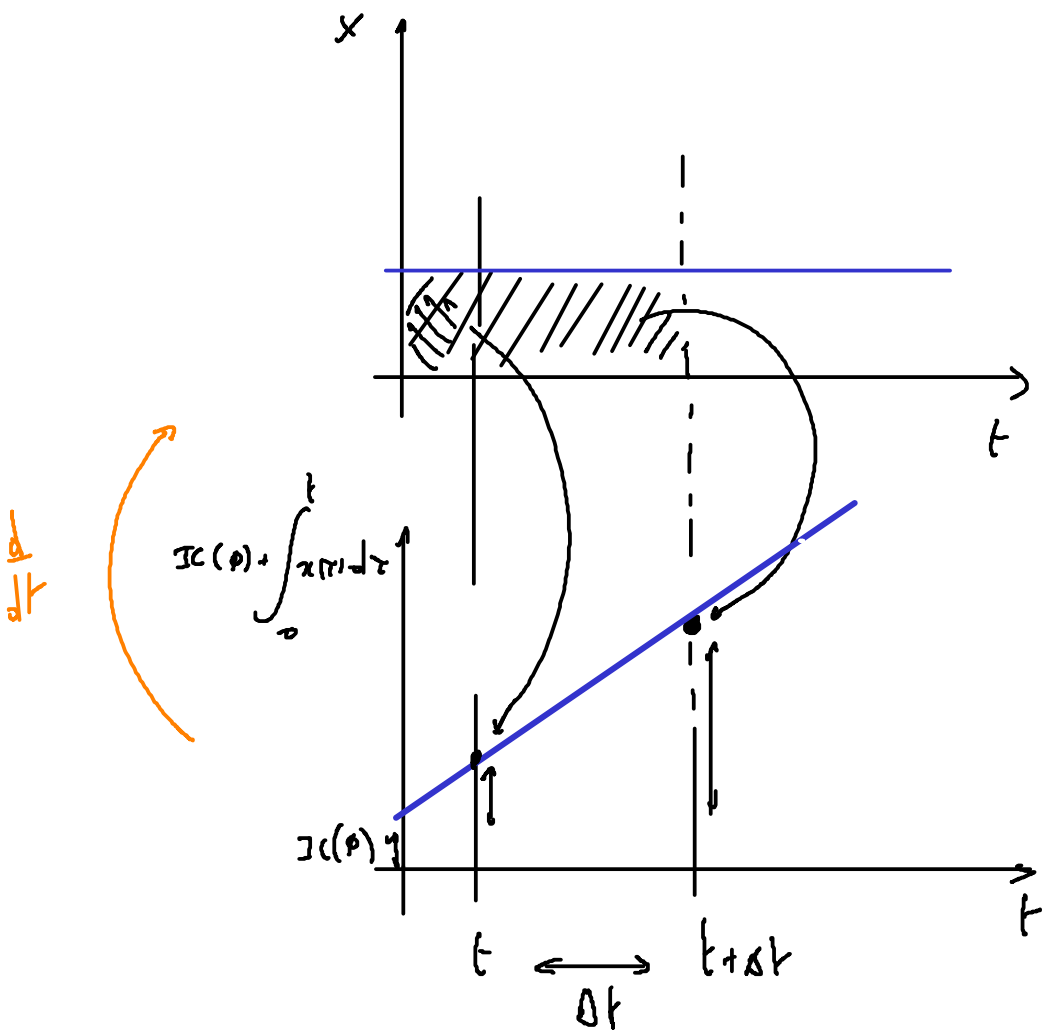
RUNTIME "

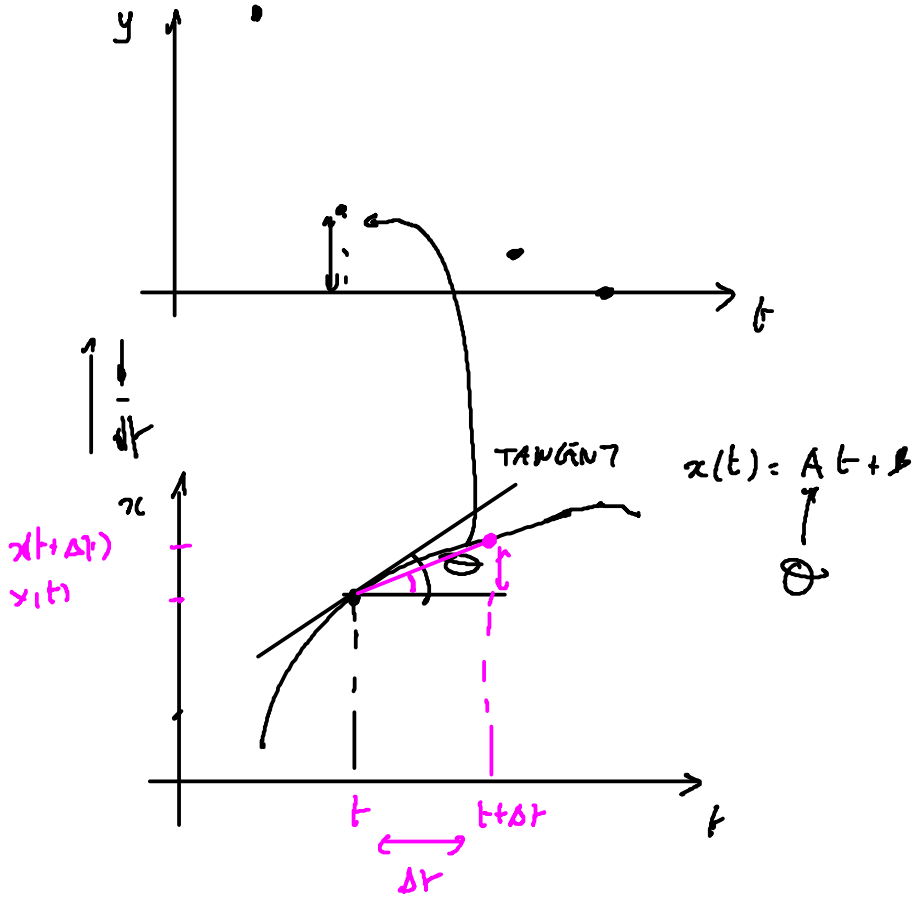
$\Delta t = 0.001$

$i : 0 \dots \left\lceil \frac{2\pi}{\Delta t} \right\rceil$.

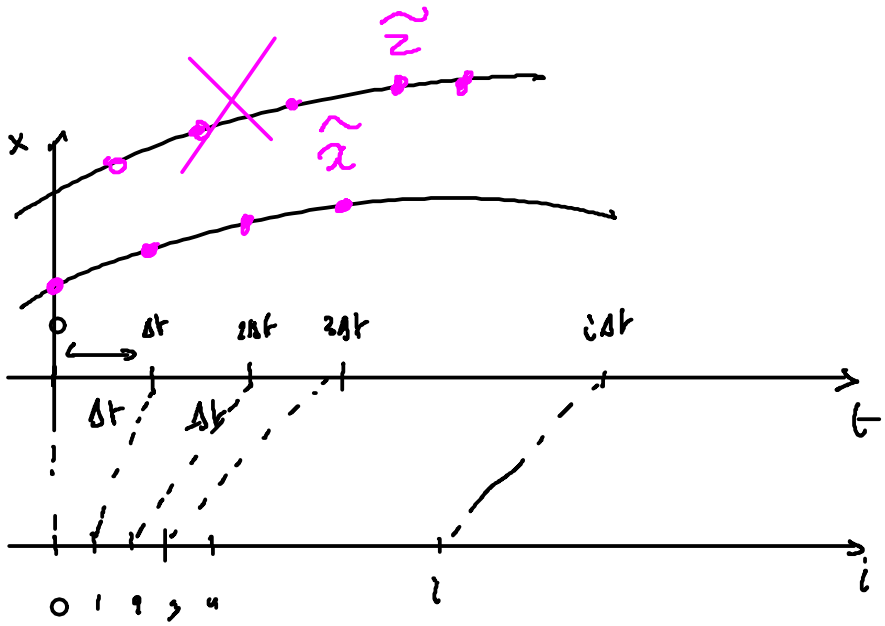
$\#\ x_i^{\text{ERR\_E}} > x_i^{\text{ERR\_I}}$

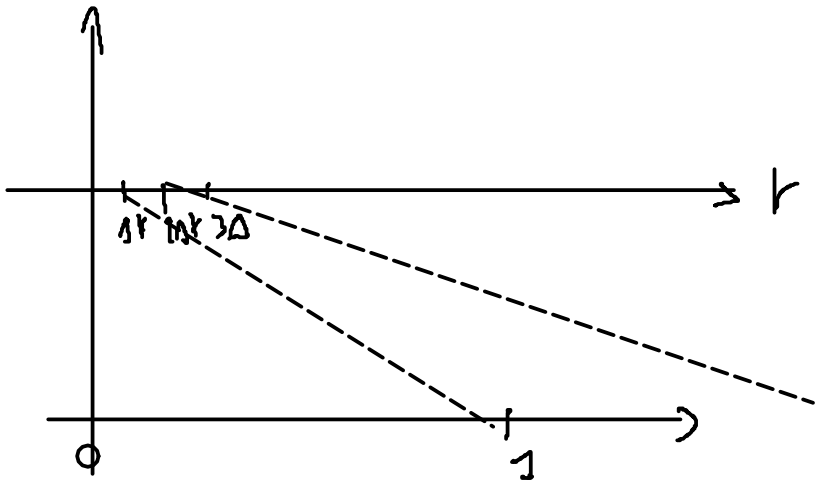
$\text{err} \downarrow \sim \Delta t \downarrow$

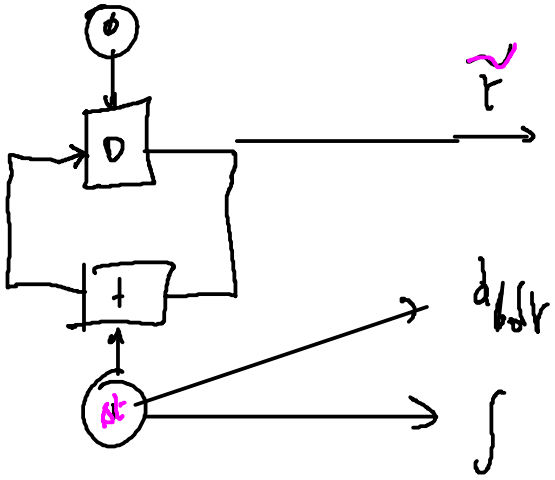




$$\lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$


x
t
i
0
1
2
3
n
l


0
1

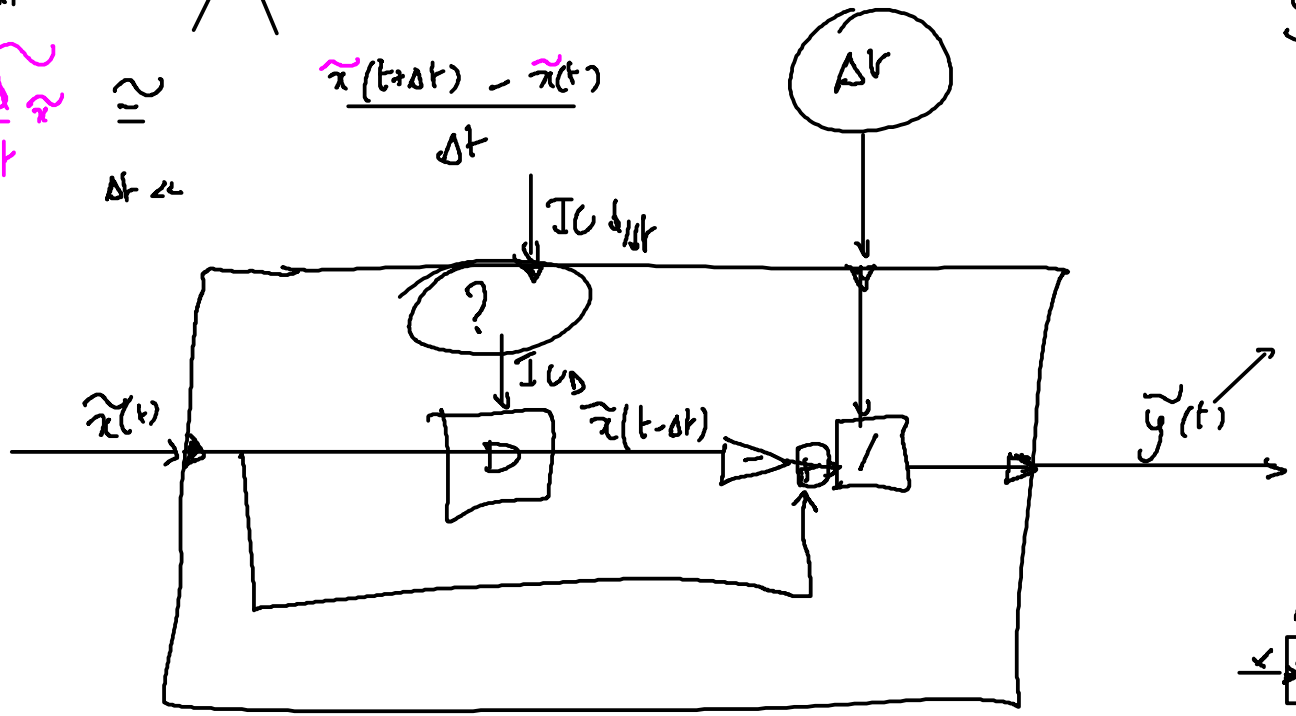

D
1

$\frac{d}{dt}x(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$

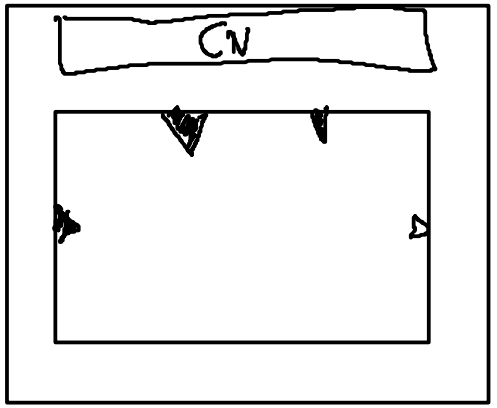
$\tilde{y}(t) \cong \frac{d}{dt}\tilde{x} \cong \frac{\tilde{x}(t+\Delta t) - \tilde{x}(t)}{\Delta t}$ $\Delta t \ll$

$\tilde{y}(t+\Delta t) = \frac{x(t+\Delta t) - x(t)}{\Delta t}$

$\tilde{y}(t) = \frac{\tilde{x}(t) - \tilde{x}(t-\Delta t)}{\Delta t}$



$y = \frac{d}{dt}x \qquad y(0) = IC(0)$



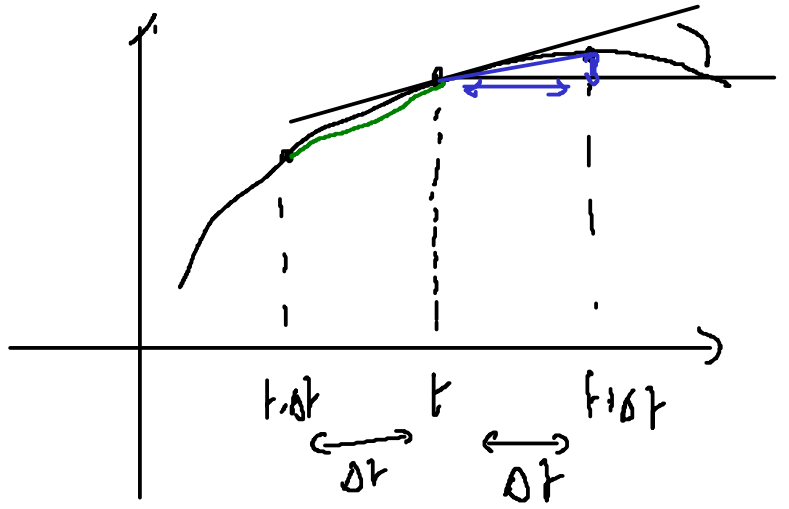
All blocks : $= \Delta t$

SYNCHRONOUS BLOCK DIAGRAMS

$$\tilde{y}(t) = \frac{\tilde{x}(t) - \tilde{x}(t-\Delta t)}{\Delta t}$$

$$\tilde{y}(t) = \frac{\tilde{x}(t+\Delta t) - \tilde{x}(t)}{\Delta t}$$

$\Delta t \ll$

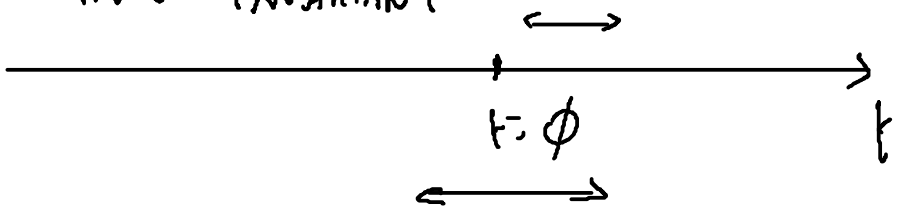


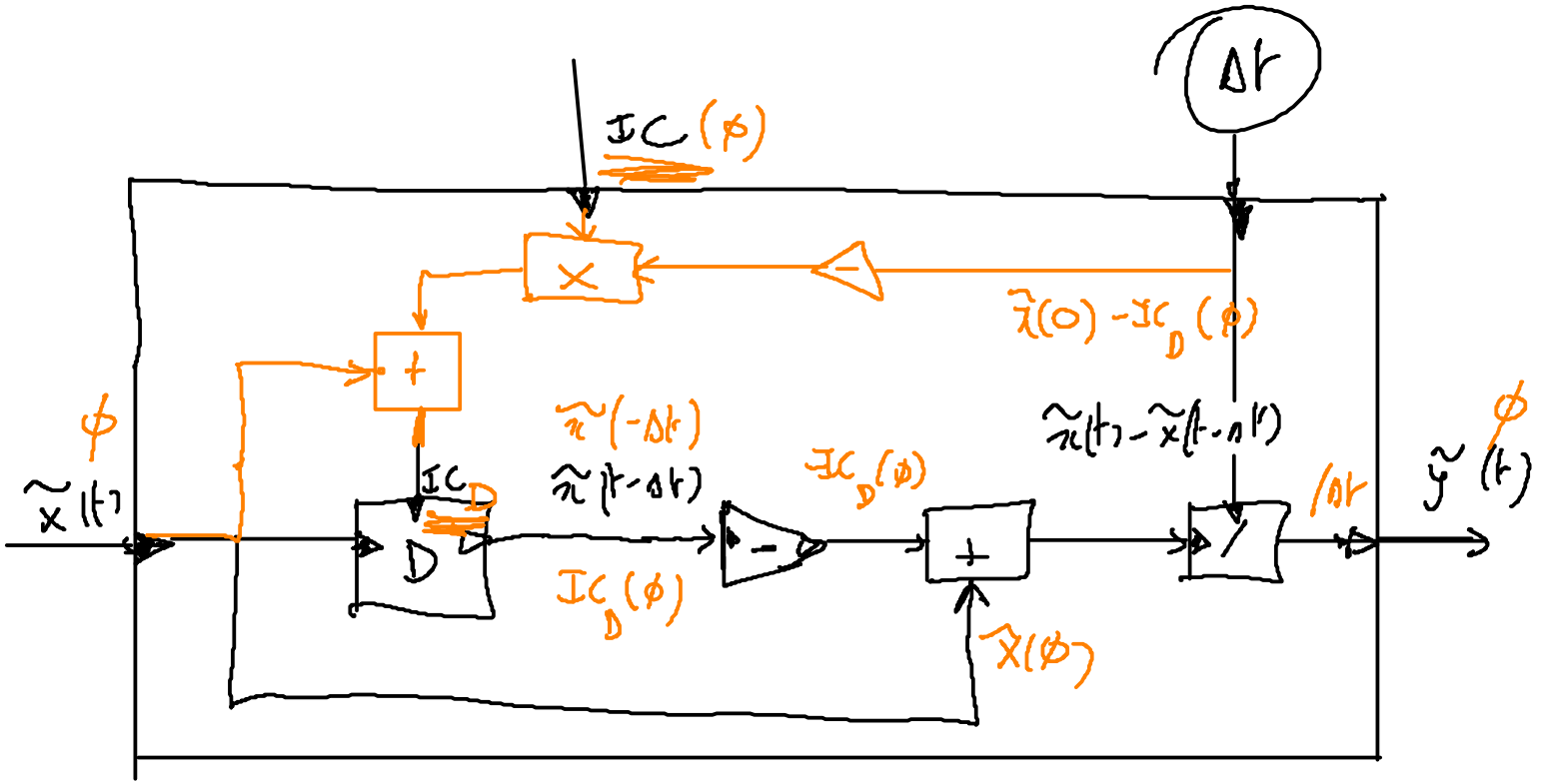
$$\lim_{\Delta t \to 0}$$

$$\bar{y}(t) = \frac{d\bar{x}}{dt}(t) = \frac{\bar{x}(t+\Delta t) - \bar{x}(t)}{\Delta t}$$

$$\hat{y}(t) = \frac{d\hat{x}}{dt}(t) = \frac{\hat{x}(t) - \hat{x}(t-\Delta t)}{\Delta t}$$

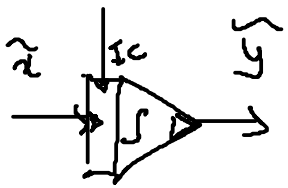
TIME - INVARIANT

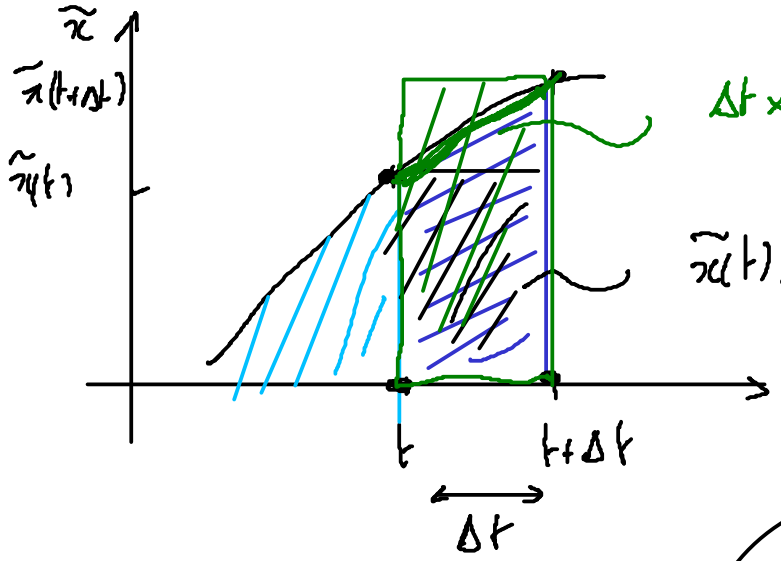




$$\frac{\tilde{x}(0) - IC_D(0)}{\Delta t} = IC_{\dot{D}}(0) \qquad \tilde{y}(\phi) = IC_{\dot{D}}(0)$$

$$\tilde{x}(0) - IC_{\dot{D}}(0) \cdot \Delta t = IC_D(0)$$



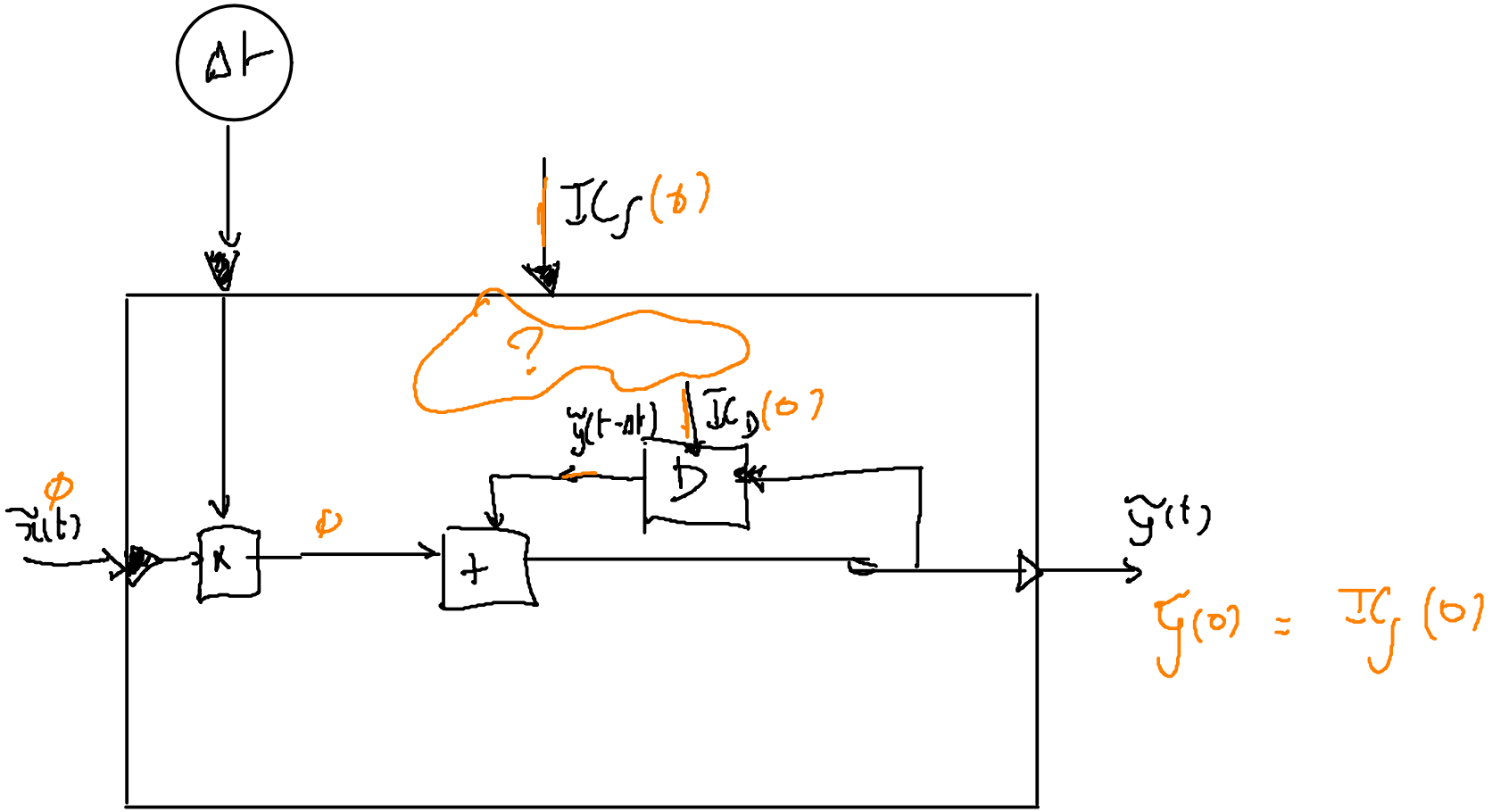


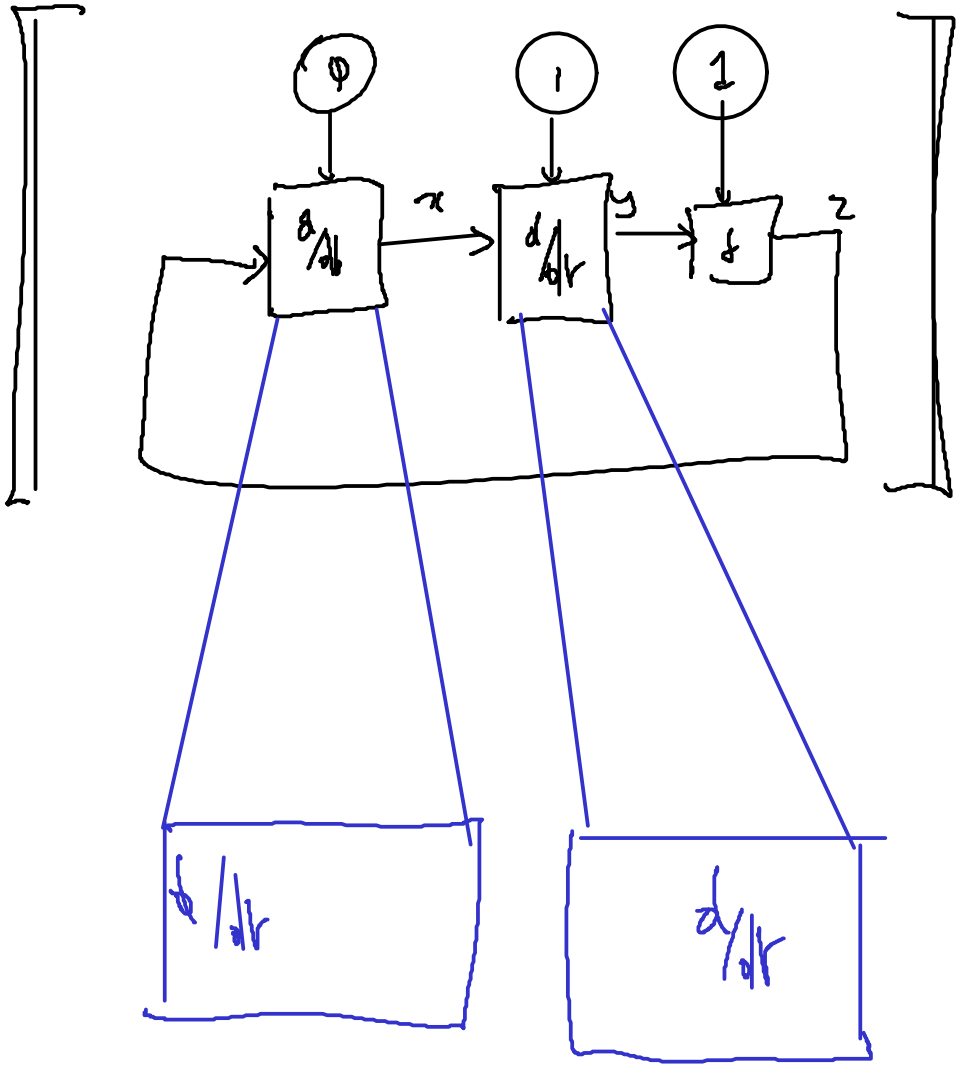
$$\tilde{y}(t+\Delta t) = \tilde{y}(t) + \int_{t}^{t+\Delta t} \tilde{x}(\tau)\, d\tau$$

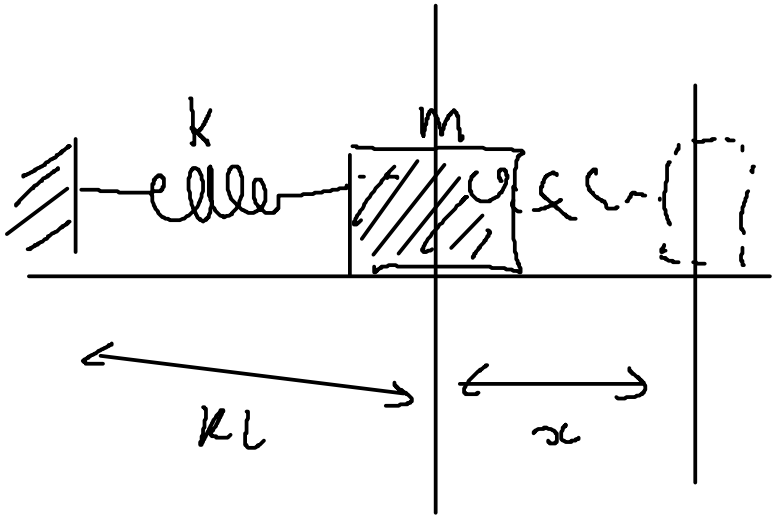
$$\lim_{\Delta t \to 0} \Delta t\ \tilde{x}(t) \qquad \Delta t\ \tilde{x}(t+\Delta t)$$

$$\tilde{y}(t) \approx y(t-\Delta t) + \Delta t\ \tilde{x}(t)$$

Approx


Δt
IC_f(0)
?
ỹ(t-Δt)
IC_D(0)
D
x̃(t)
ϕ
K
ϕ
+
ỹ(t)
y(0) = IC_f(0)


0
1
2
x
y
z



HOOKE'S LAW

$$F = -kx \qquad x \ll L$$

$$F = ma$$

$$F = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$$

$$v = \frac{dx}{dt}$$

$$v = \frac{d}{dt}x$$

$$a = \frac{d}{dt}v = \frac{d^2}{dt^2}x$$

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv)$$

$$= \underbrace{\frac{d}{dt}m}\cdot v + m\overbrace{\frac{dv}{dt}}^{a}$$

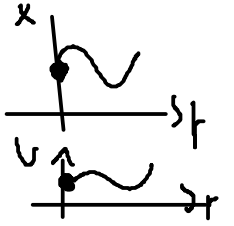
$$\frac{dv}{dt} = -\frac{k}{m}x, \; v(0)$$

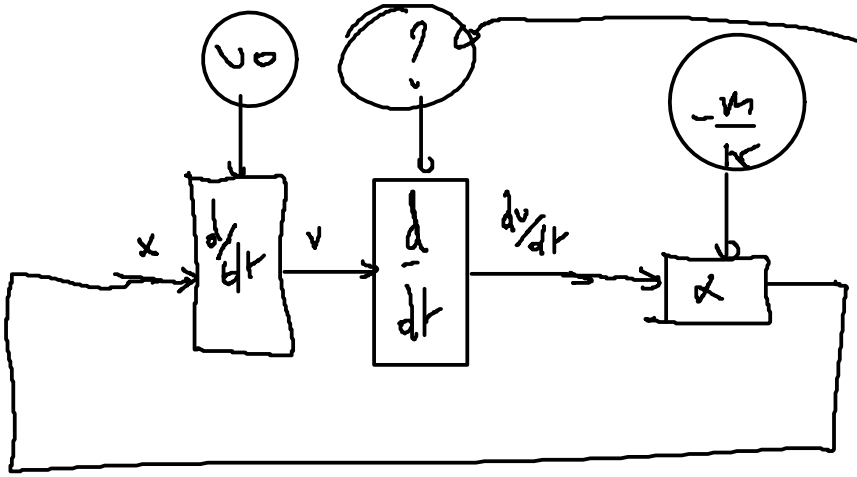
$$\frac{dx}{dt} = v, \; x(0)$$

ODE
ORDINARY
DIFFERENTIAL
EQNS

PDE

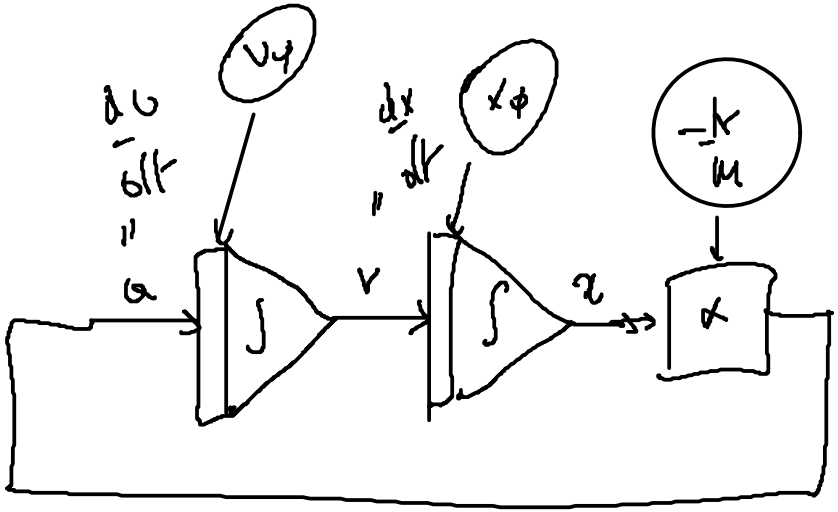
$x(t)$, $v(t)$



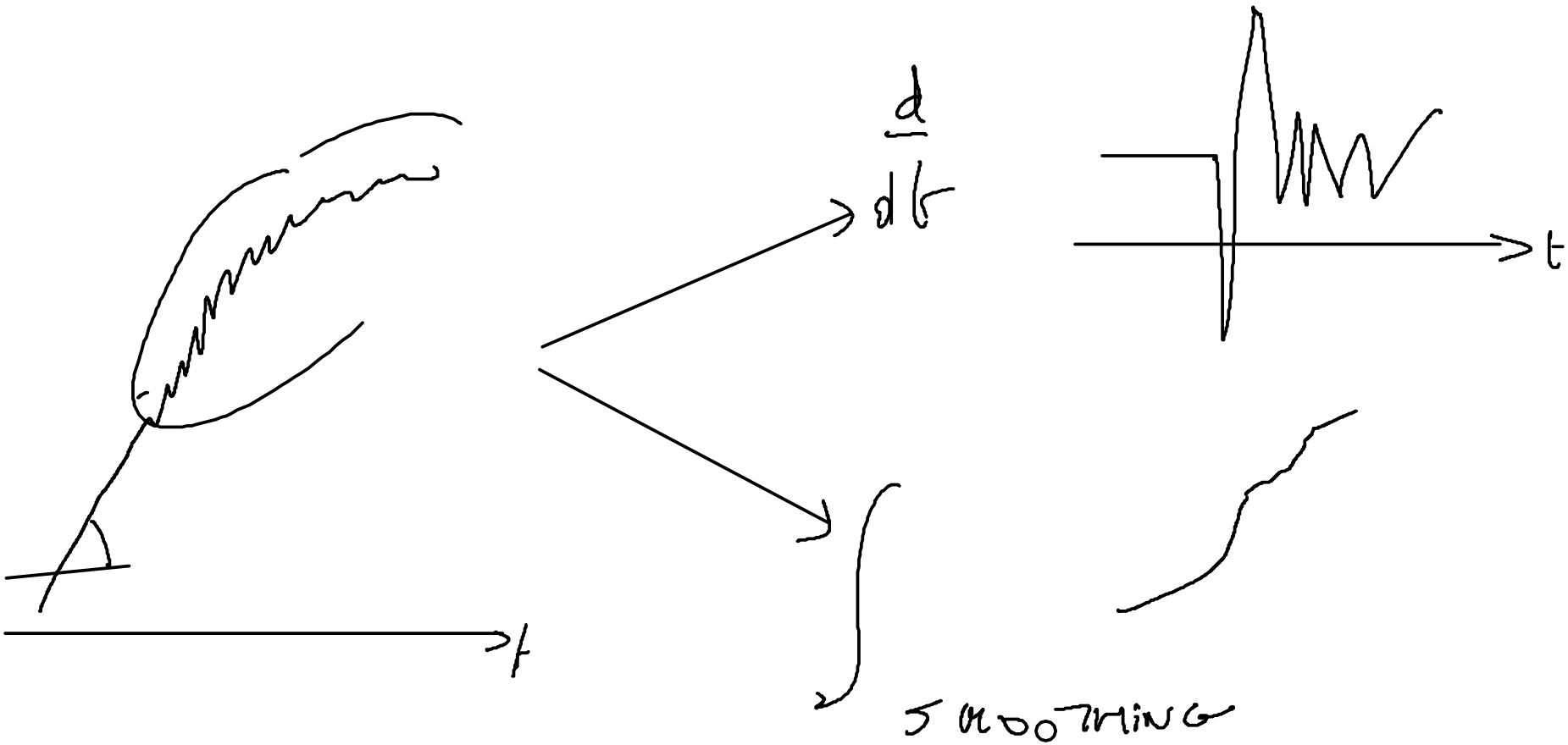


$$x = -\frac{m}{k}\frac{dv}{dt}$$

$$\frac{dv}{dt}(0) = -\frac{k}{m}x(0)$$



$$\lim_{\Delta t \to \phi}$$


d/dt
t
SMOOTHING