

Population Dynamics

- Deductive modelling: based on physical laws
- Inductive modelling: based on observation + intuition
- Single species:
Birth (in migration) Rate, Death (out migration) Rate

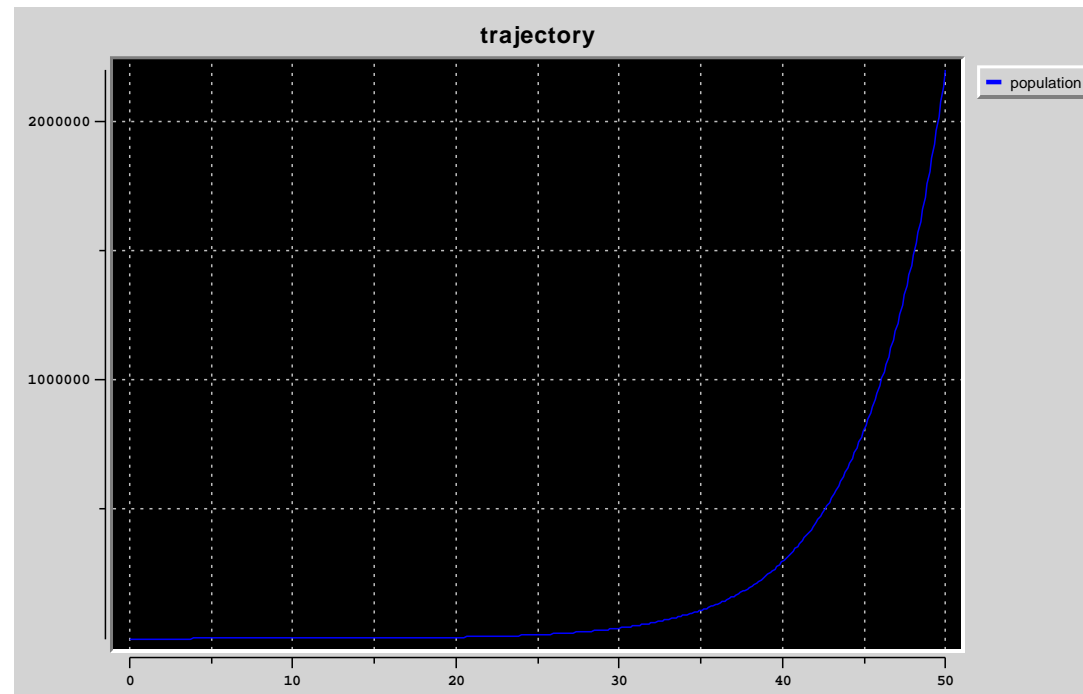
$$\frac{dP}{dt} = BR - DR$$

- Rates proportional to population

$$BR = k_{BR} \times P; DR = k_{DR} \times P$$

$$\frac{dP}{dt} = (k_{BR} - k_{DR})P$$

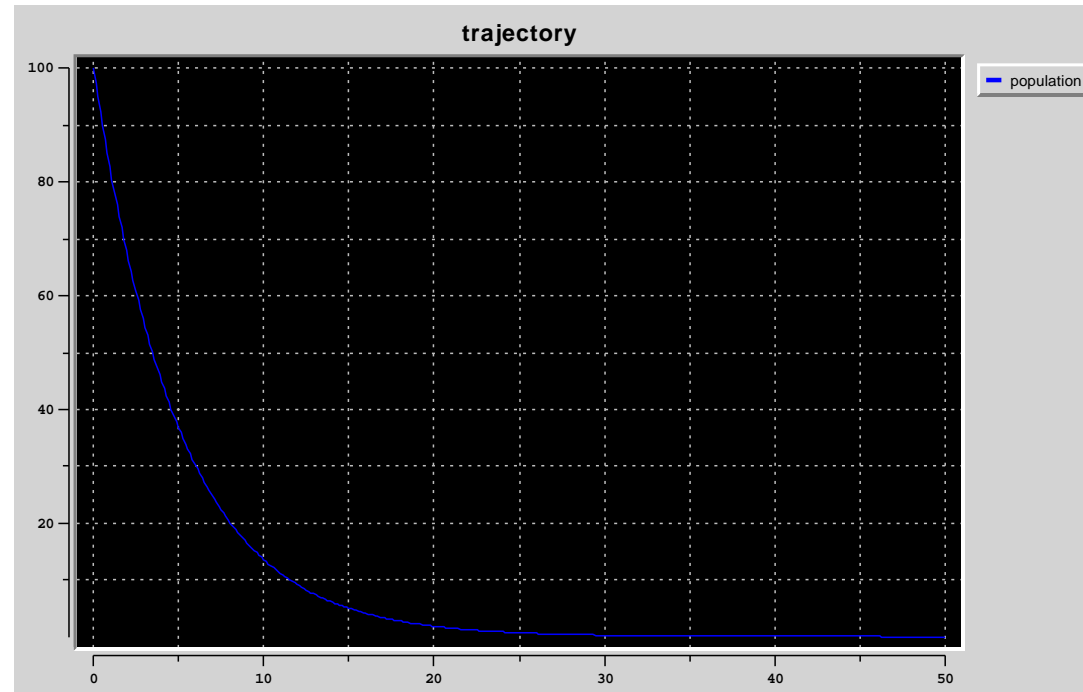
$k_{BR} = 1.4, k_{DR} = 1.2$: Exponential Growth



$$k_{BR} = 1.4, k_{DR} = 1.2 : \log(\text{Exponential Growth})$$



$k_{BR} = 1.2, k_{DR} = 1.4$: Exponential Decay



Logistic Model

- Are k_{BR} and k_{DR} really constant ?
- Energy consumption in a *closed* system \rightarrow limits growth

$$E_{pc} = \frac{E_{tot}}{P}$$

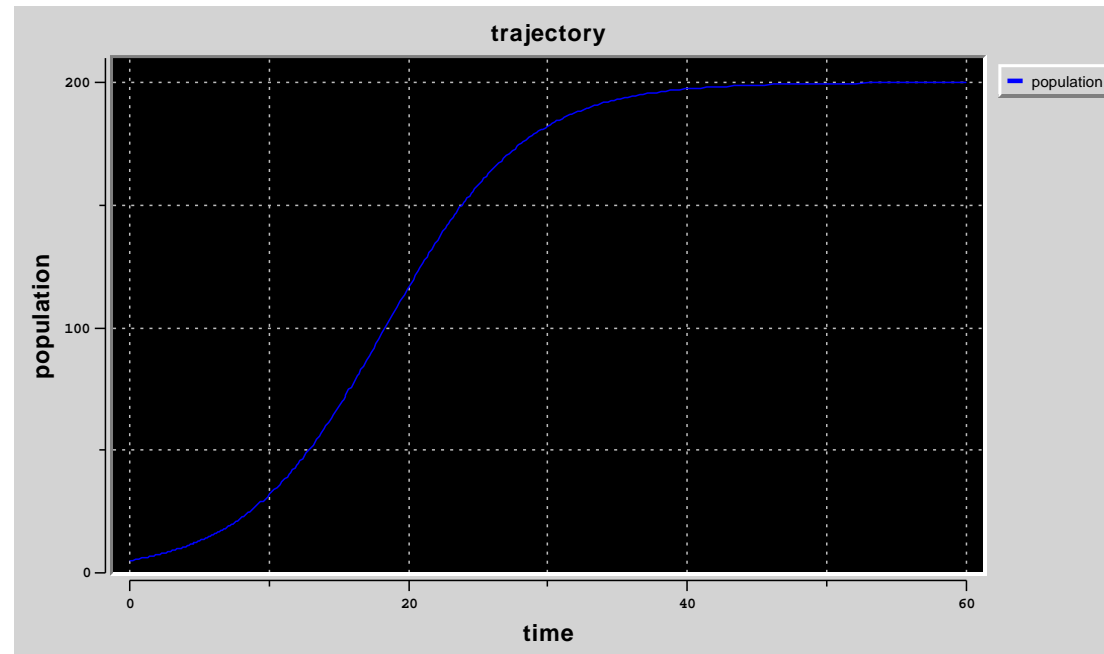
$P \uparrow \rightarrow E_{pc} \downarrow \rightarrow k_{BR} \downarrow$ and $k_{DR} \uparrow$ until equilibrium

- “crowding” effect:
ecosystem can support maximum population P_{max}

$$\frac{dP}{dt} = k \times \left(1 - \frac{P}{P_{max}}\right) \times P$$

- crowding is a quadratic effect

$$k_{BR} = 1.2, k_{DR} = 1.4, \textit{crowding} = 0.001$$



Disadvantages

- *NO* physical evidence for model structure !
- But, many phenomena can be well *fitted* by logistic model.
- P_{max} can only be *estimated* once steady-state has been reached.
Not suitable for control, optimisation, ...
- Many-species system: P_{max} , steady-state ?

Multi-species: Predator-Prey

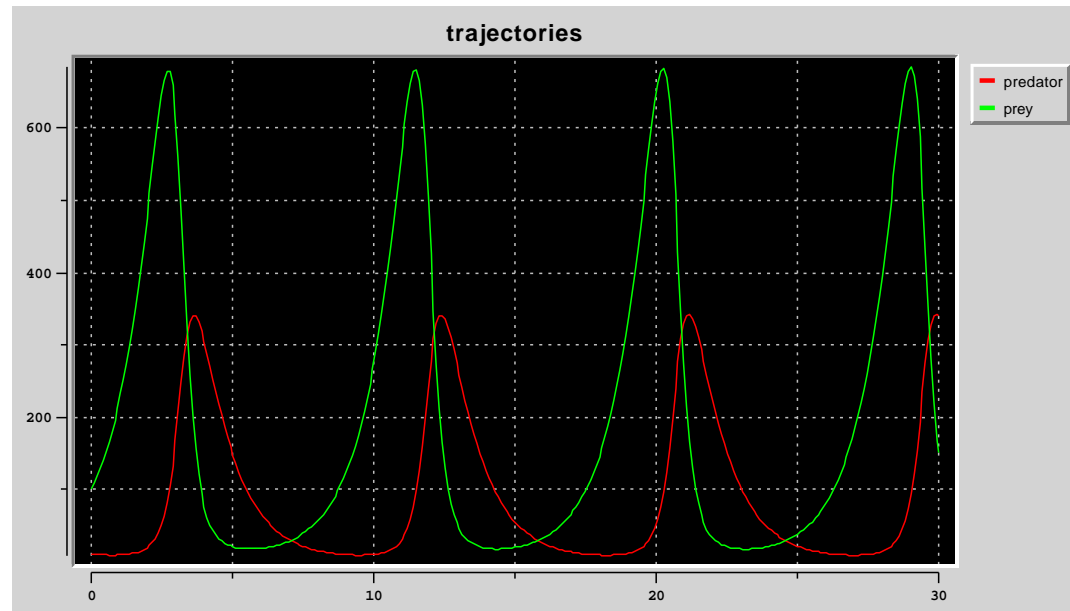
- Individual species behaviour + *interactions*
- Proportional to species, no interaction when one is extinct:
product interaction $P_{pred} \times P_{prey}$

$$\frac{dP_{pred}}{dt} = -a \times P_{pred} + k \times b \times P_{pred} \times P_{prey}$$

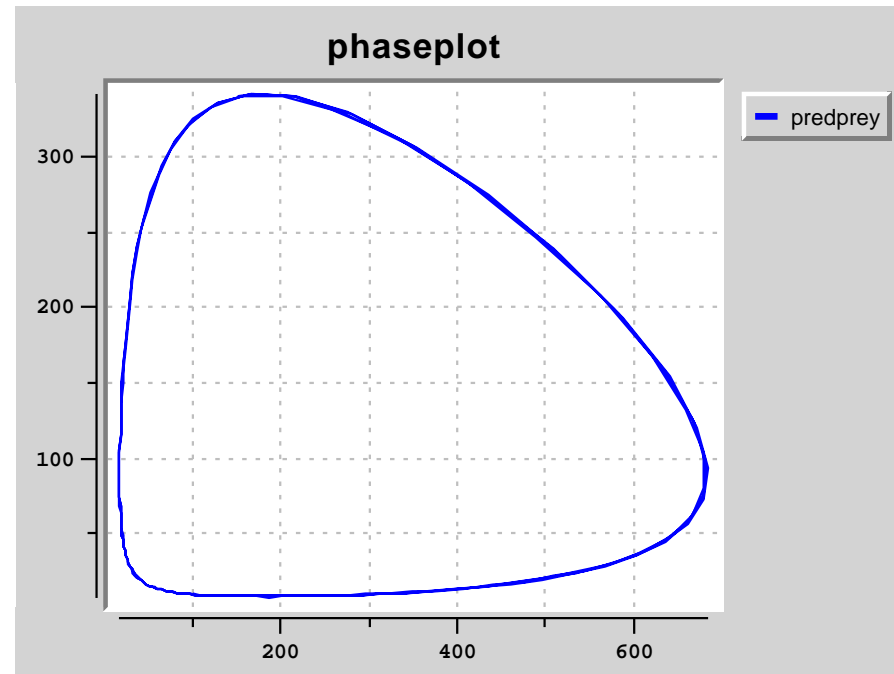
$$\frac{dP_{prey}}{dt} = c \times P_{prey} - b \times P_{pred} \times P_{prey}$$

- Excess death rate $a > 0$, excess birth rate $c > 0$,
grazing factor $b > 0$, efficiency factor $0 < k \leq 1$
- *Lotka-Volterra* equations (1956): periodic steady-state

Predator Prey (population)



Predator Prey (phase)



Competition and Cooperation

- Several species competing for the *same* food source

$$\begin{aligned}\frac{dP_1}{dt} &= a \times P_1 - b \times P_1 \times P_2 \\ \frac{dP_2}{dt} &= c \times P_2 - d \times P_1 \times P_2\end{aligned}$$

- Cooperation of different species (symbiosis)

$$\begin{aligned}\frac{dP_1}{dt} &= -a \times P_1 + b \times P_1 \times P_2 \\ \frac{dP_2}{dt} &= -c \times P_2 + d \times P_1 \times P_2\end{aligned}$$

Grouping and general n -species Interaction

- Grouping (opposite of crowding)

$$\frac{dP}{dt} = -a \times P + b \times P^2$$

- n -species interaction

$$\frac{dP_i}{dt} = \left(a_i + \sum_{j=1}^n b_{ij} \times P_j \right) \times P_i, \forall i \in \{1, \dots, n\}$$

- Only *binary* interactions,
no $P_1 \times P_2 \times P_3$ interactions

Forrester System Dynamics

- based on observation + physical insight
- semi-physical, semi-inductive *methodology*

Methodology

1. levels/stocks and rates/flows

Level	Inflow	Outflow
population	birth rate	death rate
inventory	shipments	sales
money	income	expenses

2. laundry list: levels, rates, and *causal relationships*

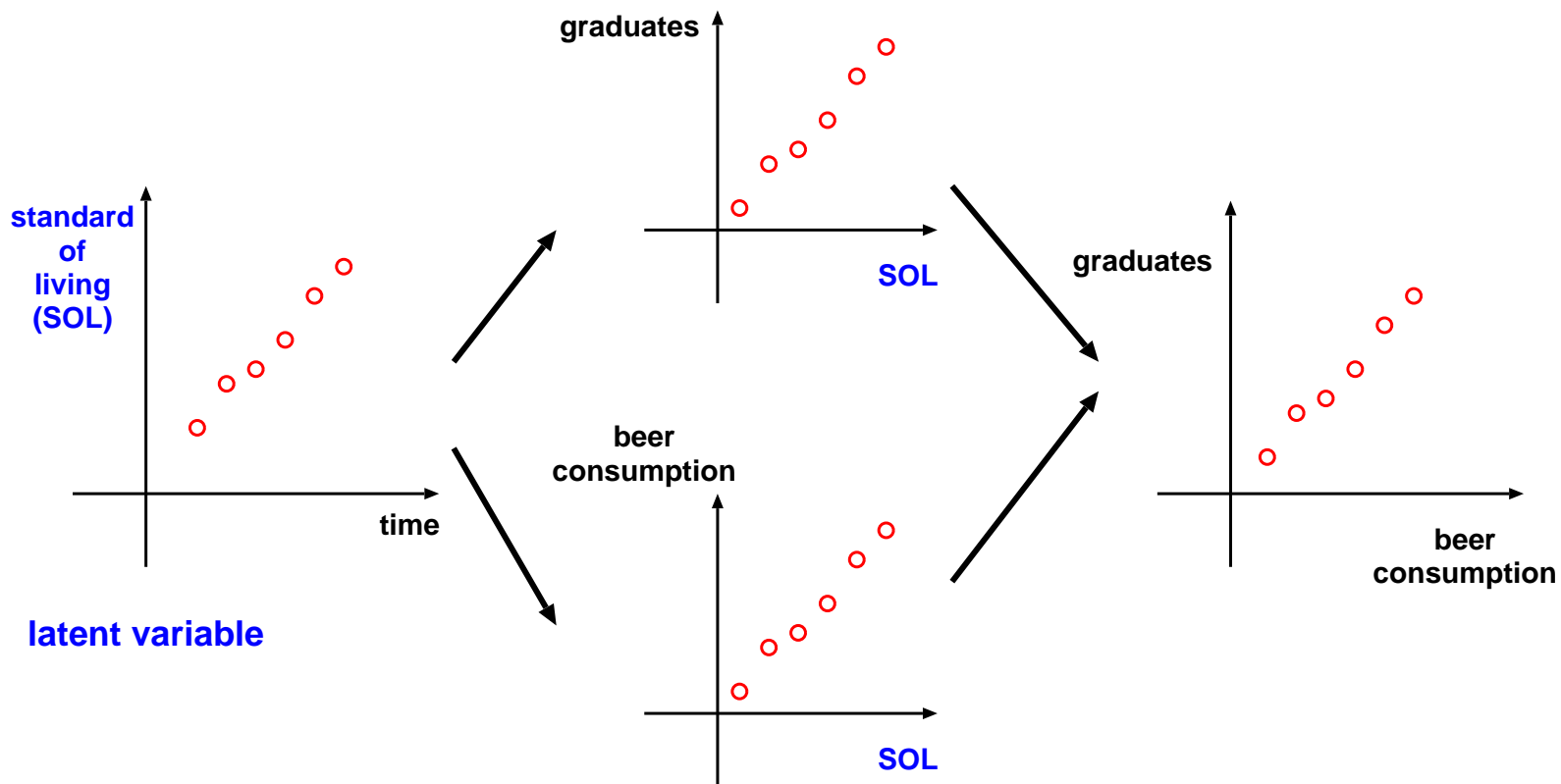
birth rate → birth → population

3. Influence Diagram (+ and -)

4. Structure Diagram (functional relationships)

$$\frac{dP}{dt} = BR - DR$$

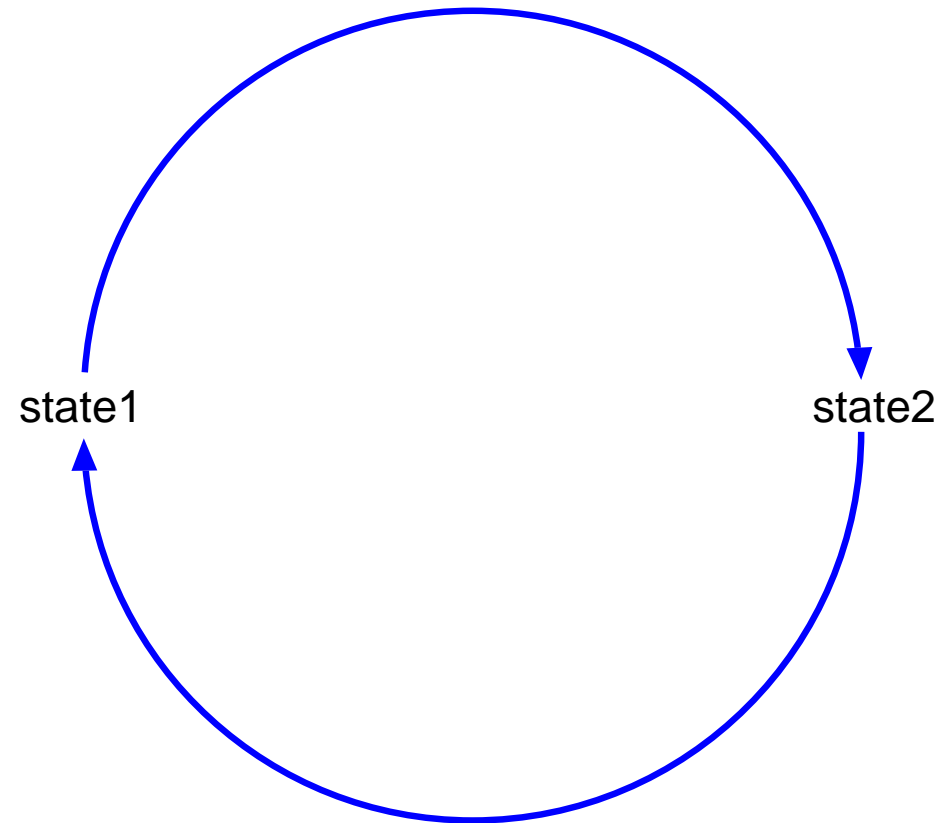
Causal Relationships



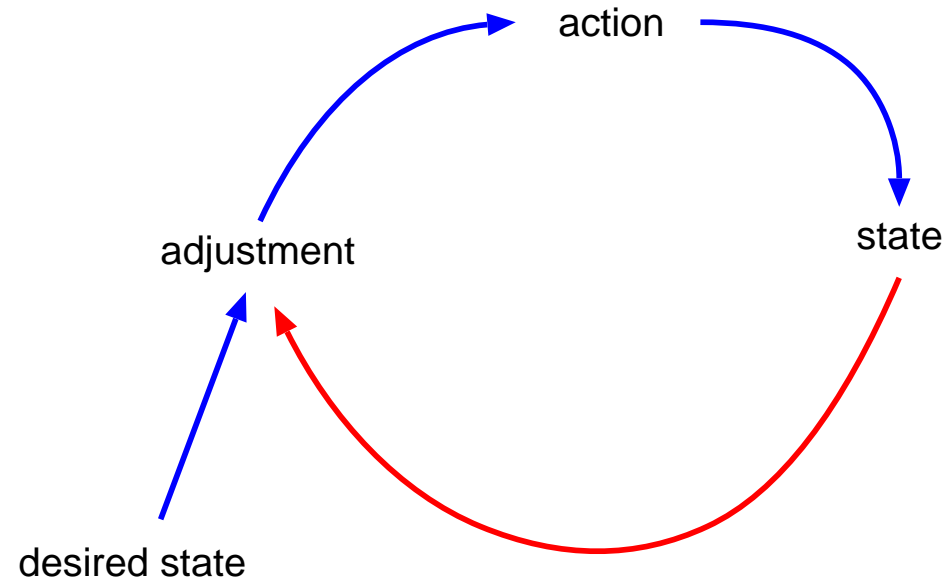
Archetypes

- Bellinger <http://www.outsights.com/systems/>
- influence diagrams
- Common combinations of reinforcing and balancing structures

Archetypes: Reinforcing Loop



Archetypes: Balancing Loop



Inductive Modelling: World Dynamics

- *BR*: BirthRate
- *P*: Population
- *POL*: Pollution
- *MSL*: Mean Standard of Living
- ...

Inductive Modelling: Structure Characterization

$$BR = f(P, POL, MSL, \dots)$$

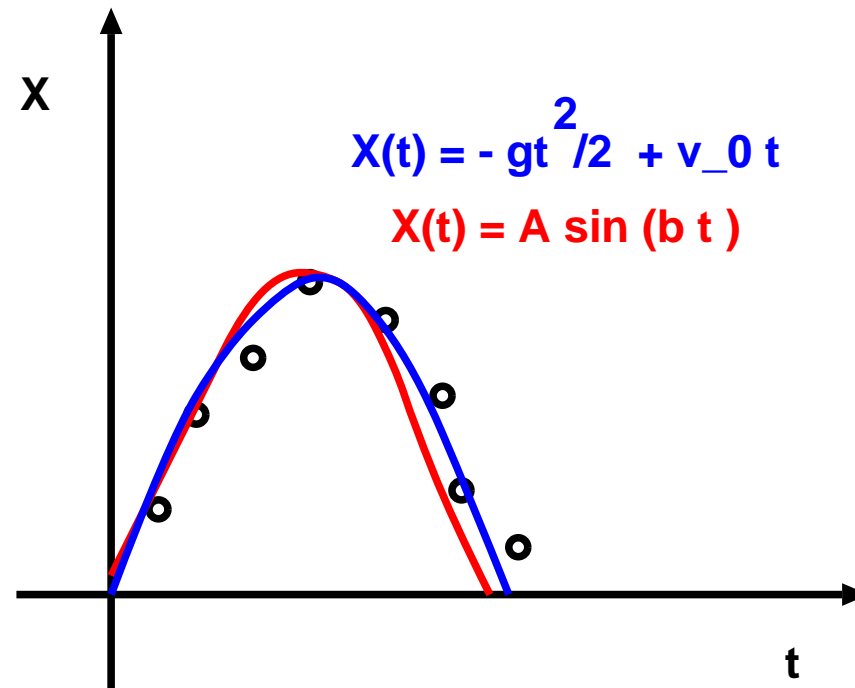
$$BR = BRN \times f^{(1)}(P, POL, MSL, \dots)$$

$$BR = BRN \times P \times f^{(2)}(POL, MSL, \dots)$$

$$BR = BRN \times P \times f^{(3)}(POL) \times f^{(4)}(MSL) \dots$$

- $f^{(3)}(POL)$ inversely proportional
- $f^{(4)}(MSL)$ proportional
- compartmentalize to find correlations
- ... Structure Characterization !

Structure Characterisation: LSQ fit

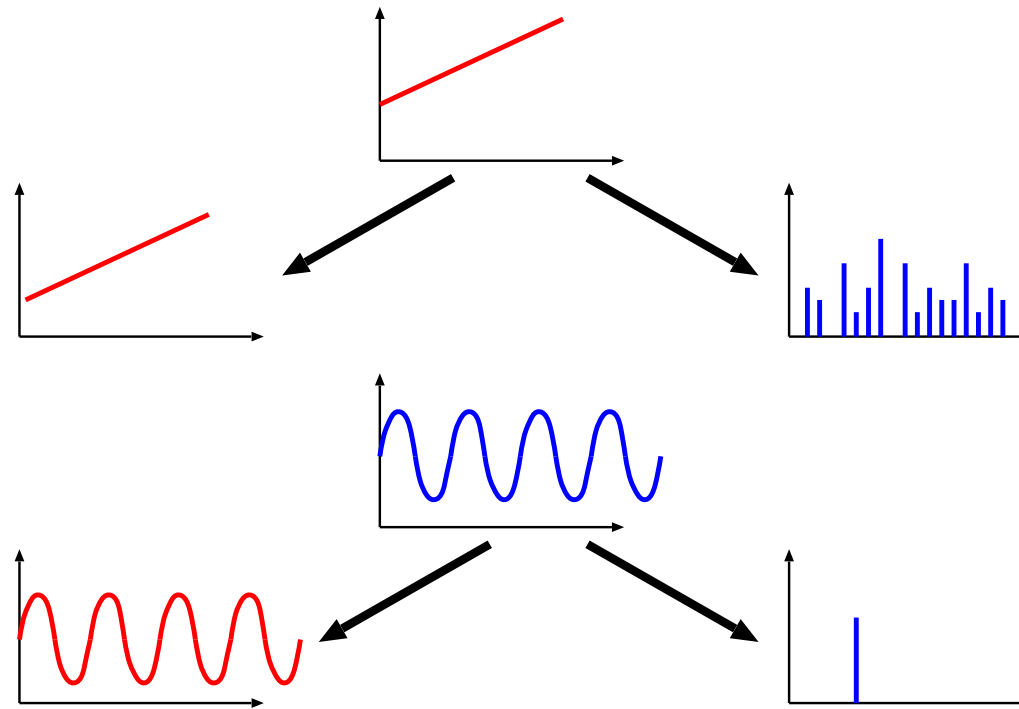


$$\text{LSQ}(\sin) < \text{LSQ}(t^2)$$

Feature Extraction

1. Measurement data *and* model candidates
2. Structure selection and validation
3. Parameter estimation
4. Model use

Feature Rationale



Minimum Sensitivity to Noise
Maximum Discriminating Power

Throwing Stones

Candidate Models

1. $x = -\frac{1}{2}gt^2 + v_0t$
2. $x = A\sin(bt)$

Feature 1 (quadratic model)

$$g_i = \frac{2x_i}{t_i^2} - \frac{2\dot{x}_i}{t_i}, i = A, B$$

$$F1 = g_A/g_B$$

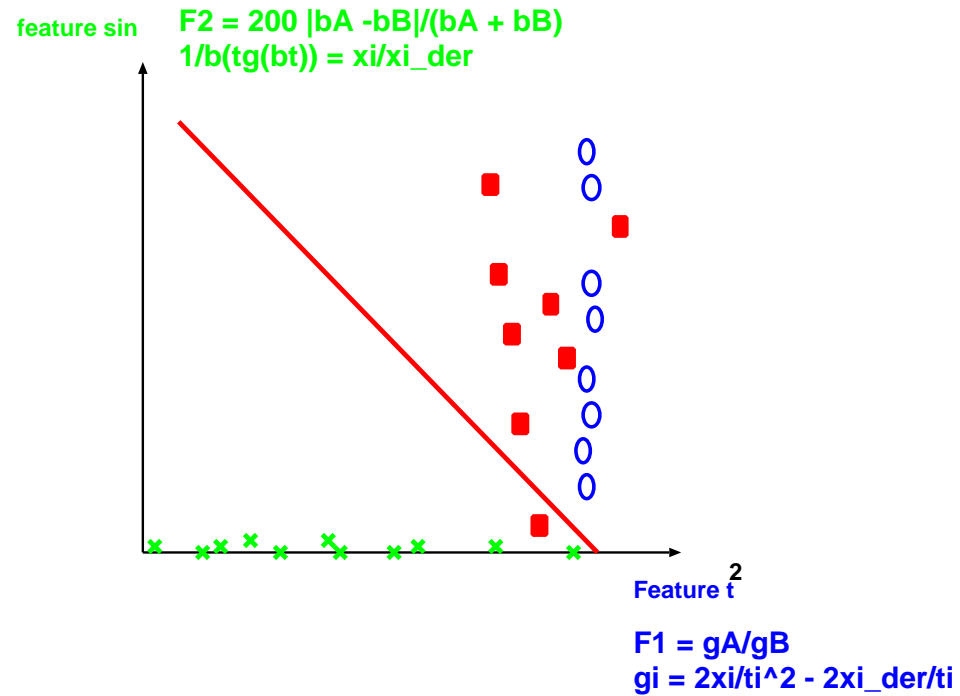
Feature 2 (sin model)

$$\frac{1}{b} \operatorname{tg}(bt) = \frac{x_i}{\dot{x}_i}$$

solve numerically for b

$$F2 = 200 \frac{|b_A - b_B|}{b_A + b_B}$$

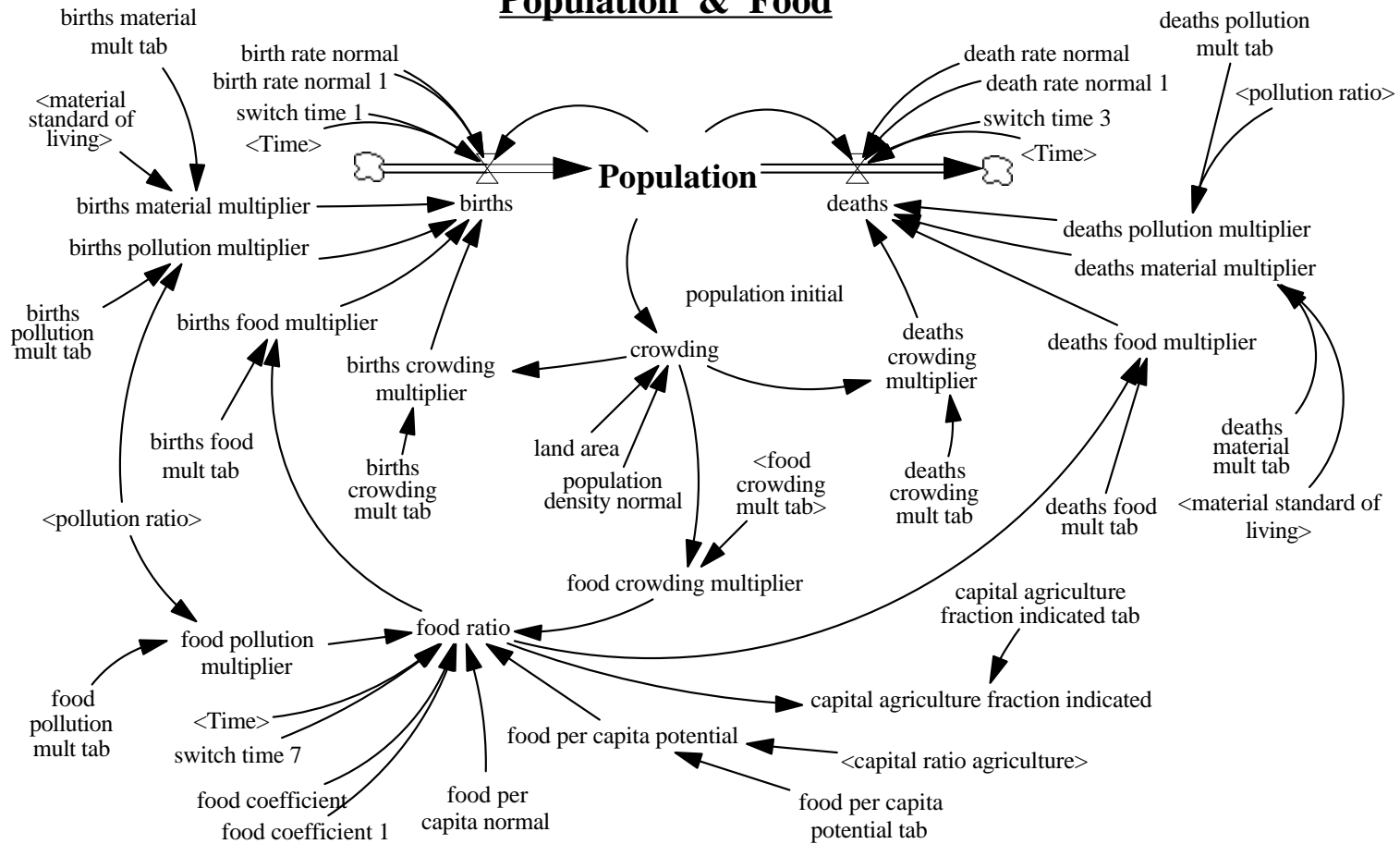
Feature Space Classification



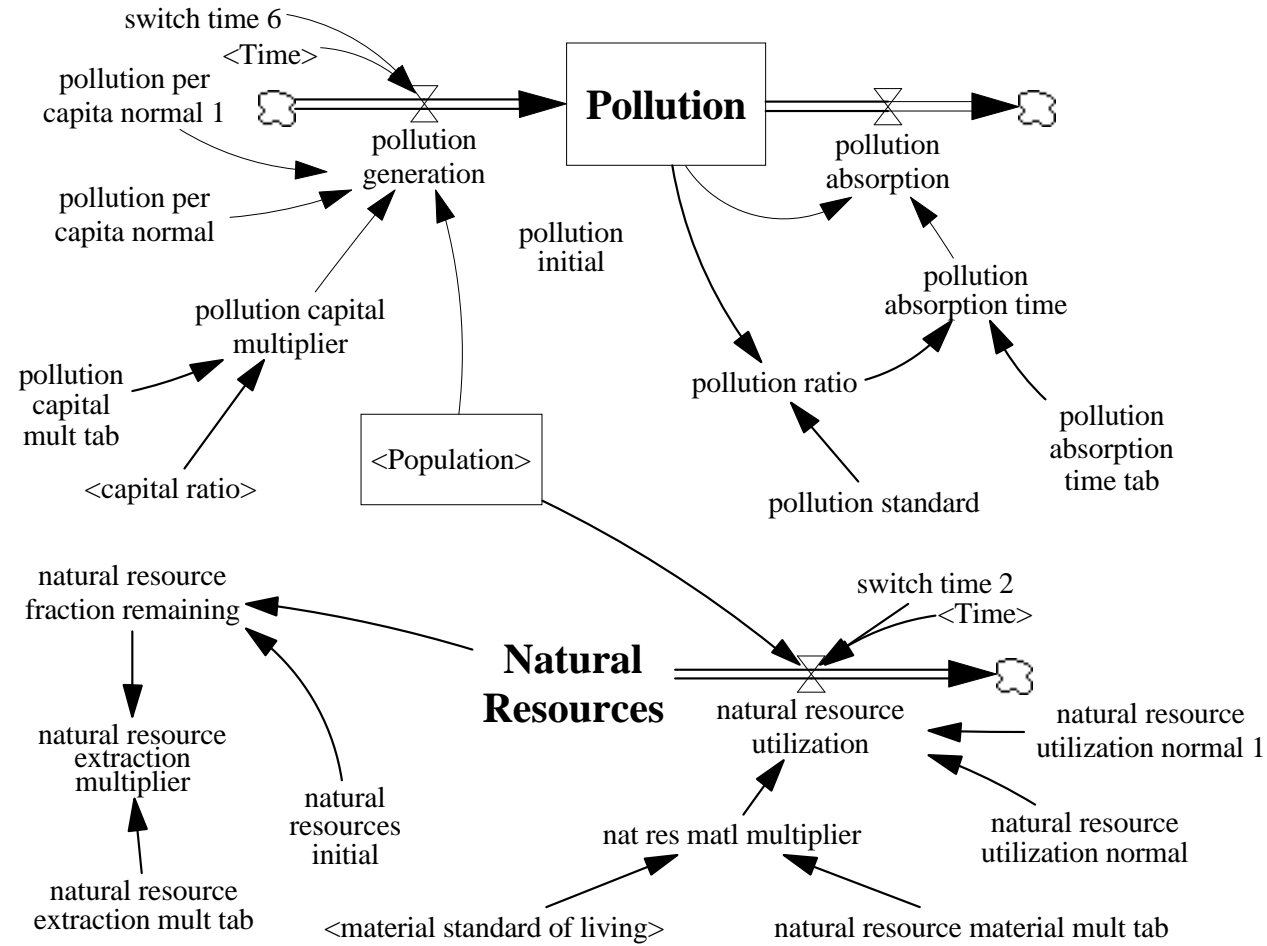
Forrester's World Dynamics model

- “Club of Rome” World Dynamics model
- Few “levels”, note the depletion of natural resources
- implemented in Vensim PLE (www.vensim.com)

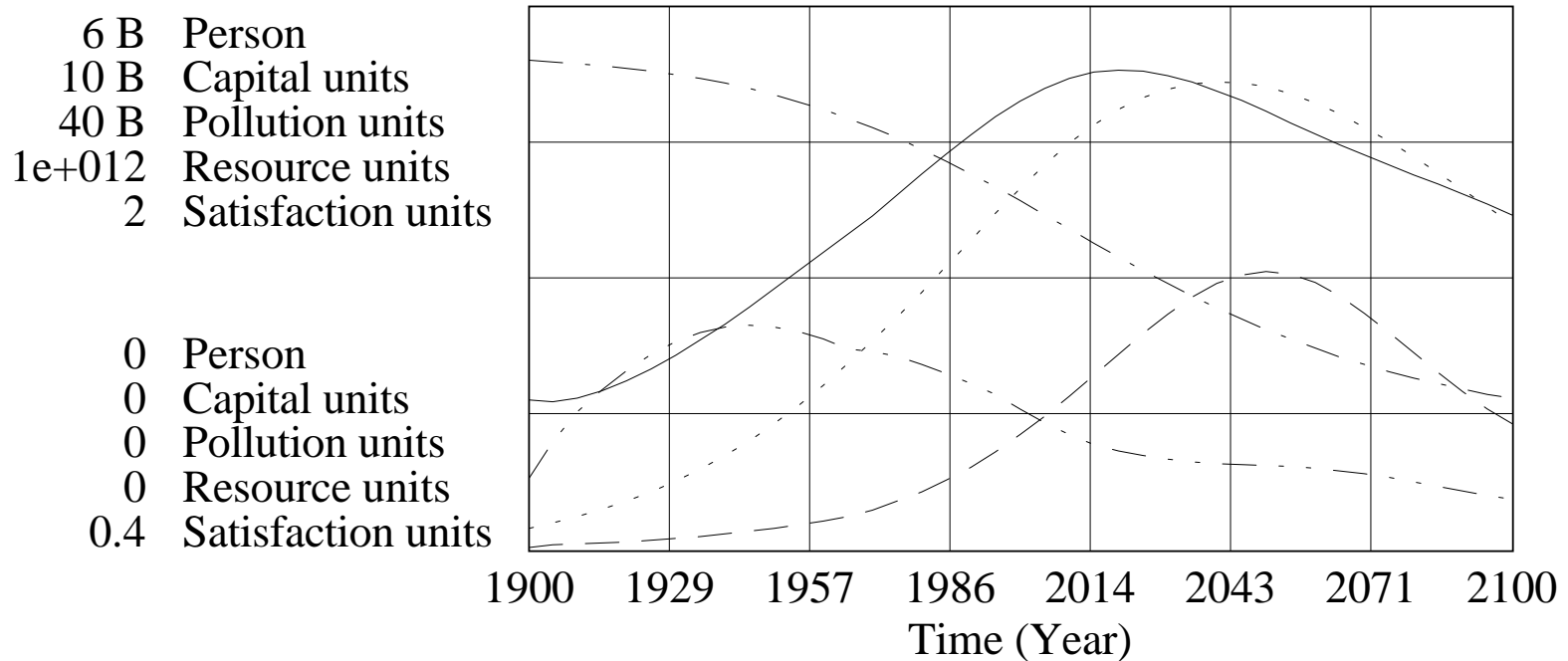
Population & Food



Pollution & Natural Resources

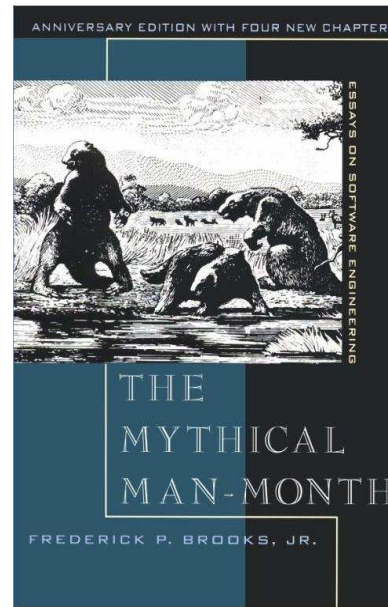


World Model Results



Population : run1 _____ Person
 Capital : run1 - - - - - Capital units
 Pollution : run1 - - - - - Pollution units
 Natural Resources : run1 - - - - - Resource units
 quality of life : run1 - - - - - Satisfaction units

The Process influences Productivity

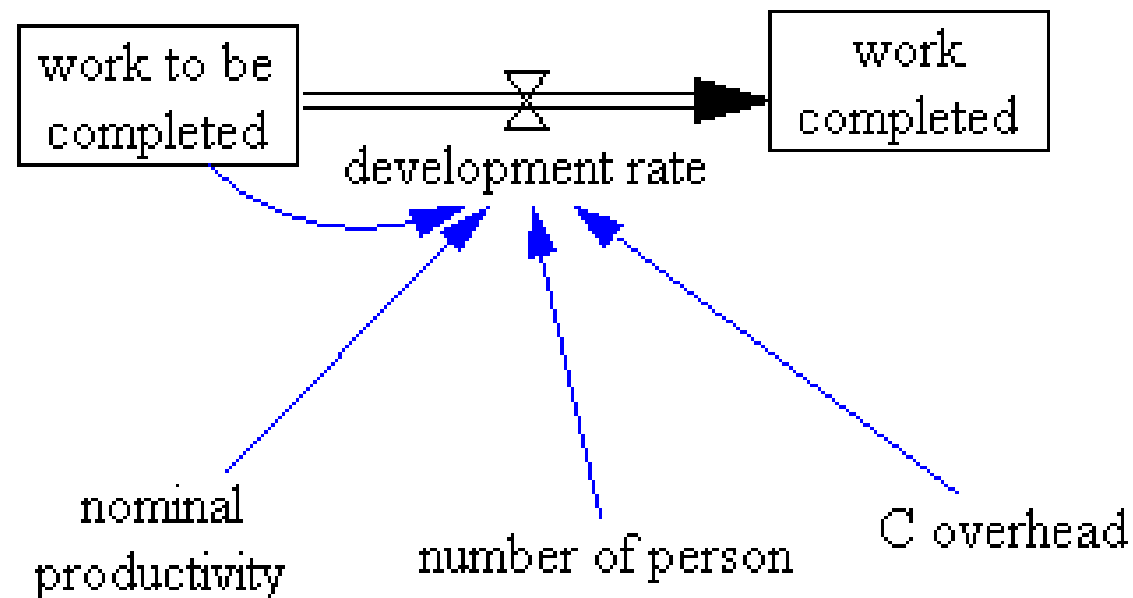


“Adding manpower to a late software project makes it later”

Fred Brooks. The Mythical Man-Month.

(www.ercb.com/feature/feature.0001.html)

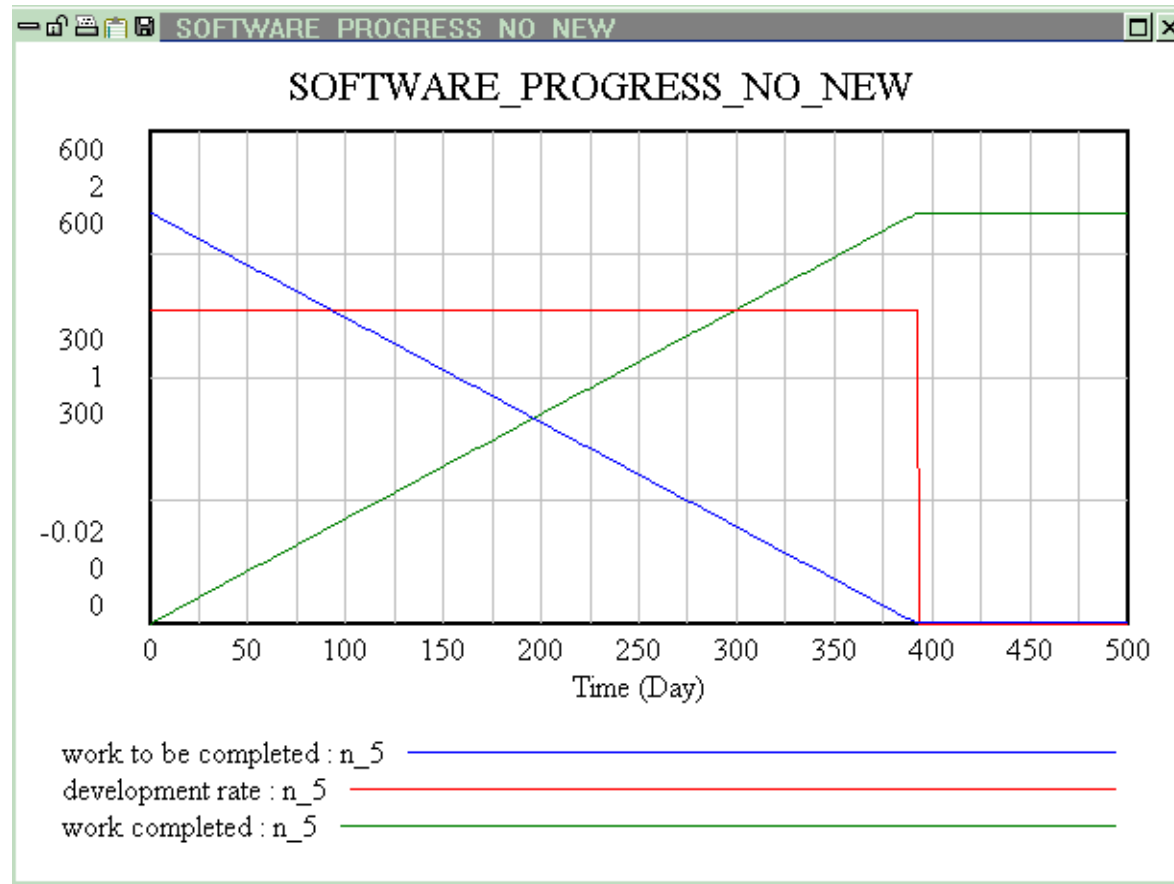
Why Brooks' Law ? Team Size.



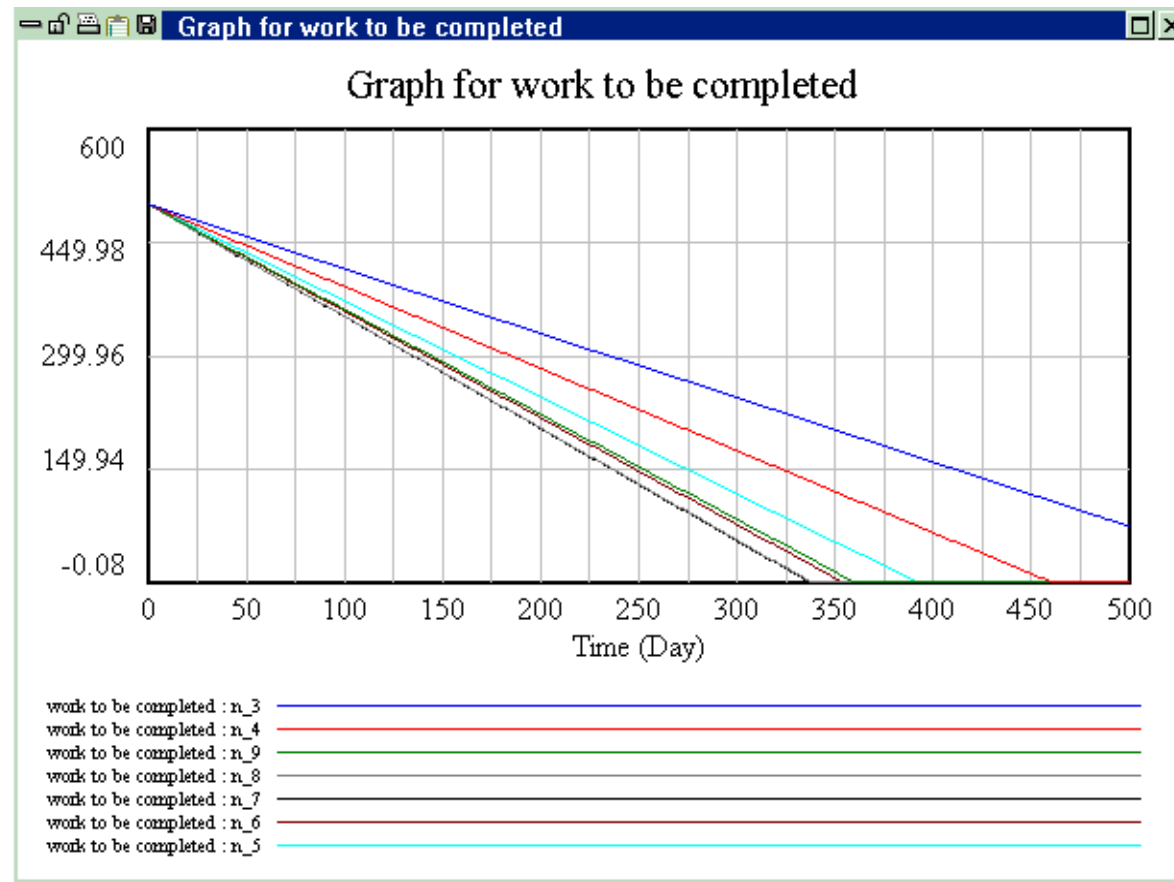
Model in **Forrester System Dynamics**
using Vensim PLE (www.vensim.com)

$$\text{development rate} = \text{nominal_productivity} * (1 - \text{C_overhead} * (\text{N} * (\text{N} - 1))) * \text{N}$$

Team Size $N = 5$

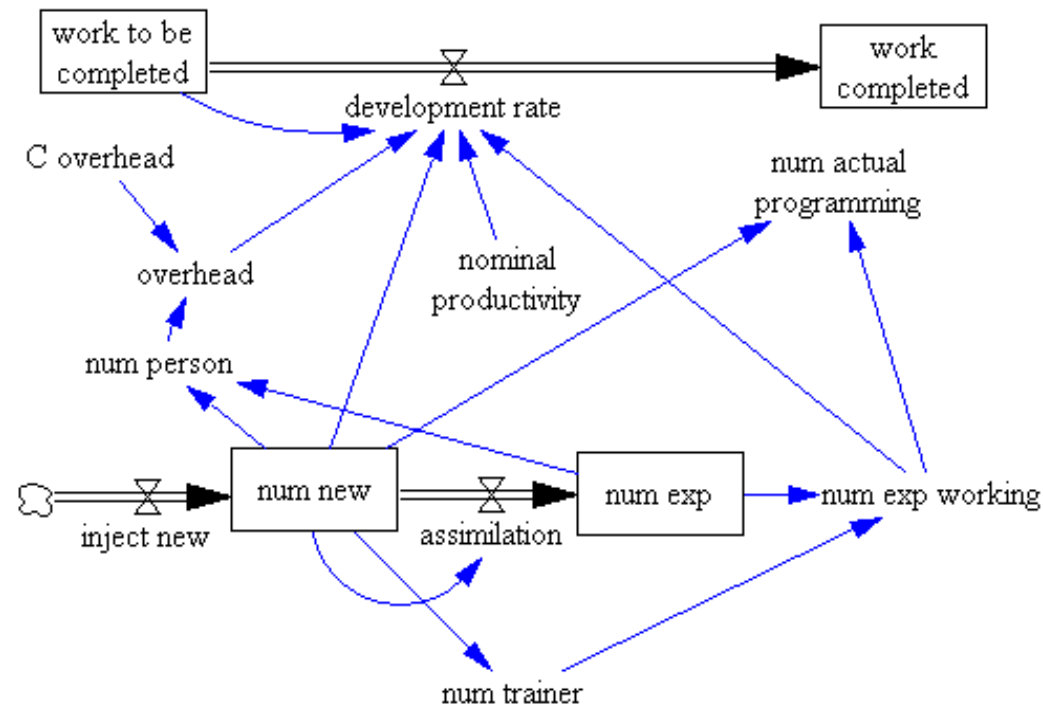


Team Size $N = 3 \dots 9$



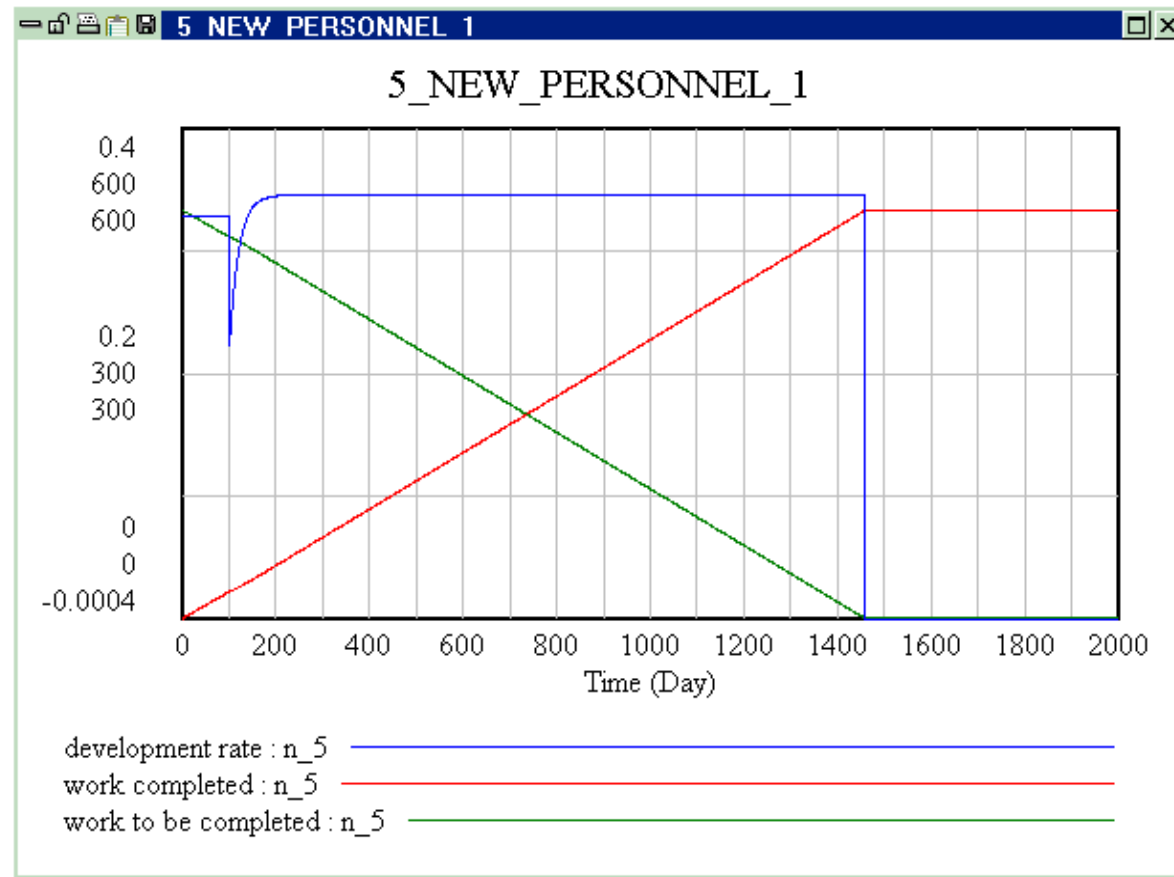
Optimal Team Size between 7 and 8

The Effect of Adding New Personnel (FSD model)

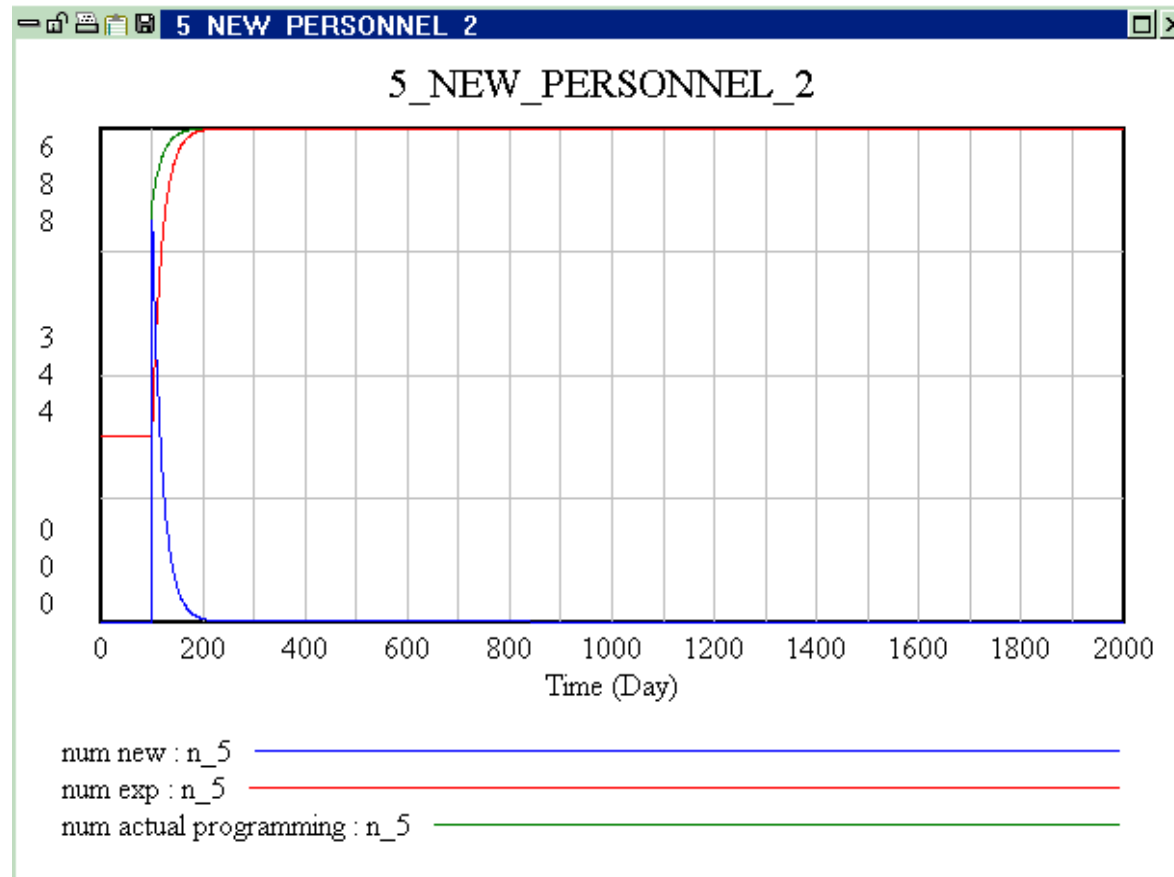


$$\text{development rate} = \text{nominal_productivity} * (1 - C_overhead * (N * (N - 1))) * (1.2 * \text{num_exp_working} + 0.8 * \text{num_new})$$

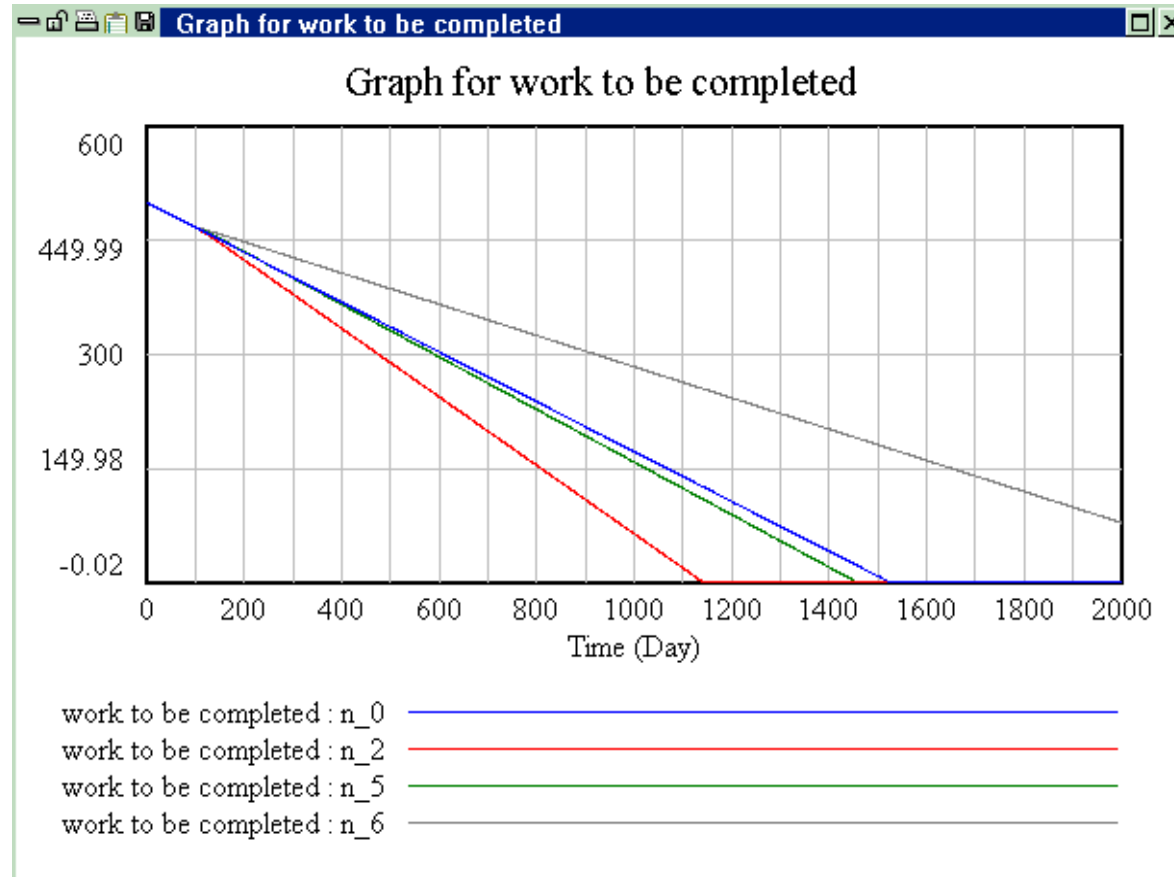
5 New Programmers after 100 days



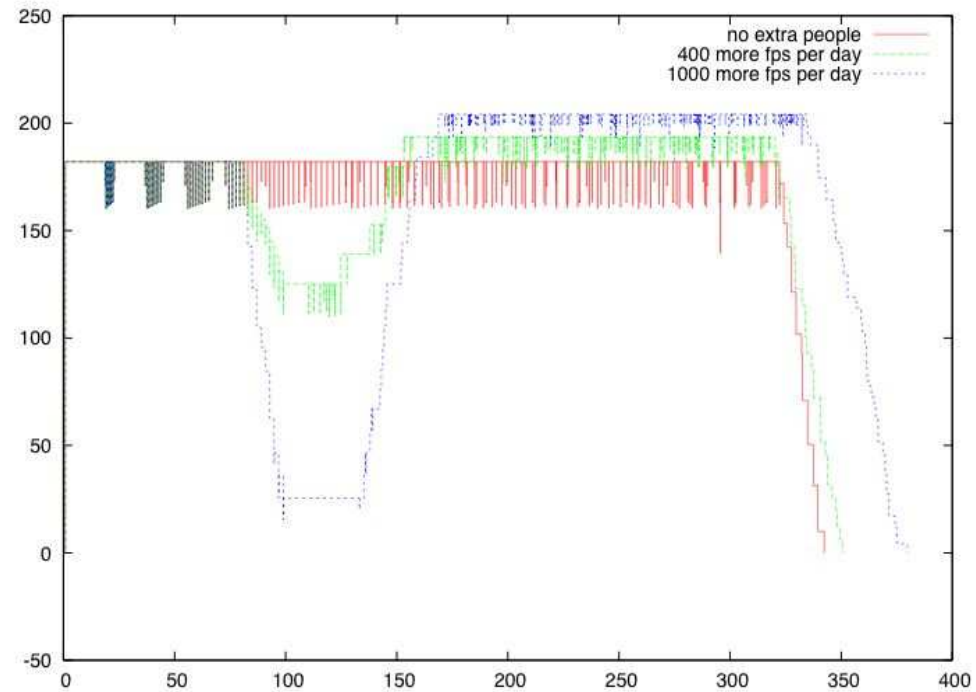
5 New Programmers after 100 days



0 ... 6 New Programmers after 100 days



Individual-based (discrete-event) model: Productivity.



Individual-based (discrete-event) model: Remaining work.

