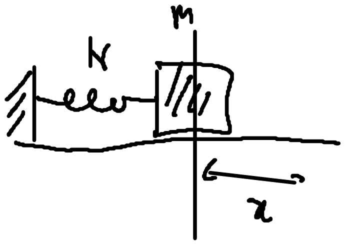


HIGHER-ORDER ODES

$$\frac{d^k x}{dt^k} = \underbrace{\frac{d}{dt}\left(\frac{d}{dt}\left(\cdots \frac{d}{dt}x\right)\right)}_{k}$$

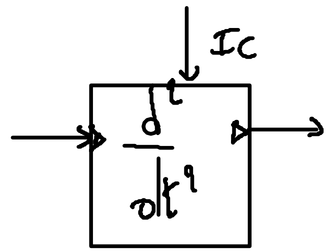
$$\begin{cases} \dfrac{dx}{dt} = x\_der\_1 \\ \dfrac{d\,x\_der\_1}{dt} = x\_der\_2 \\ \vdots \\ \dfrac{d\,x\_der\_k{-}1}{dt} = x\_der\_k \end{cases}$$

x_der_der

HARMONIC EQN.

$$\frac{d^2x}{dt^2} = -x, \quad x(0) = x_0, \quad \frac{dx}{dt}(0) = v(0) = v_0 \qquad \text{ODE}$$

$$\begin{cases} \dfrac{dx}{dt} = v, & x(0) = x_0 \\ \dfrac{dv}{dt} = -x, & v(0) = v_0 \end{cases}$$



$$\frac{d^2x}{dt^2} = -x \quad , \quad x(0) = \phi \quad , \quad \frac{dx}{dt}(0) = 1$$

$$\frac{dx}{dt} = x$$

$$x(t) = e^x + c$$

$$\begin{cases} x(t) = A \sin(t) + B \cos(t) \; (+ \; c) \\ x(0) = \phi \quad , \quad \frac{dx}{dt}(0) = 1 \end{cases}$$

$$\frac{dx}{dt} = A \cos(t) - B \sin(t)$$

$$\frac{d^2x}{dt^2} = -A \sin(t) - B \cos(t) = -\left(A \sin(t) + B \cos(t)\right) = -x(t) \qquad \text{q.e.d.}$$

$$x(0) = B = \phi$$

$$\frac{dx}{dt}(0) = A = 1$$

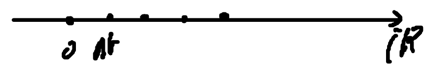
$$\begin{cases} x(t) = \sin(t) \\ \frac{dx}{dt}(t) = \cos(t) \end{cases}$$

$$\tilde{x}(\hat{t})$$

$$\widehat{\frac{dx}{dt}} = \tilde{v}(\tilde{t})$$

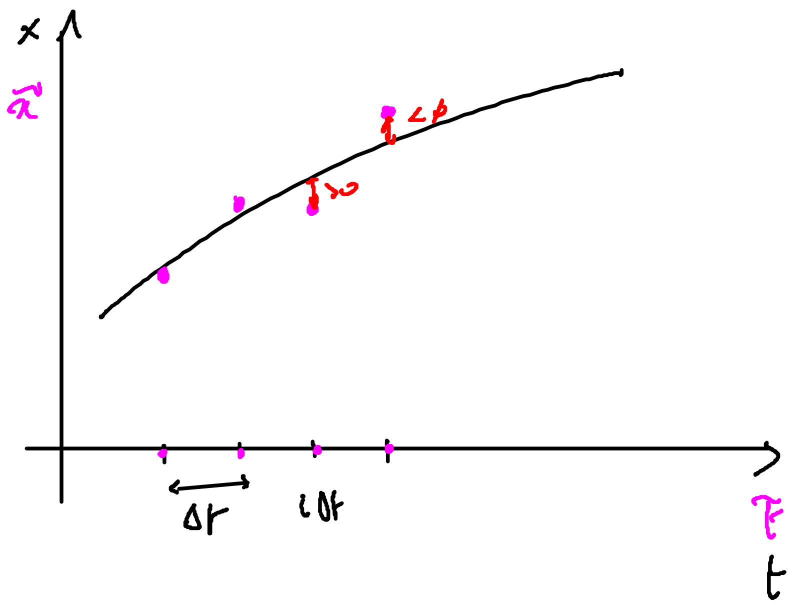
$$\tilde{t} = i \times \Delta t$$

$$\wedge$$



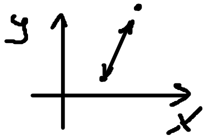
ERROR ~ $e(i\Delta t) = x(i\Delta t) - \tilde{x}(i\Delta t)$

TOTAL ERROR ~ $\sum_{i=0}^{i\_END} e(i\Delta t)$



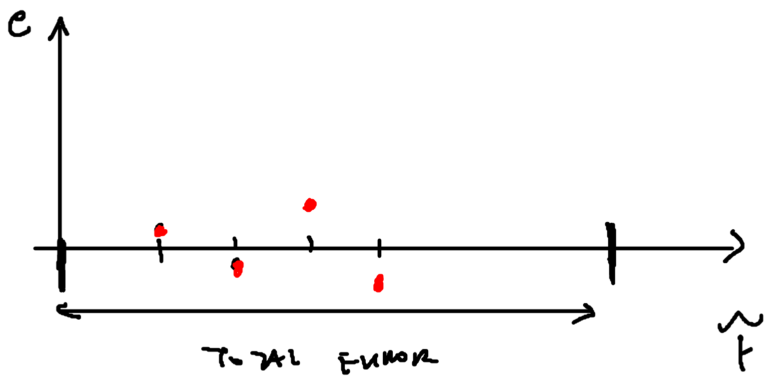
$e(i\Delta t) = \left| x(i\Delta t) - \tilde{x}(i\Delta t) \right|$

$e(i\Delta t) = \left(x(i\Delta t) - \tilde{x}(i\Delta t) \right)^2$



"SUM OF SQUARES"

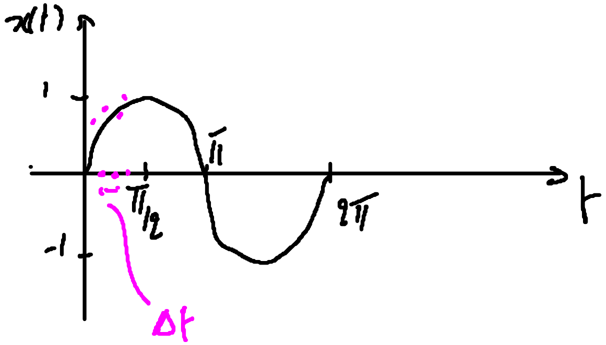
TOTAL ERROR = $\sum_{i=\phi}^{i\_END} \left(x(i\Delta t) - \tilde{x}(i\Delta t) \right)^2$

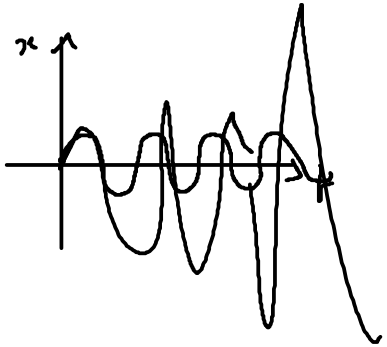


$= \int \left(x(\tau) - \tilde{x}(\tau) \right)^2 d\tau$

$x_{in}(\tau)$ CBD

CBD h =



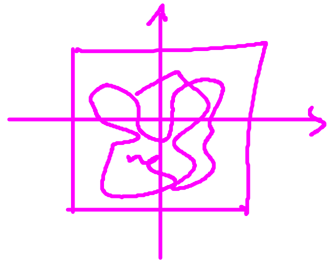


UNSTABLE

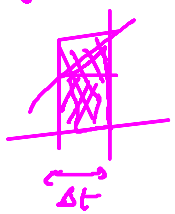
SYSTEM ?

NUMERICAL ?

BOUNDED REGION



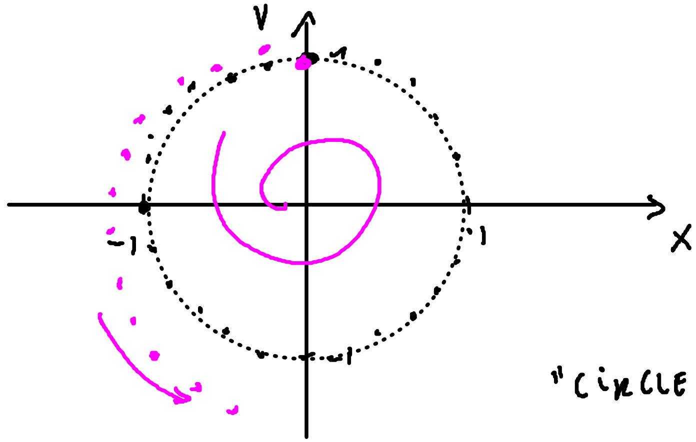
~ Δt , SCHEME



PHASE PLOT

~ STABILITY

- SYSTEM (SUS)
- NUMERICAL

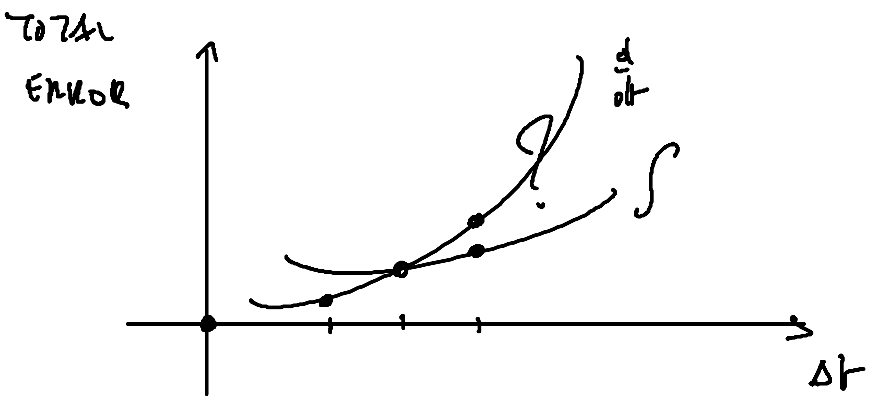


$(x(t), v(t))$

$x(0) = 0$, $v(0) = 1$

$x(\frac{\pi}{2}) =$, $v(\frac{\pi}{2}) =$

"CIRCLE TEST"



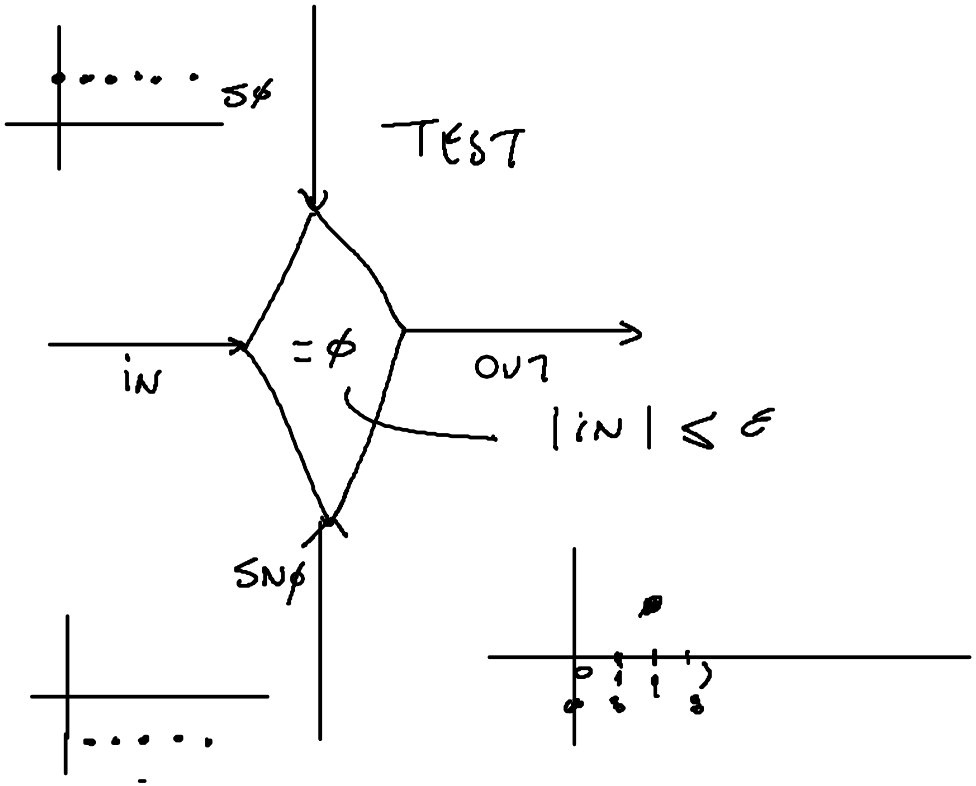
DISCRETIZATION SCHEME FIXED

$$\text{TOTAL ERROR}(\Delta t) \overset{?}{=} f(\Delta t)$$

$$x(t+\Delta t) = x(t) + \frac{x'(t)\,\Delta t}{1!} + \frac{x''(t)}{2!}\Delta t^2 + \frac{x^{(3)}(t)}{3!}\Delta t^3 + \dots$$

TAYLOR EXPANSION

$$\longrightarrow \mathcal{O}(\Delta t^2)$$



$\mathbb{N}, \mathbb{Z}$ int — $i_1 == i_2$

$\mathbb{R}$ float, double — $r_1 == r_2$

$|r_1 - r_2| \leq \varepsilon$