(Place/Transition) Petri Nets

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Finite State Automaton

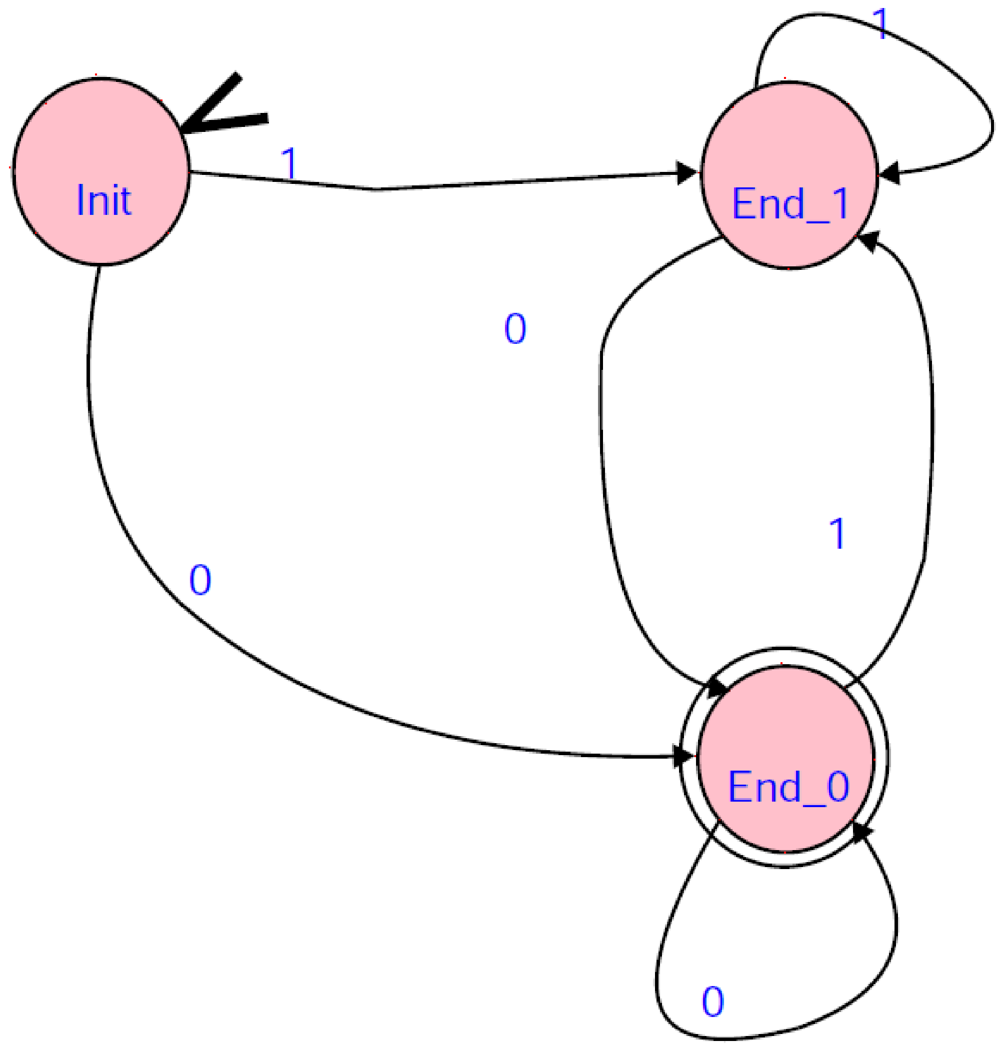
$$(E, X, f, x_0, F)$$

- E is a finite alphabet
- X is a finite state set
- f is a state transition function,
  $f : X \times E \rightarrow X$
- x_0 is an initial state, $x_0 \in X$
- F is the set of final states

Dynamics (x' is next state):

$$x' = f(x, e)$$

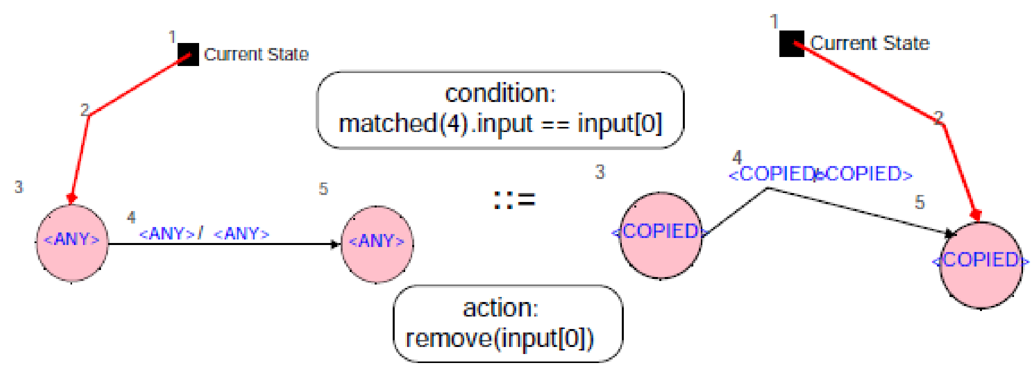
FSA graphical/visual notation: State Transition Diagram



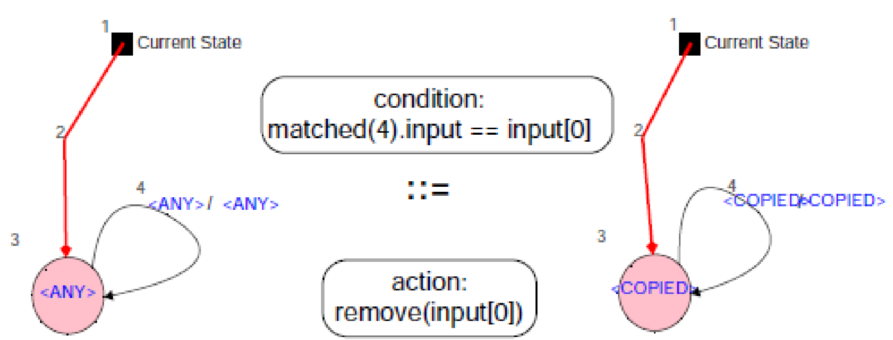
FSA Operational Semantics

Rule 1 (priority 3) **Locate Initial Current State**

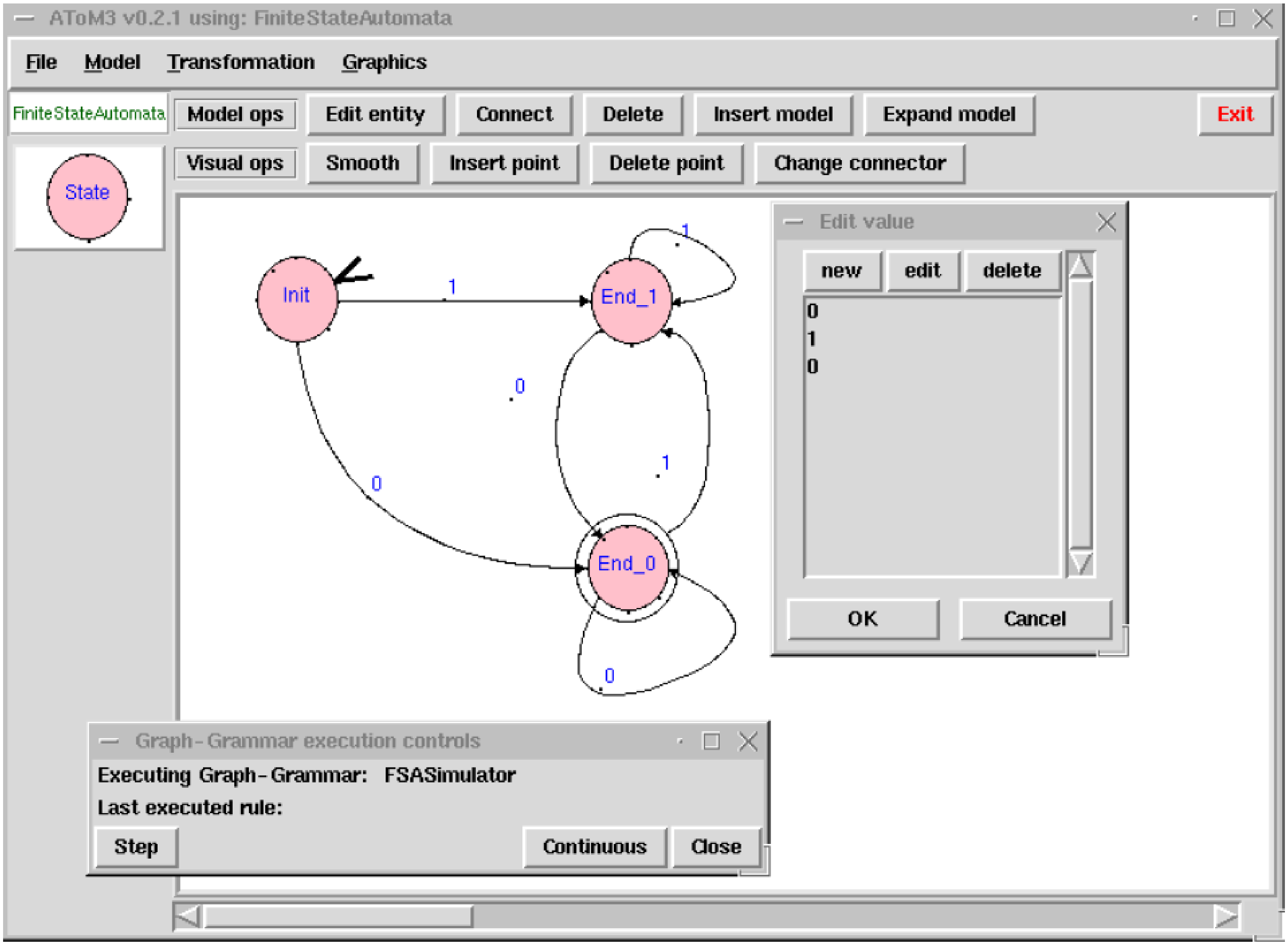
Rule 2 (priority 1) **State Transition**

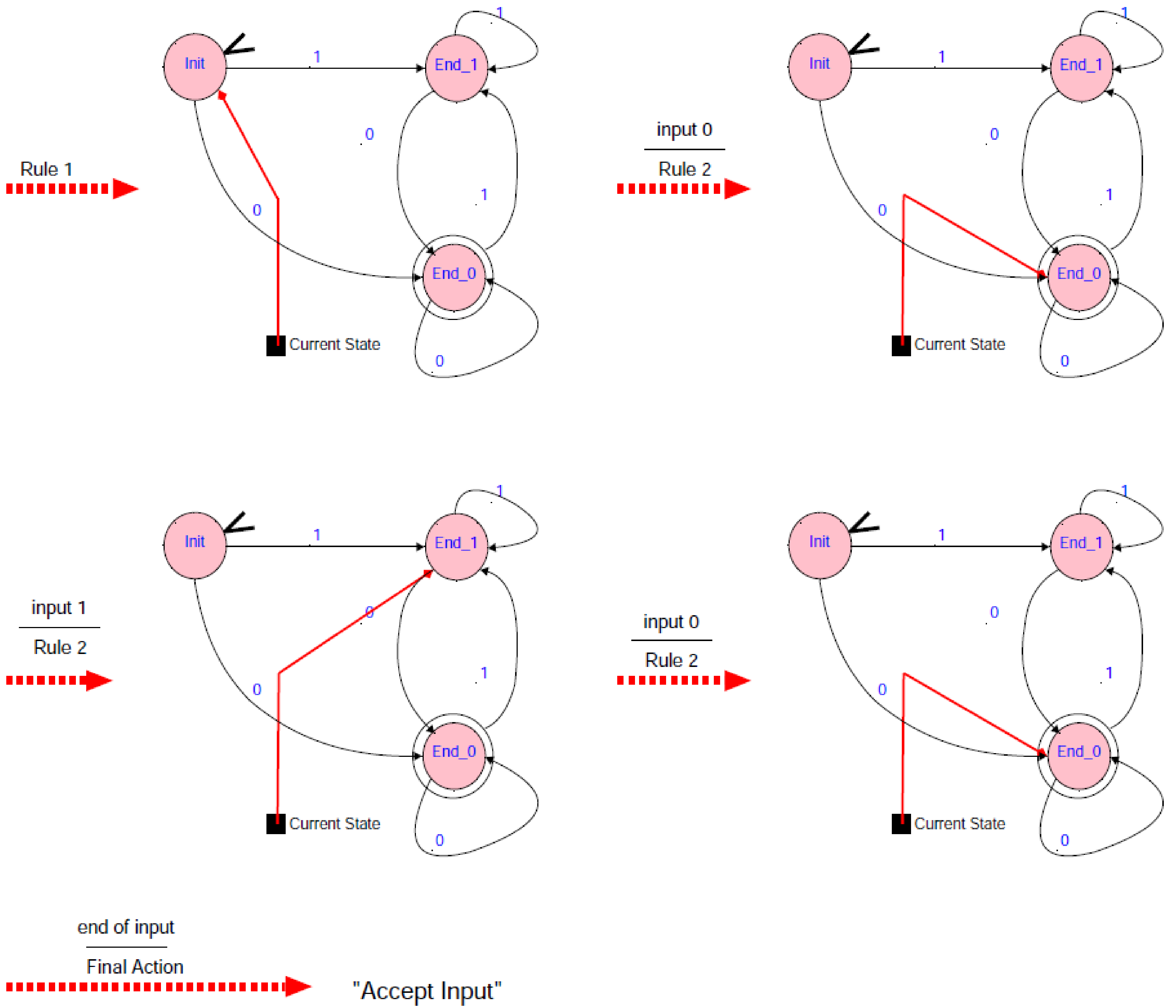


Rule 3 (priority 2) **Local State Transition**



Simulation steps




Rule 1
Init
End_1
End_0
Current State
input 0
Rule 2
input 1
Rule 2
input 0
Rule 2
end of input
Final Action
"Accept Input"

State Automaton

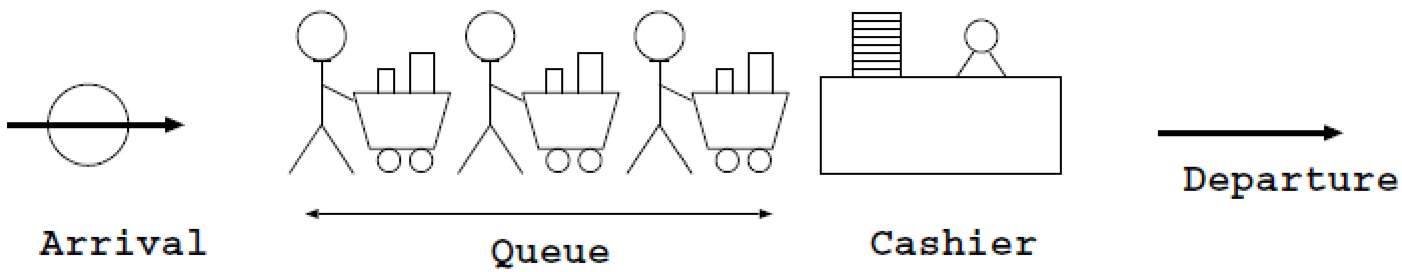
$$(E, X, \Gamma, f, x_0)$$

- E is a countable event set
- X is a countable state space
- $\Gamma(x)$ is the set of feasible or enabled events
  $x \in X, \Gamma(x) \subseteq E$
- f is a state transition function,
  $f : X \times E \to X$, only defined for $e \in \Gamma(x)$
- x_0 is an initial state, $x_0 \in X$

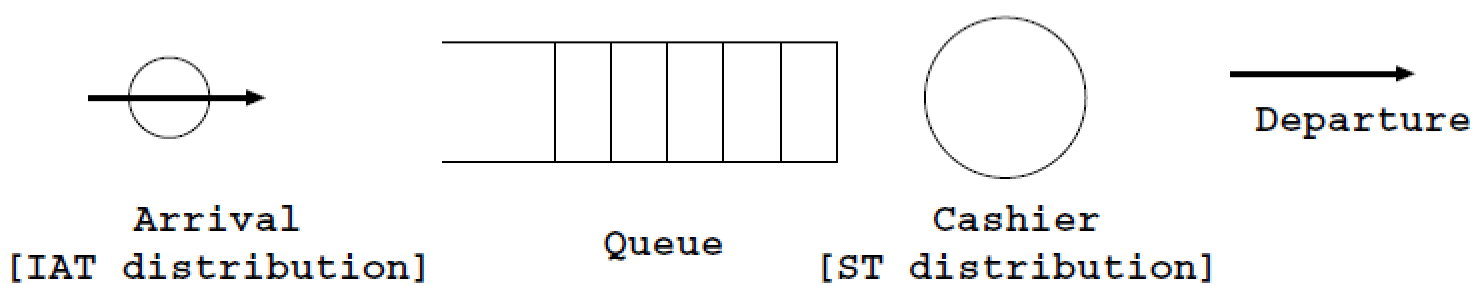
$$(E, X, \Gamma, f)$$

omits x_0 and describes a class of State Automata.

State Automata for Queueing Systems

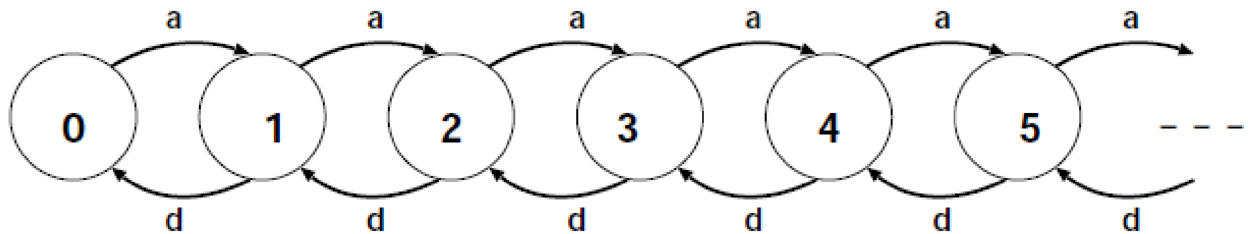


Physical View



Abstract View

State Automata for Queueing Systems: customer centered



$$E = \{a, d\}$$

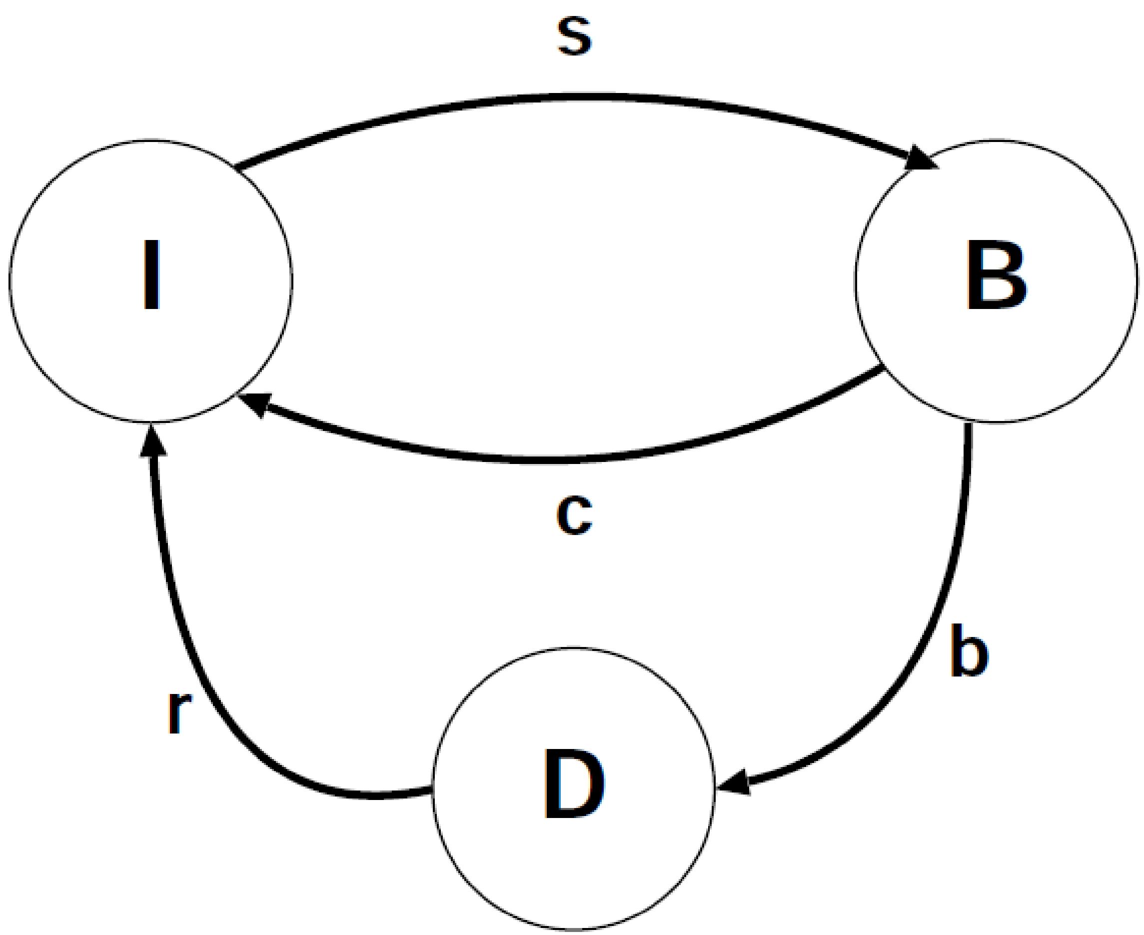
$$X = \{0, 1, 2, \dots\}$$

$$\Gamma(x) = \{a, d\}, \forall x > 0; \Gamma(0) = \{a\}$$

$$f(x, a) = x + 1, \forall x \geq 0$$

$$f(x, d) = x - 1, \forall x > 0$$

State Automata for Queueing Systems: server centered (with breakdown)



Limitiations/extensions of State Automata

- Adding time ?
- Hierarchical modelling ?
- Concurrency by means of $\times$
- States are represented explicitly
- Specifying control logic, synchronisation ?

Petri nets

- Formalism similar to FSA
- Graphical/Visual notation
- C.A. Petri 1960s
- Additions to FSA:
  - Explicitly (graphically/visually) represent when event is enabled $\rightarrow$ describe control logic
  - Elegant notation of concurrency
  - Express non-determinism

Petri net notation and definition (no dynamics)

$$(P, T, A, w)$$

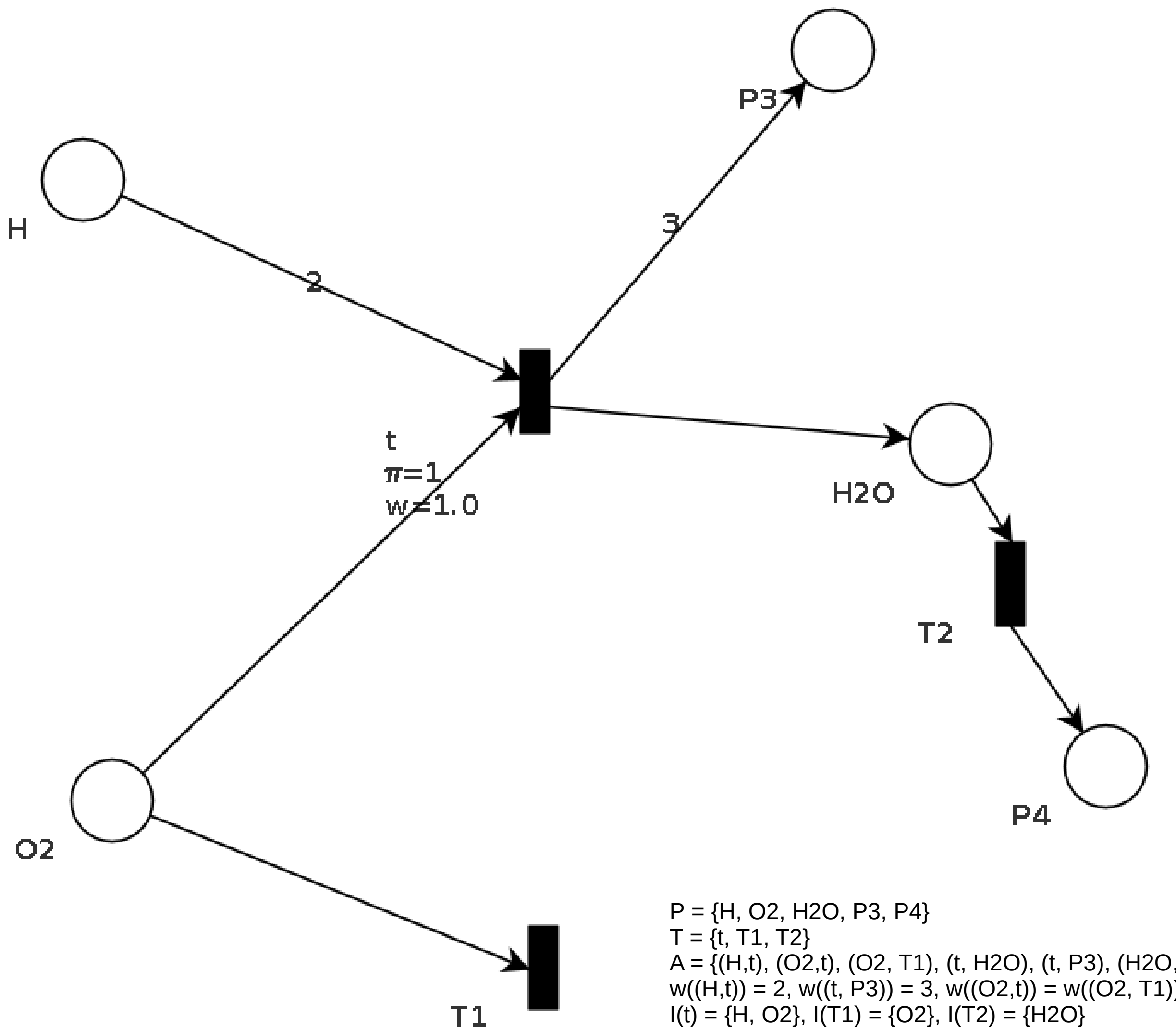
- $P = \{p_1, p_2, \ldots\}$ is a finite set of *places*
- $T = \{t_1, t_2, \ldots\}$ is a finite set of *transitions*
- $A \subseteq (P \times T) \cup (T \times P)$ is a set of *arcs*
- $w : A \to \mathbb{N}$ is a *weight function*

Note: no need for *countable* P and T.

Derived Entities

- $I(t_j) = \{p_i : (p_i, t_j) \in A\}$ set of *input places* to transition t_j
  ($\equiv$ conditions for transition)
- $O(t_j) = \{p_i : (t_j, p_i) \in A\}$ set of *output places* from transition t_j
  ($\equiv$ affected by transition)
- Transitions $\equiv$ events
- similarly: input- and output-transitions for p_i
- graphical/visual representation: *Petri net graph* (multigraph)

Example Petri Net



P = {H, O2, H2O, P3, P4}
T = {t, T1, T2}
A = {(H,t), (O2,t), (O2, T1), (t, H2O), (t, P3), (H2O, T2), (T2, P4)}
w((H,t)) = 2, w((t, P3)) = 3, w((O2,t)) = w((O2, T1)) = w((t, H2O)) = w((H2O, T2)) = w((T2, P4)) = 1
I(t) = {H, O2}, I(T1) = {O2}, I(T2) = {H2O}
O(t) = {P3, H2O}, O(T1) = {}, O(T2) = {P4}

Introducing State: Petri net Markings

- Conditions met ? Use *tokens* in places
- Token assignment $\equiv$ *marking* x

$$x : P \rightarrow \mathbb{N}$$

- A marked Petri net

$$(P, T, A, w, x_0)$$

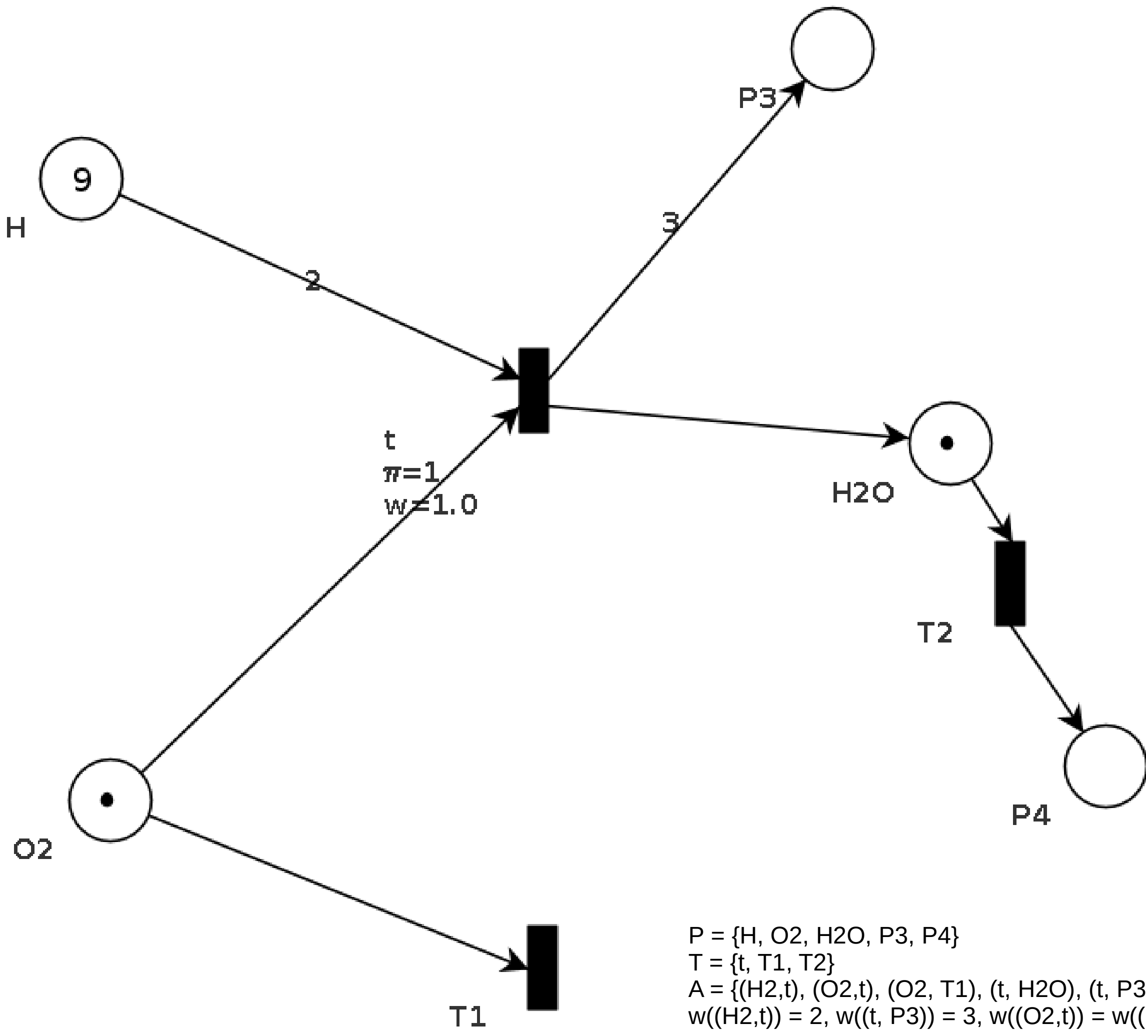
x_0 is the *initial marking*

- The *state* **x** of a marked Petri net

$$\mathbf{x} = [x(p_1), x(p_2), \ldots, x(p_n)]$$

Number of tokens need not be bounded (cfr. State Automata states).

Example Marked Petri Net



P = {H, O2, H2O, P3, P4}
T = {t, T1, T2}
A = {(H2,t), (O2,t), (O2, T1), (t, H2O), (t, P3), (H2O, T2), (T2, P4)}
w((H2,t)) = 2, w((t, P3)) = 3, w((O2,t)) = w((O2, T1)) = w((t, H2O)) = w((H2O, T2)) = w((T2, P4)) = 1
I(t) = {H, O2}, I(T1) = {O2}, I(T2) = {H2O}
O(t) = {P3, H2O}, O(T1) = {}, O(T2) = {P4}
x = [9, 1, 1, 0, 0] corresponding to places [H, O2, H2O, P3, P4]

State Space of Marked Petri net

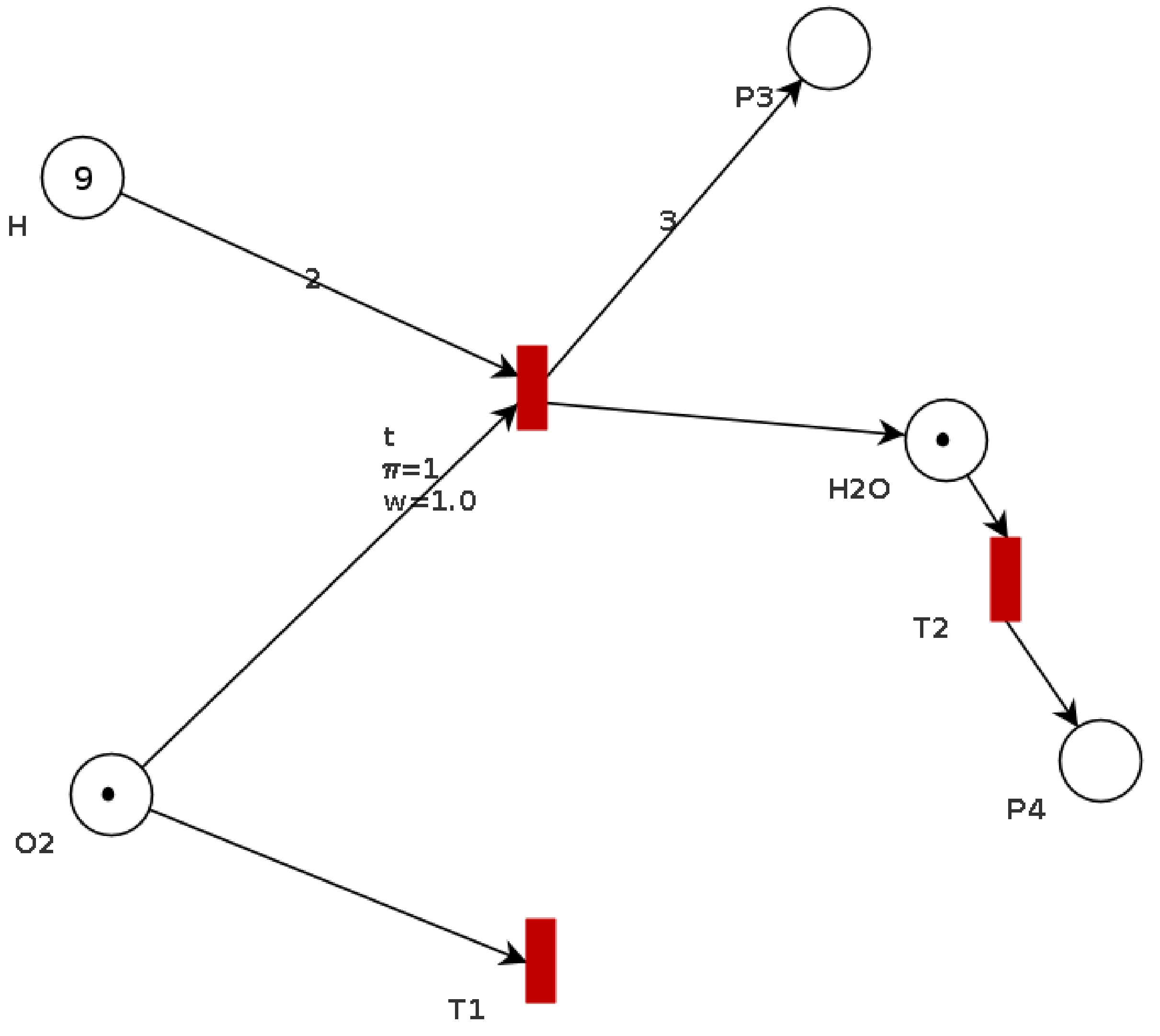
- All n-dimensional vectors of nonnegative integer markings

$$X = \mathbb{N}^n$$

- Transition $t_j \in T$ is *enabled* if

$$x(p_i) \geq w(p_i, t_j), \forall p_i \in I(t_j)$$

Example Marked Petri Net
Enabled transitions in red



Petri Net Dynamics

State Transition Function f of marked Petri net (P, T, A, w, x_0)

$$f : \mathbb{N}^n \times T \rightarrow \mathbb{N}^n$$

is defined for transition $t_j \in T$ if and only if

$$x(p_i) \geq w(p_i, t_j), \forall p_i \in I(t_j)$$

If $f(\mathbf{x}, t_j)$ is defined, set $\mathbf{x}' = f(\mathbf{x}, t_j)$ where

$$x'(p_i) = x(p_i) - w(p_i, t_j) + w(t_j, p_i)$$

- State transition function f based on *structure* of Petri net
- Number of tokens *need not be conserved* (but can)

Algebraic Description of Dynamics

- Firing vector $\mathbf{u}$: transition j firing

$$\mathbf{u} = [0, 0, \ldots, 1, 0, \ldots, 0]$$

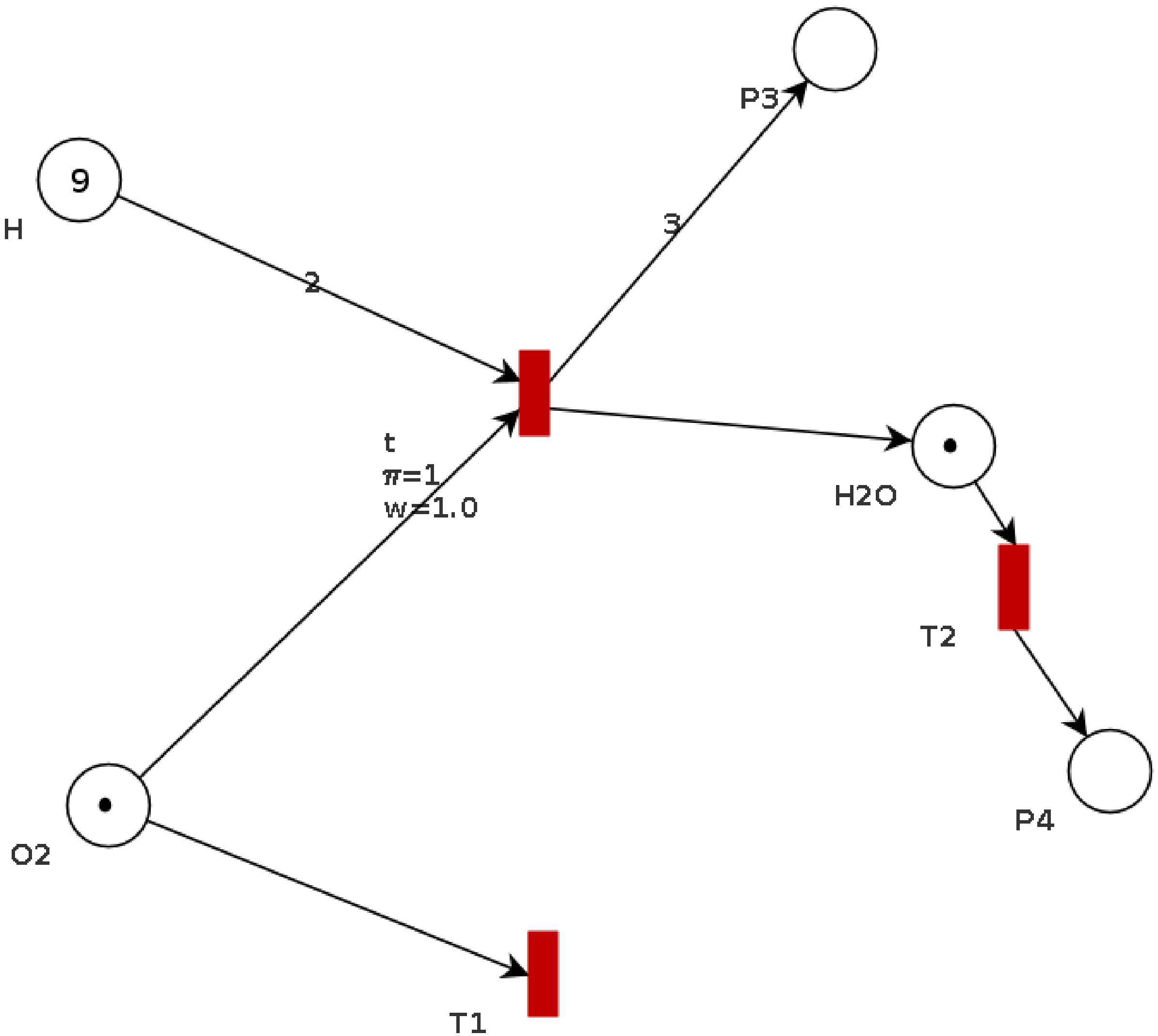
- Incidence matrix $\mathbf{A}$:

$$a_{ji} = w(t_j, p_i) - w(p_i, t_j)$$

- State Equation

$$\mathbf{x}' = \mathbf{x} + \mathbf{u}\mathbf{A}$$

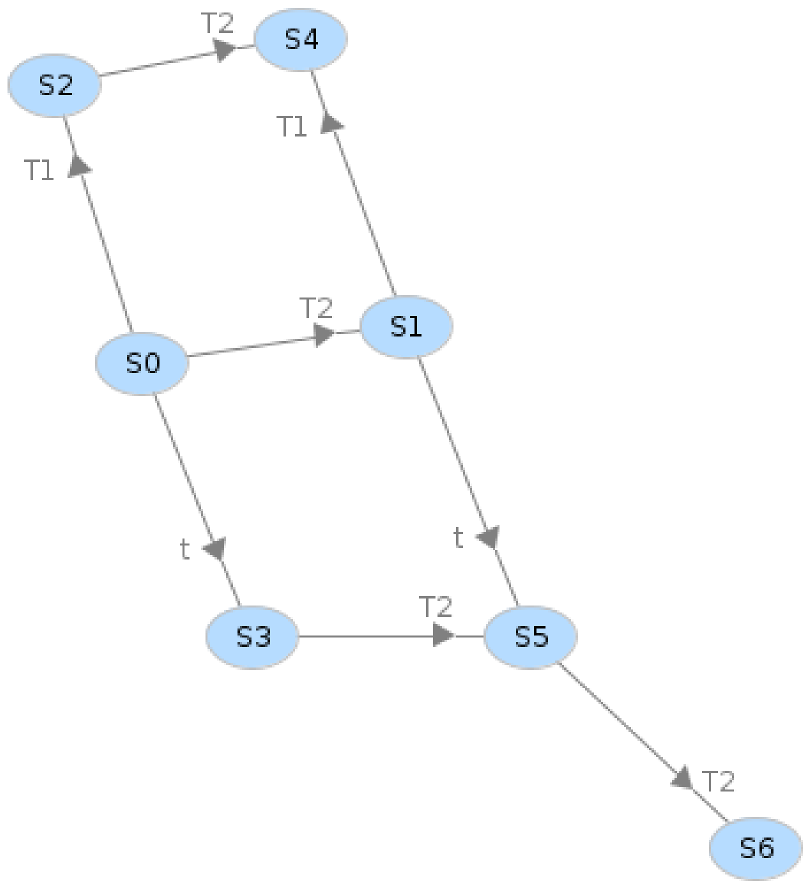
Example Marked Petri Net
Enabled transitions in red



Semantics

- *sequential* vs. *parallel*
- Handle nondeterminism:
  1. User choice
  2. Priorities
  3. Probabilities (Monte Carlo)
  4. Reachability Graph (enumerate all choices)

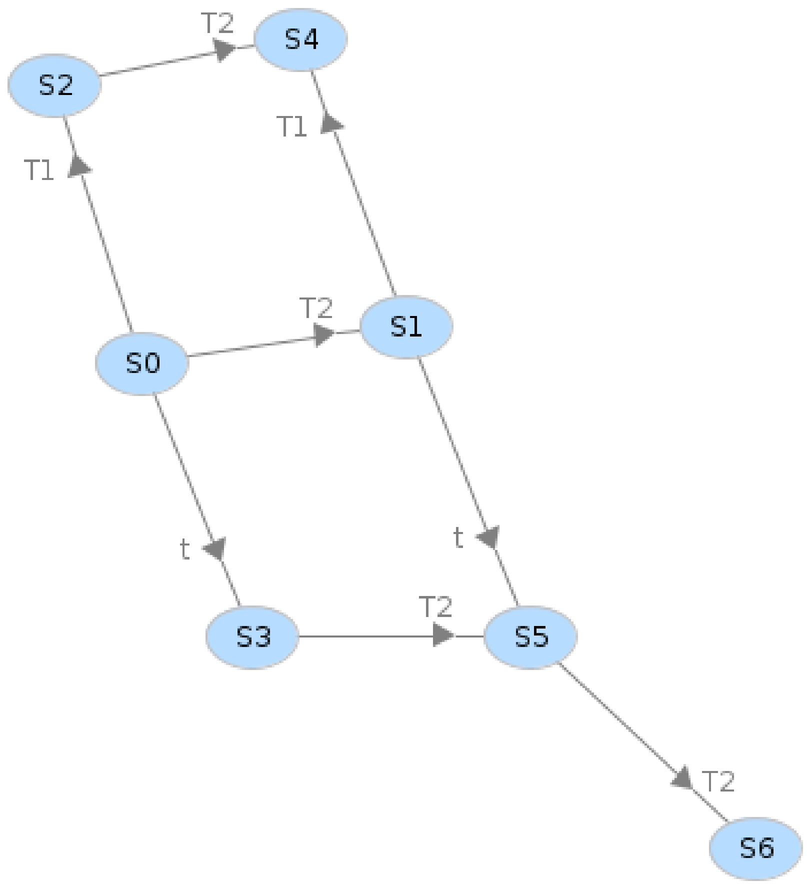
Example Marked Petri Net



S0 = [9, 1, 1, 0, 0]
S1 = [9, 1, 0, 0, 1]
S2 = [9, 0, 1, 0, 0]
S3 = [7, 0, 2, 3, 0]
S4 = [9, 0, 0, 0, 1]
S5 = [7, 0, 1, 3, 1]
S6 = [7, 0, 0, 3, 2]

Marking corresponds to [H, O2, H2O, P3, P4]

Example Marked Petri Net



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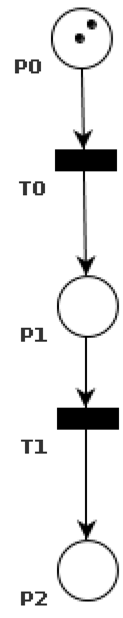
S4 = [9, 0, 0, 0, 1]
S5 = [7, 0, 1, 3, 1]
S6 = [7, 0, 0, 3, 2]

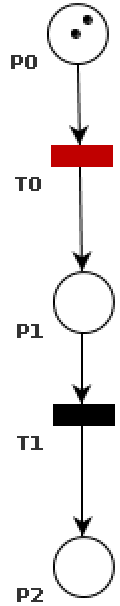
Reachability graph =
compact notation of all possible “sample paths” (behaviour traces) =

{ `S0 -T1-> S2 -T2-> S4, S0 -T2-> S1 -T1-> S4,`
`S0 -T2-> S1 -t-> S5 -T2-> S6, S0 -t-> S3 -T2-> S5 -T2-> S6`}

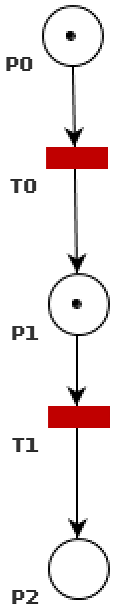
Pattern: sequence



Pattern: sequence



Pattern: sequence



Semantics

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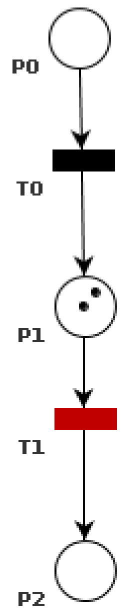
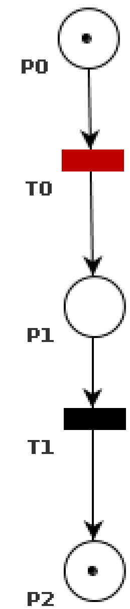
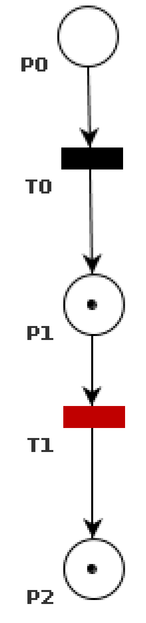
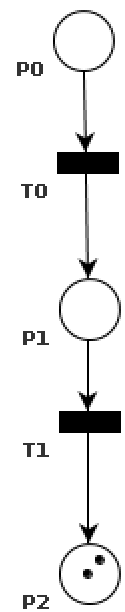




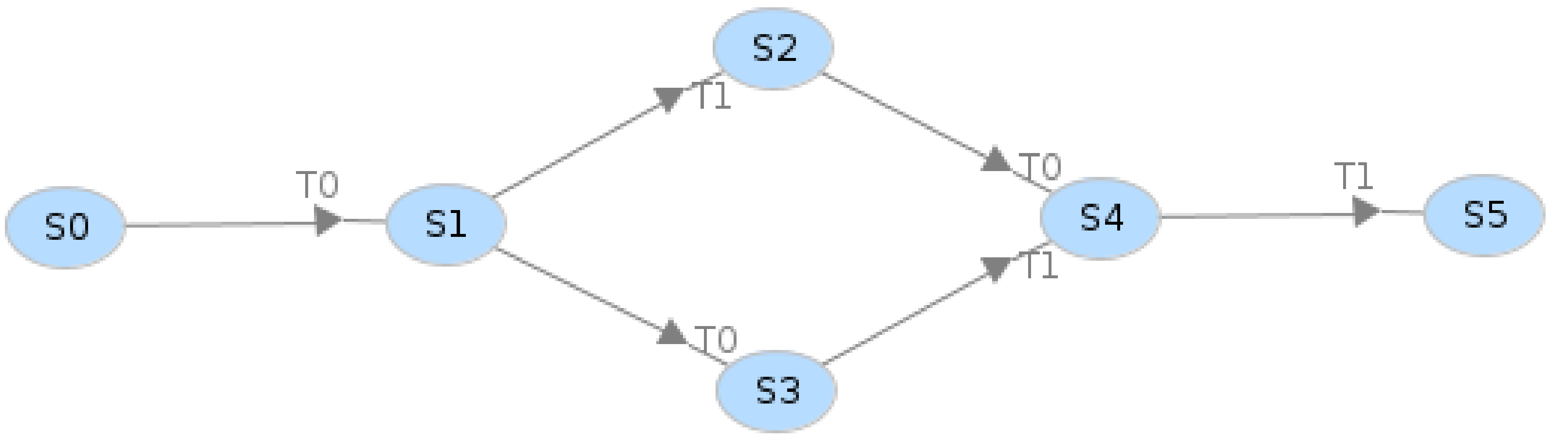
Pattern: sequence



Pattern: sequence

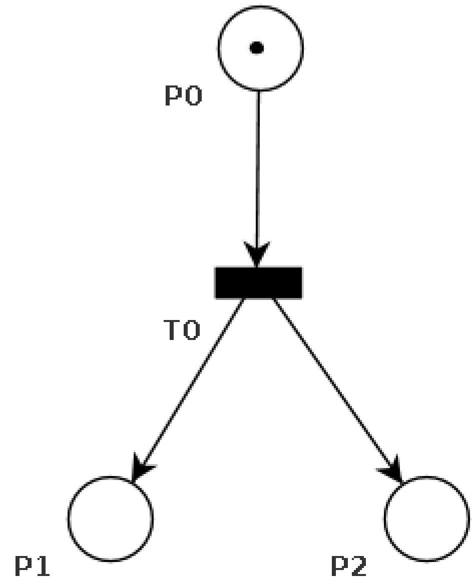


Pattern: sequence

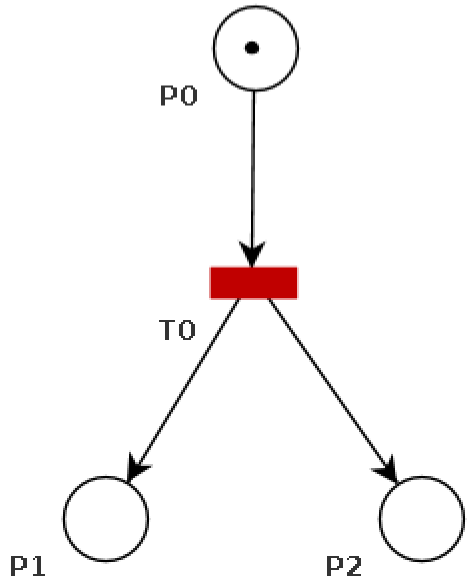


S0 = [2, 0, 0]
S1 = [1, 1, 0]
S2 = [1, 0, 1]
S3 = [0, 2, 0]
S4 = [0, 1, 1]
S5 = [0, 0, 2]

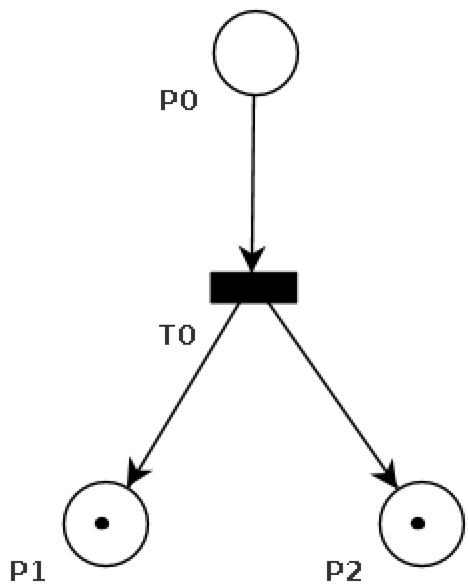
Pattern: split



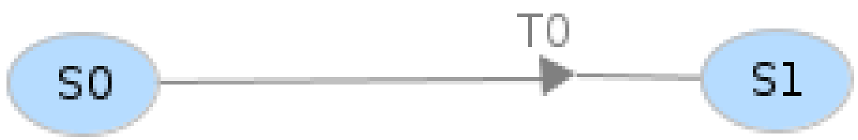
Pattern: split



Pattern: split

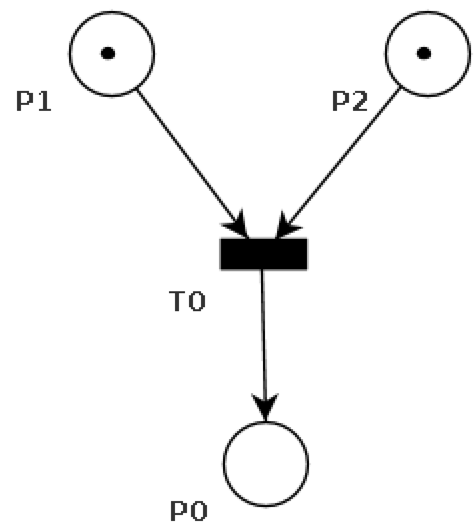


Pattern: split

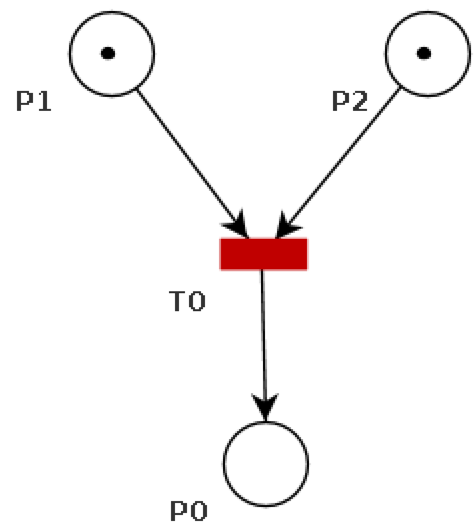


S0 = [1, 0, 0]
S1 = [0, 1, 1]

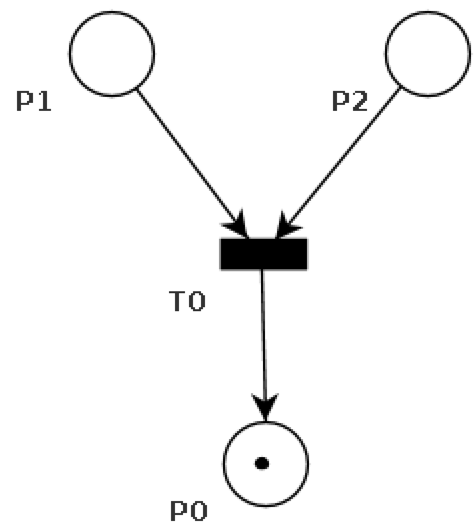
Pattern: join



Pattern: join



Pattern: join



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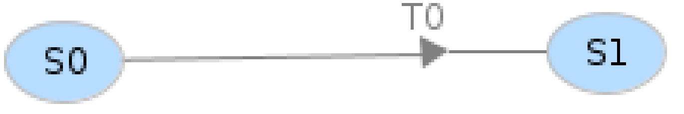
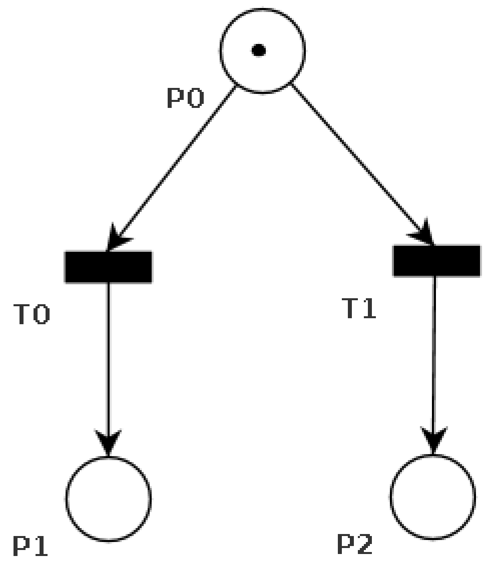
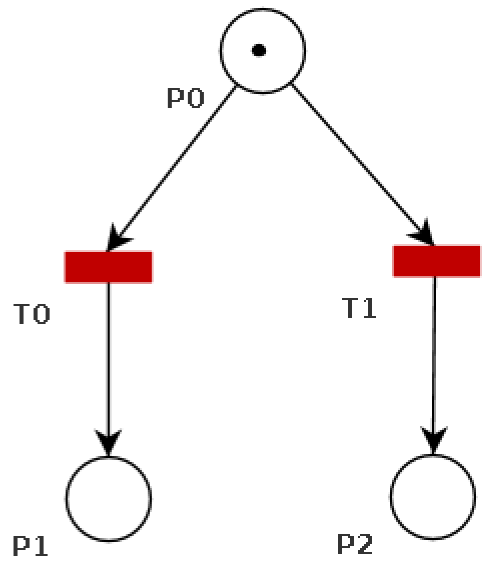


S0 = [1, 1, 0]
S1 = [0, 0, 1]

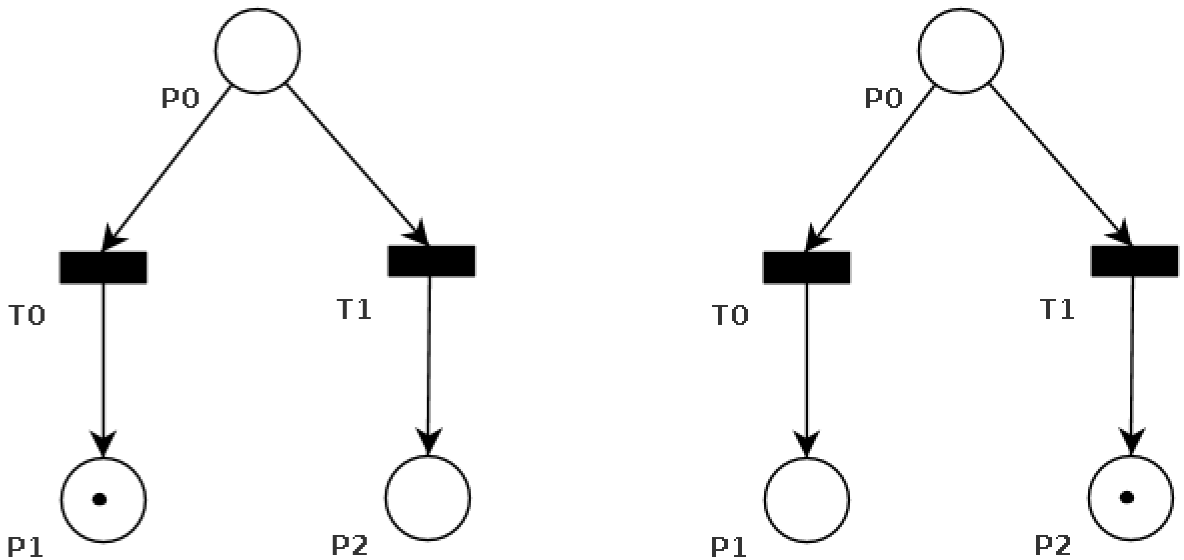
Pattern: conflict, choice, decision



Pattern: conflict, choice, decision



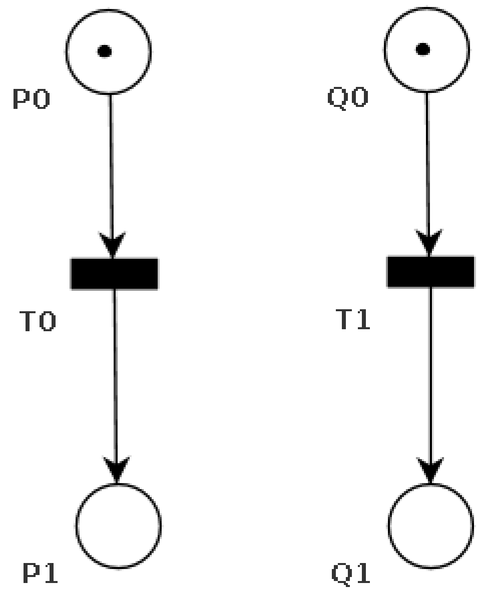
Pattern: conflict, choice, decision



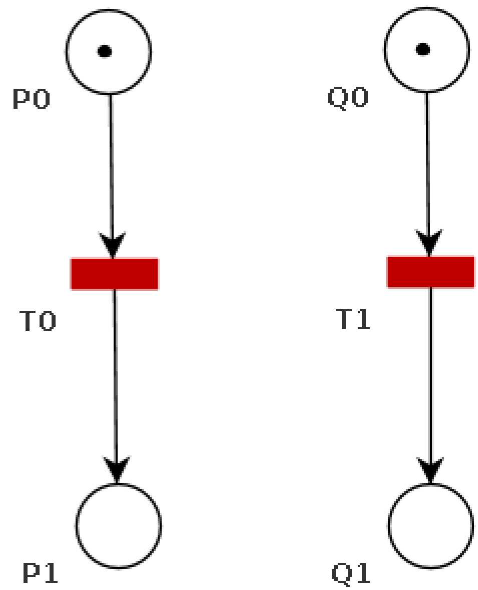
Pattern: conflict, choice, decision

S0 = [1, 0, 0]
S1 = [0, 0, 1]
S2 = [0, 1, 0]

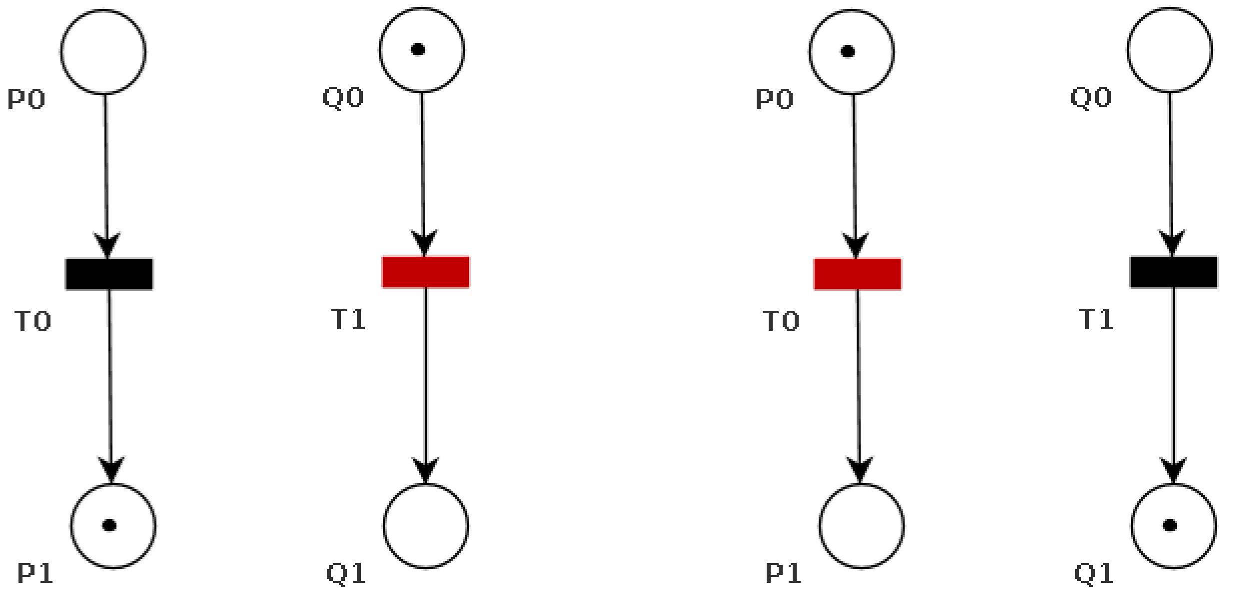
parallel indepencence, confluence



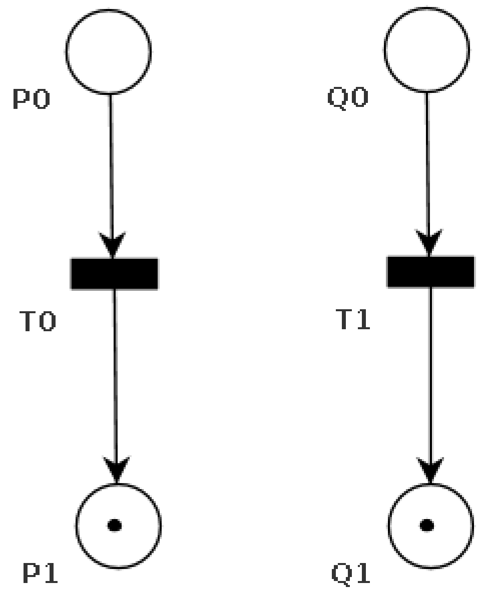
parallel indepencence, confluence



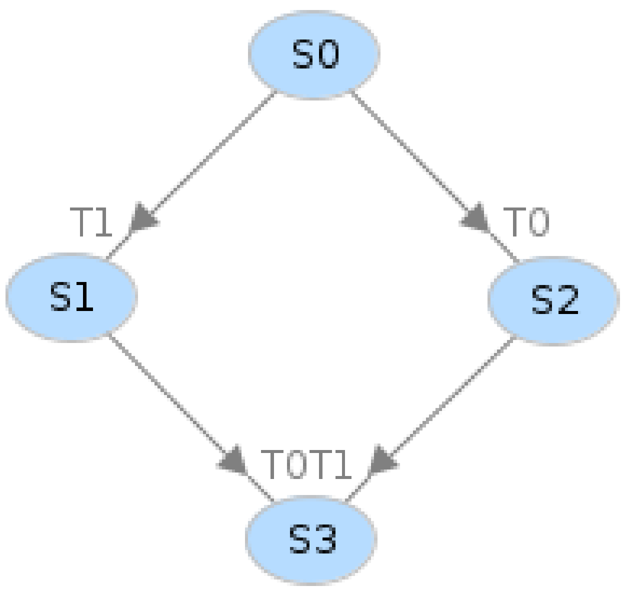
parallel indepencence, confluence



parallel indepencence, confluence

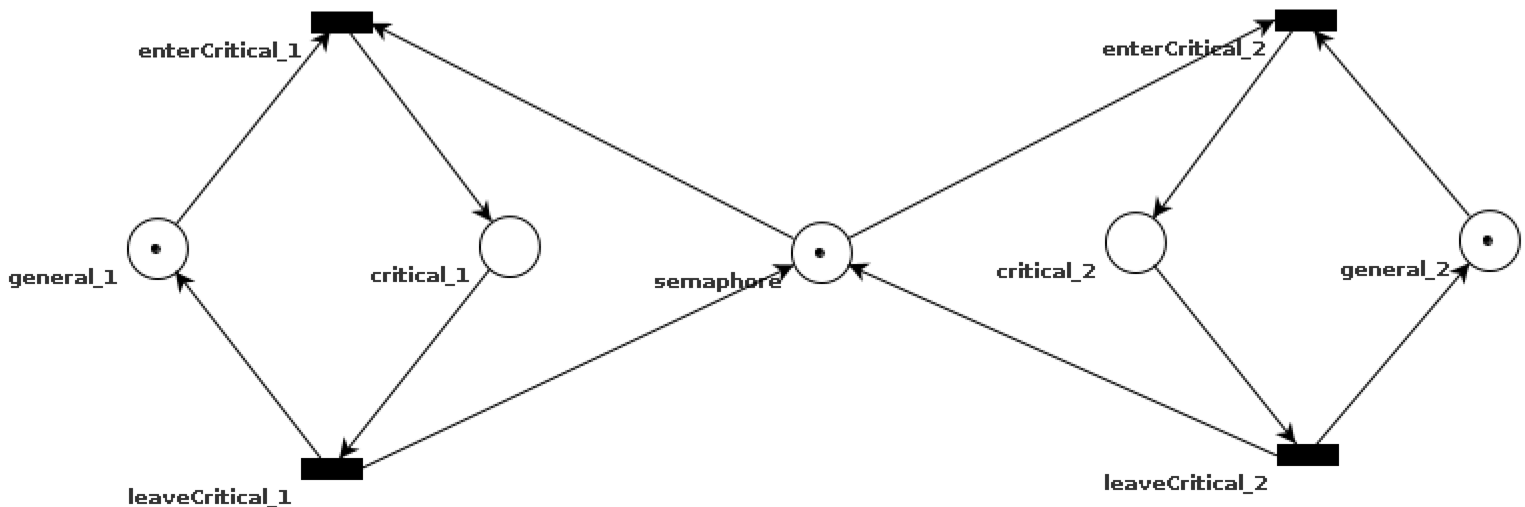


“parallel indepencence”
“confluence”
“diamond” pattern

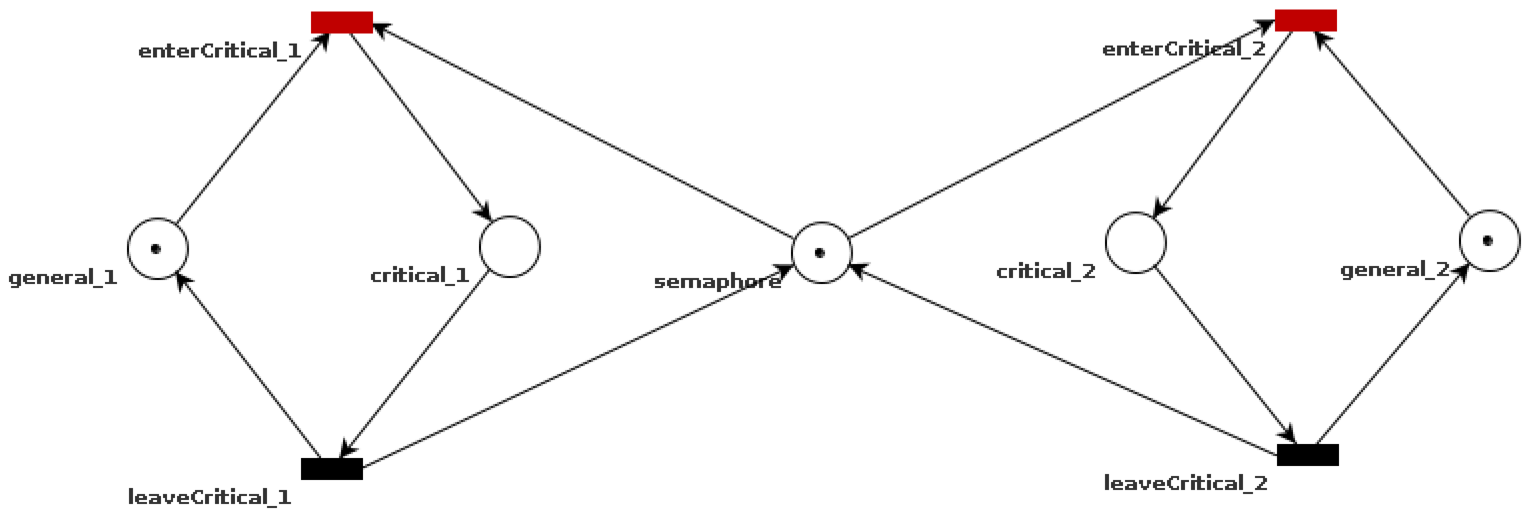


S0 = [1, 0, 1, 0]
S1 = [0, 1, 1, 0]
S2 = [1, 0, 0, 1]
S3 = [0, 1, 0, 1]

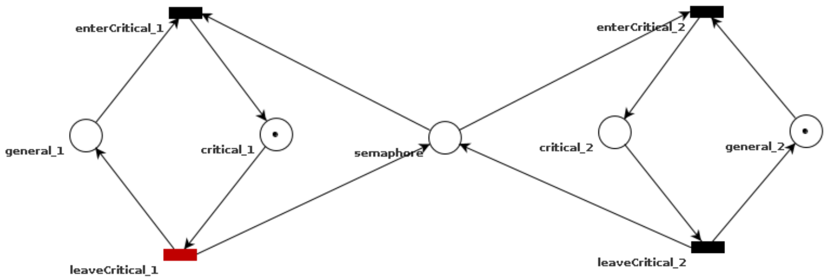
critical section, semaphore, mutex

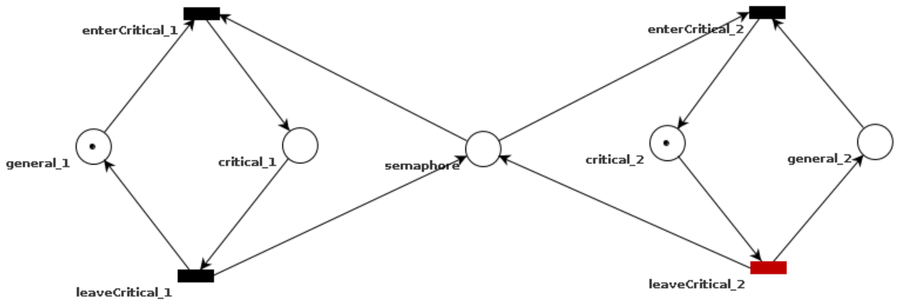


critical section, semaphore, mutex

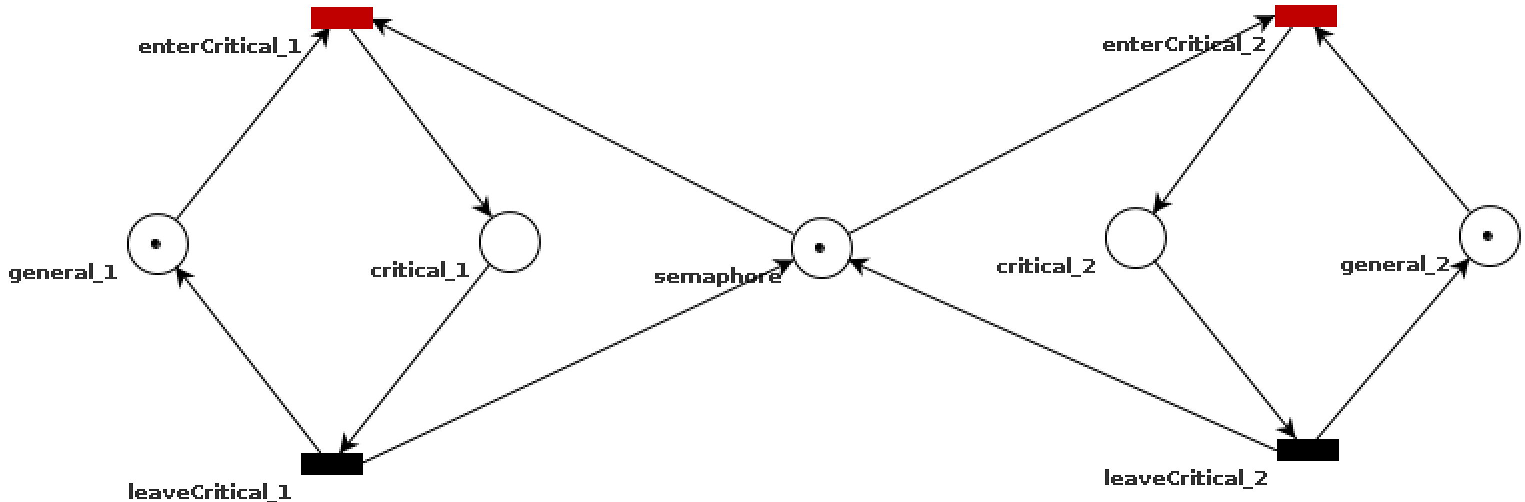


critical section, semaphore, mutex

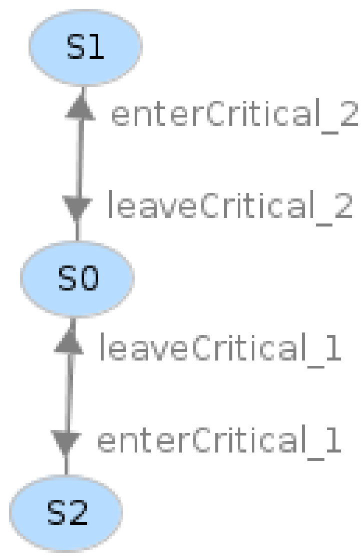




critical section, semaphore, mutex

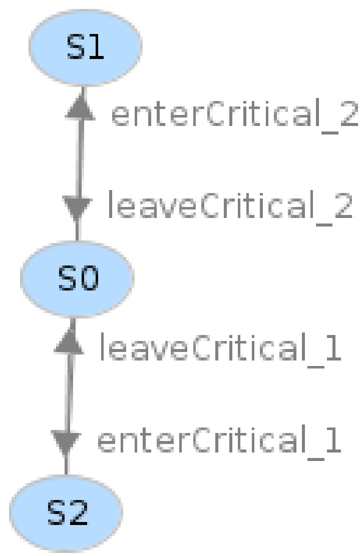


critical section, semaphore, mutex



S0 = [1, 0, 1, 0, 1]
S1 = [1, 0, 0, 1, 0]
S2 = [0, 1, 0, 0, 1]

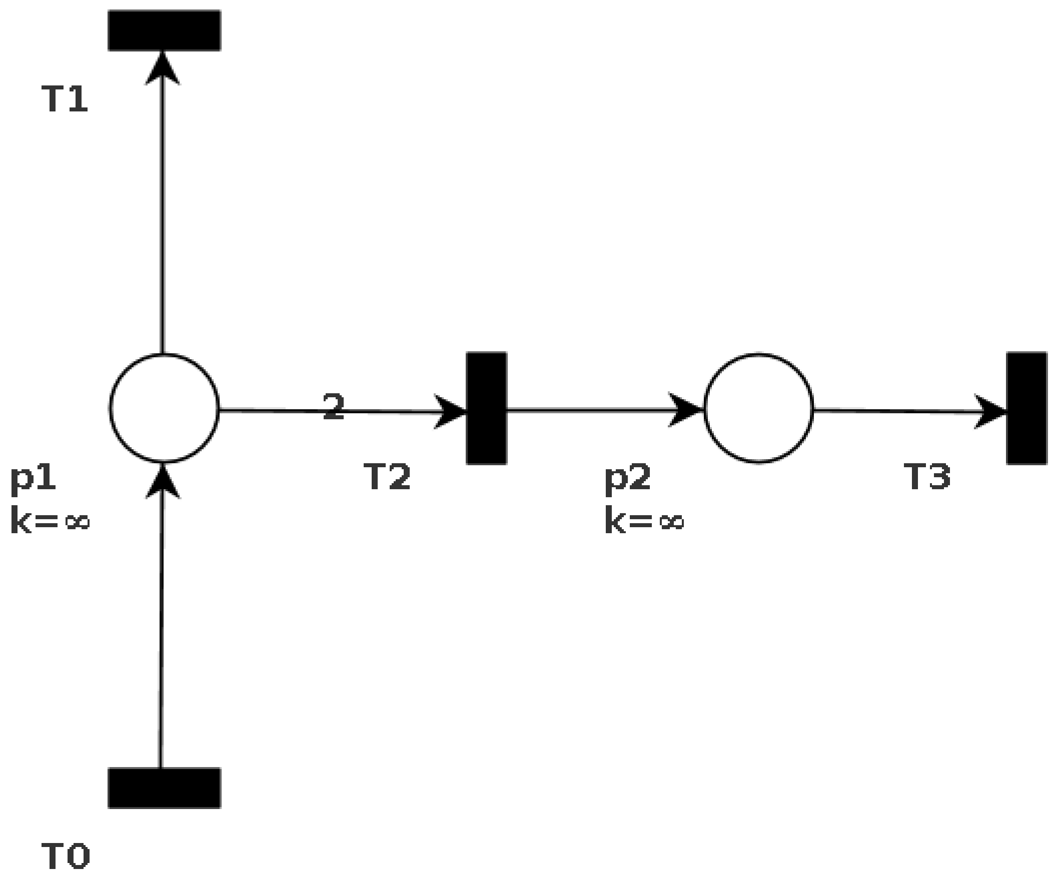
critical section, semaphore, mutex



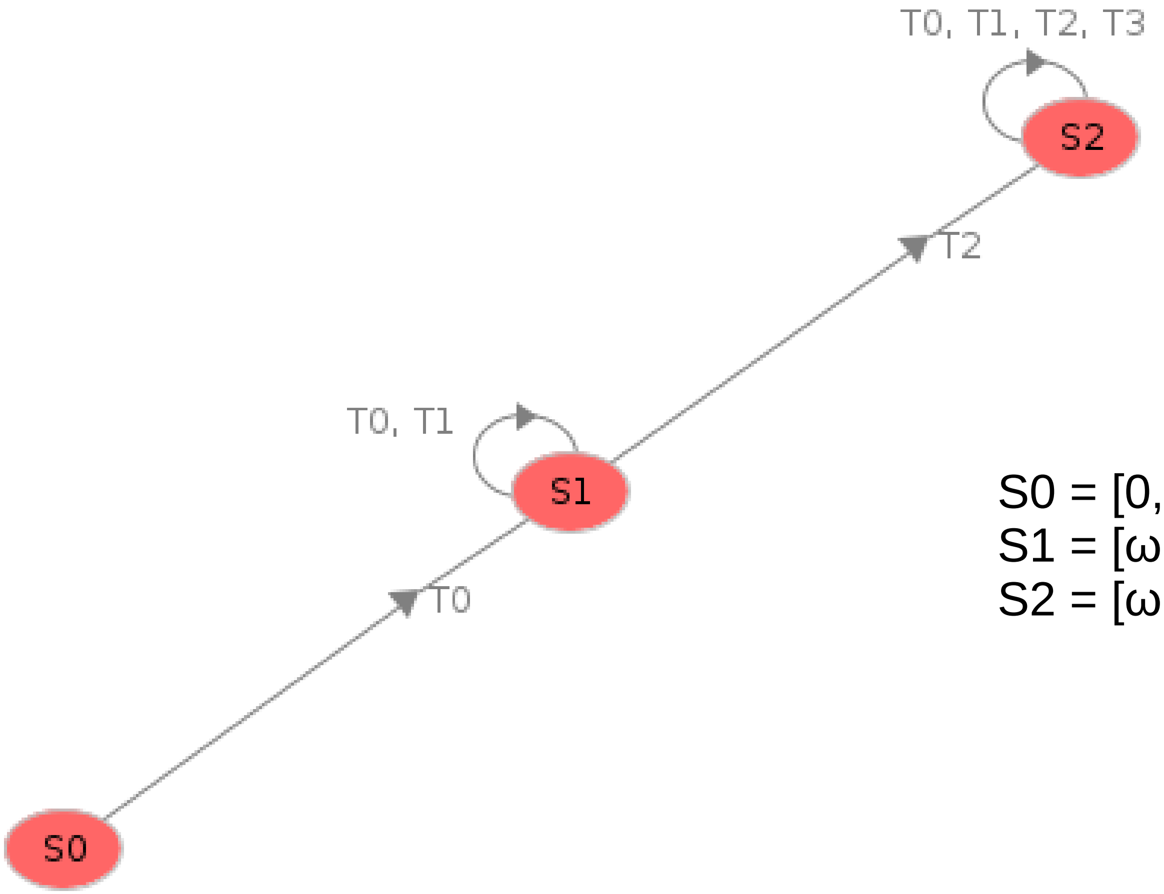
S0 = [1, 0, 1, 0, 1]
S1 = [1, 0, 0, 1, 0]
S2 = [0, 1, 0, 0, 1]

[*, 1, *, 1, *]
reachable in some path?

Infinite Capacity Petri net

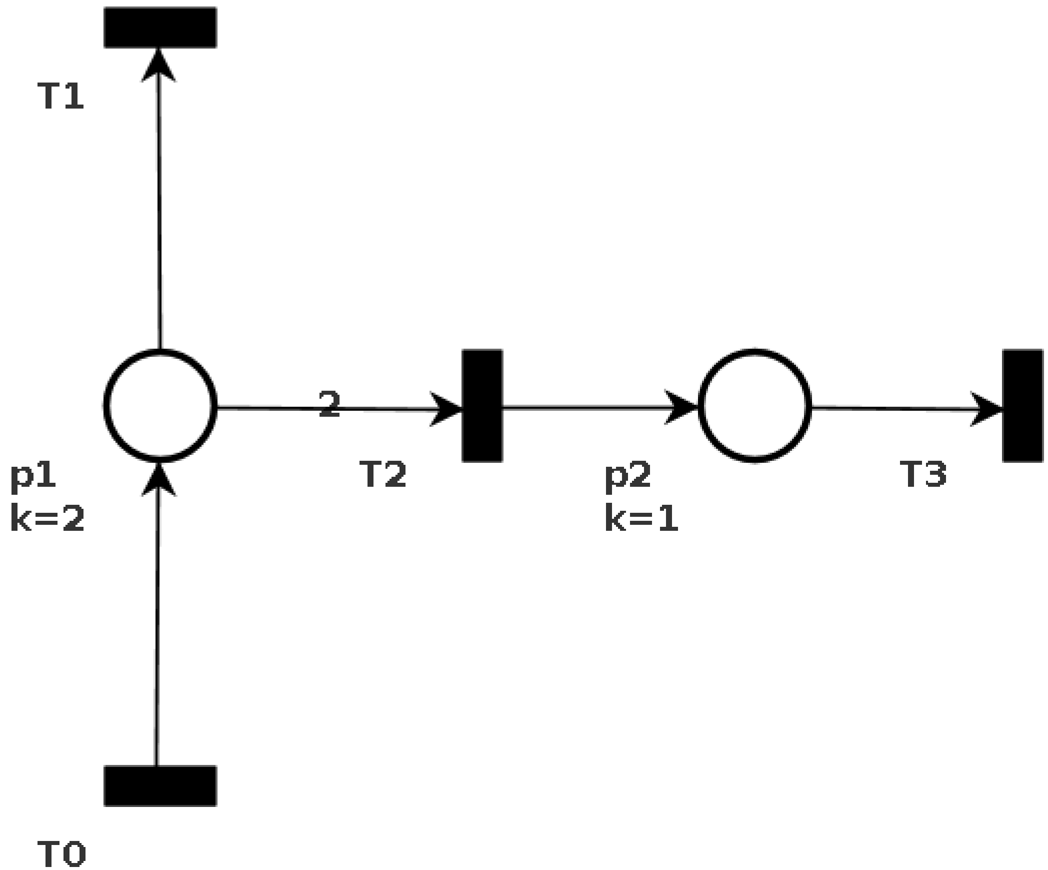


Infinite Capacity Petri net

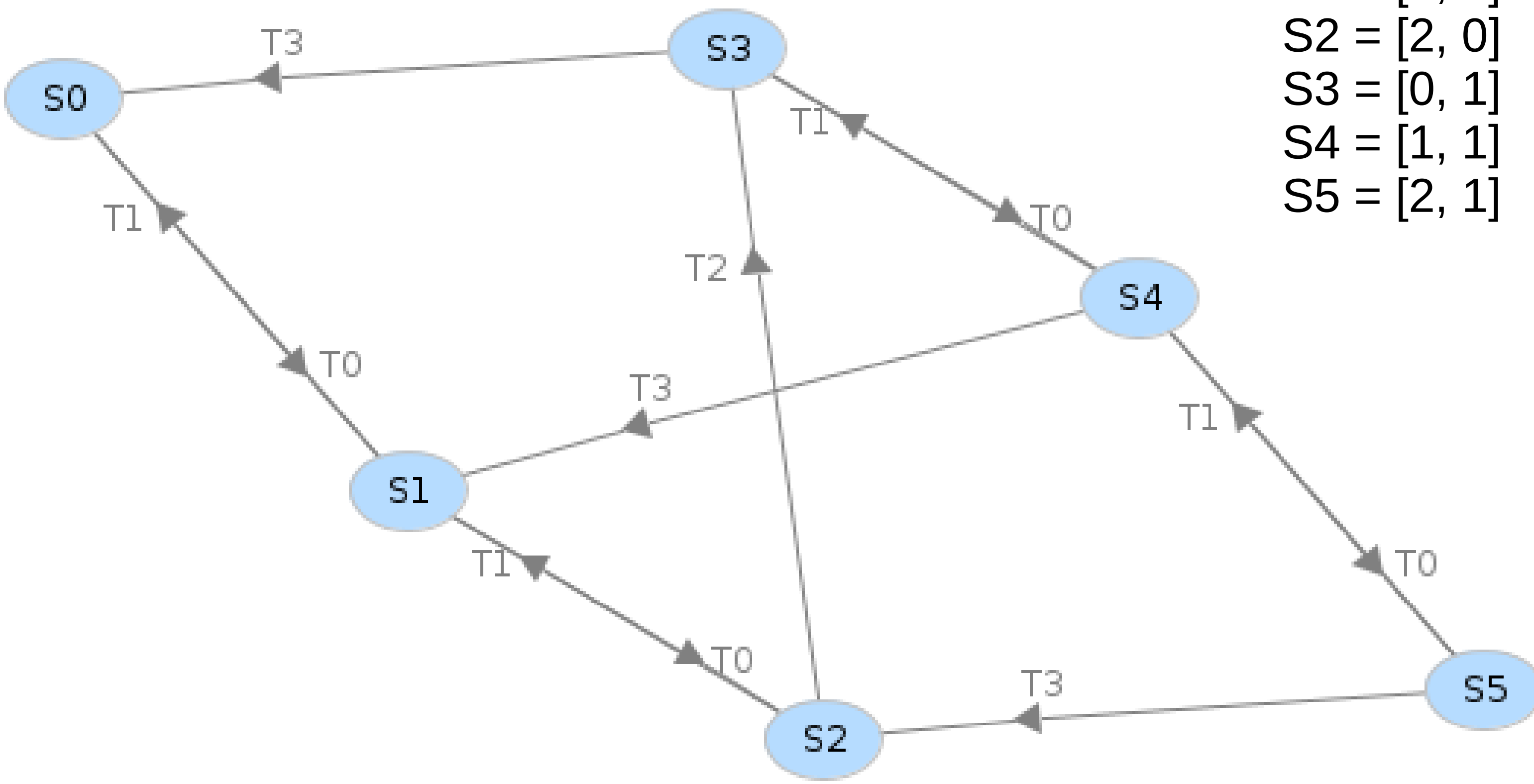


S0 = [0, 0]
S1 = [ω, 0]
S2 = [ω, ω]

Finite Capacity Petri net (FC P/T PN)

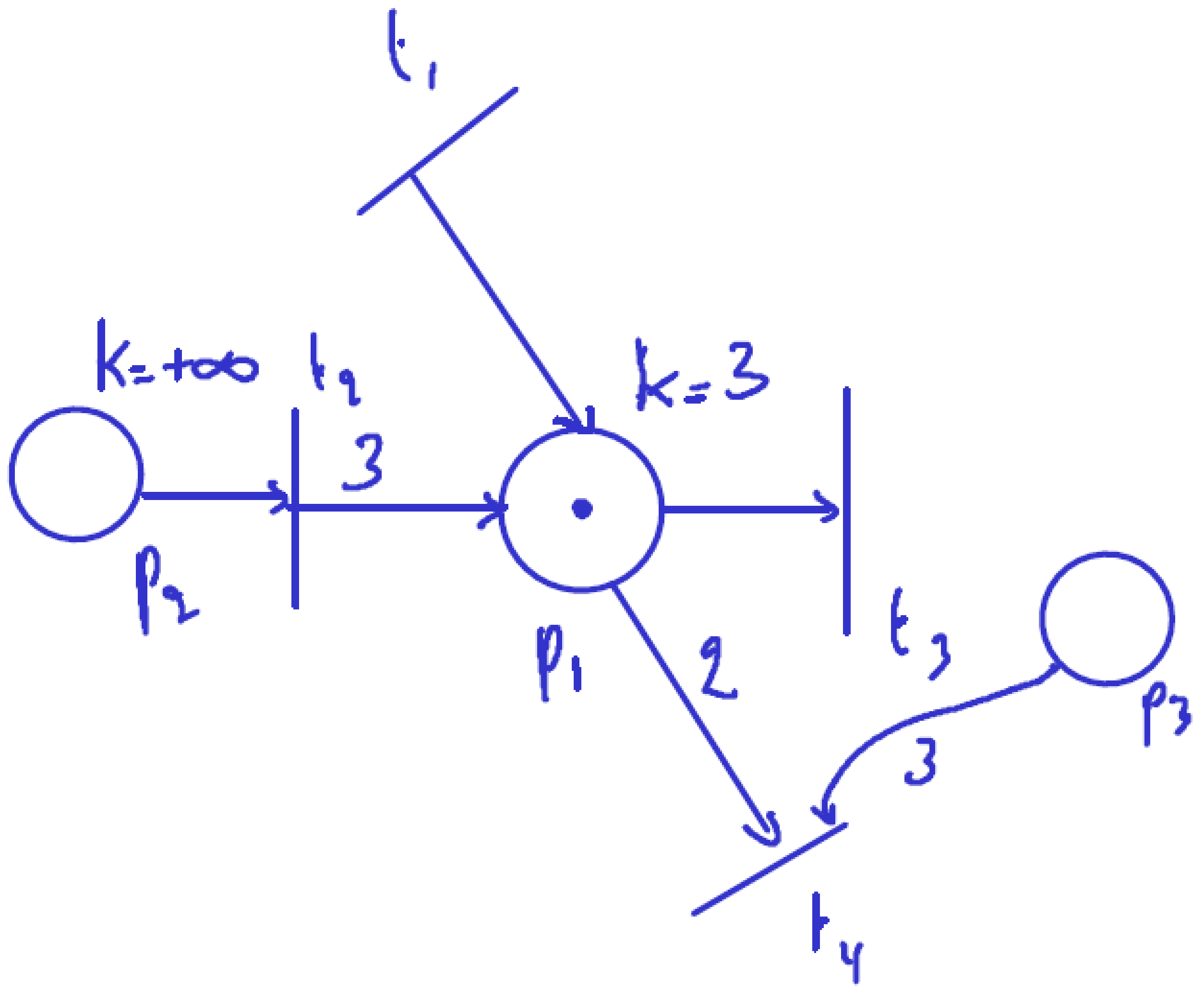


Finite Capacity Petri net (FC P/T PN)

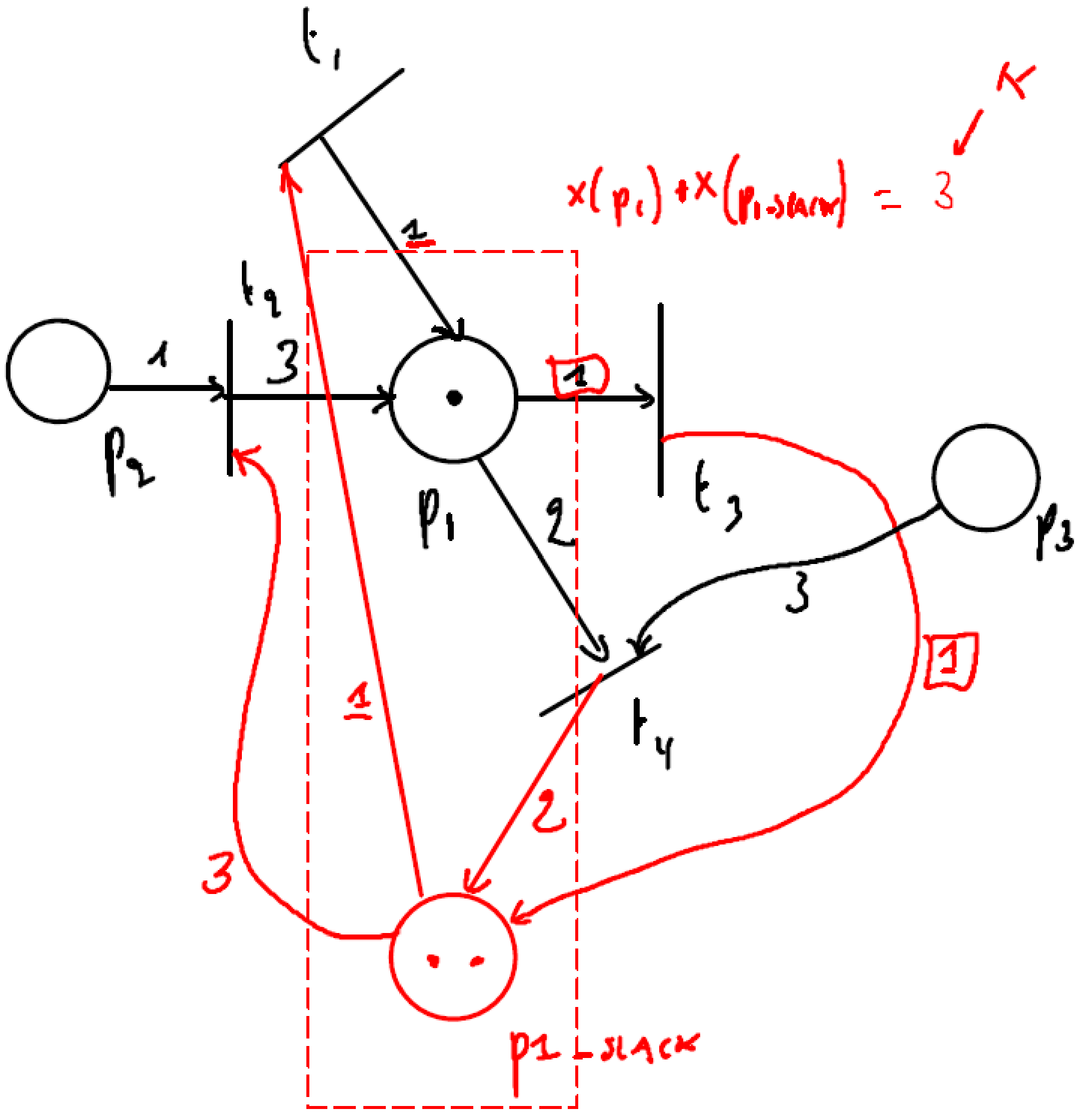


S0 = [0, 0]
S1 = [1, 0]
S2 = [2, 0]
S3 = [0, 1]
S4 = [1, 1]
S5 = [2, 1]

Finite Capacity Petri net
as augmented Infinite Capacity Petri net?



Finite Capacity Petri net
as augmented Infinite Capacity Petri net

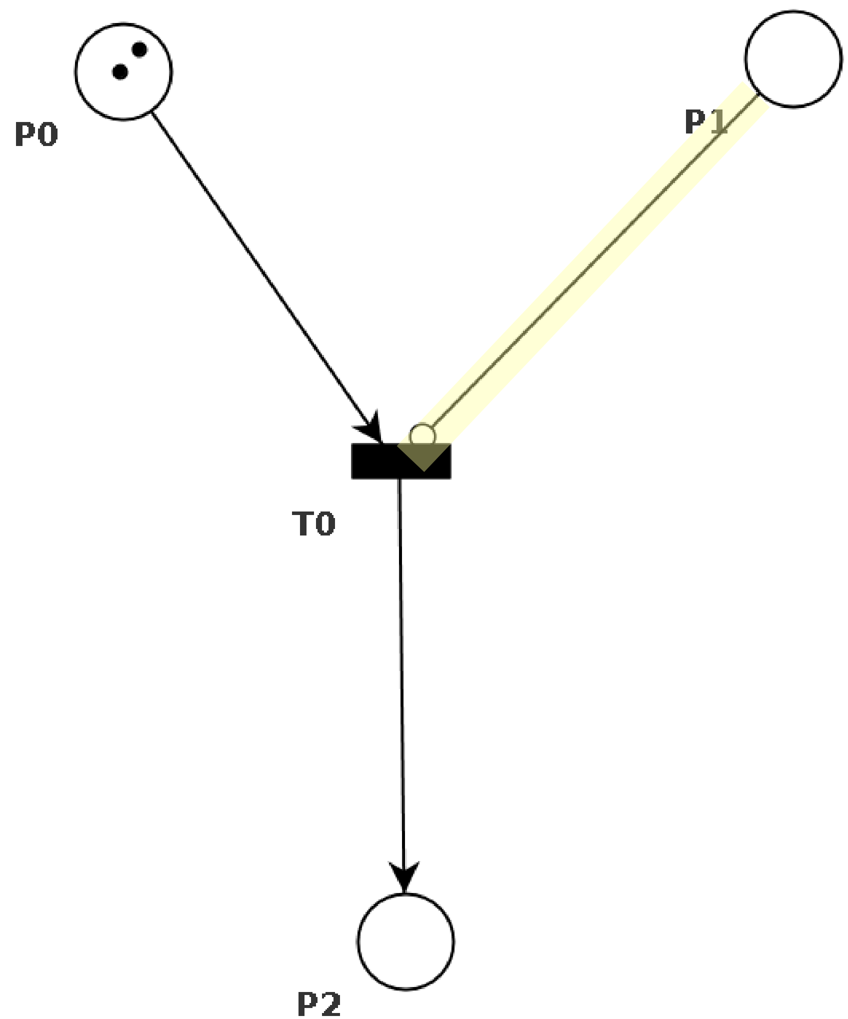


Finite Capacity Petri net
as augmented Infinite Capacity Petri net

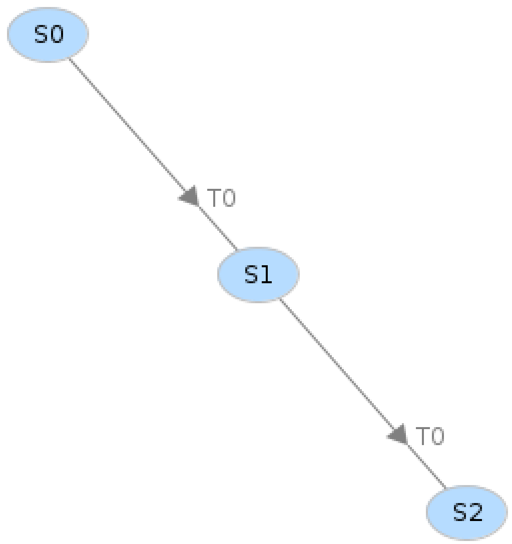
1. Add complimentary place p' with initial marking $x_0(p') = K(p)$ - $x_0(p)$
2. Between each transition t and complimentary places p'
   - add arcs (t, p') or (p', t) where
   - $w(t, p') = w(p, t)$
   - $w(p', t) = w(t, p)$

→ same “expressiveness”
→ Finite Capacity is “syntactic sugar”

P/T PN with **Inhibitor Arc** (makes Turing equiv.)

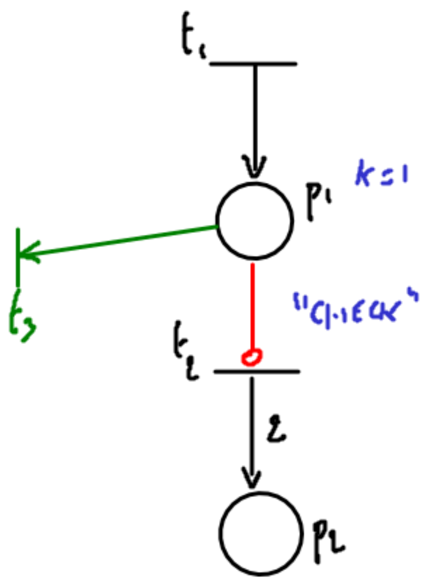


P/T PN with Inhibitor Arc (makes Turing equiv.)

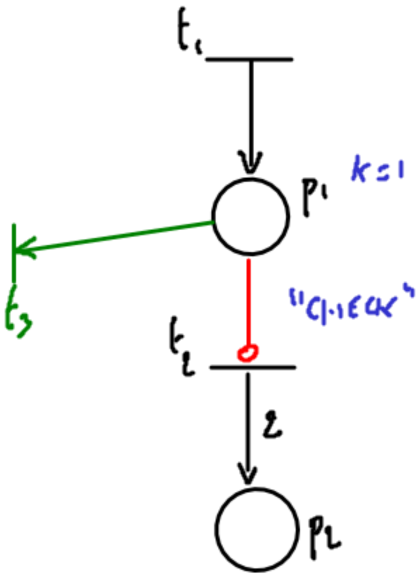


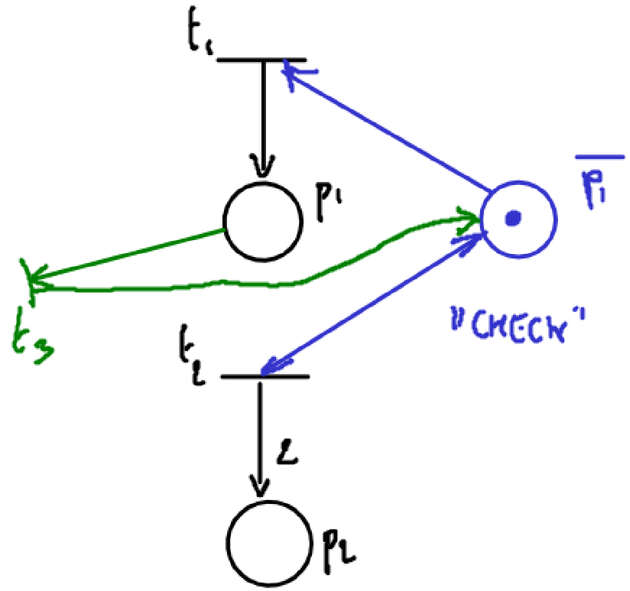
S0 = [2, 0, 0]
S1 = [1, 0, 1]
S2 = [0, 0, 2]

P/T PN with Inhibitor Arc (finite capacity)



P/T PN with Inhibitor Arc (finite capacity)



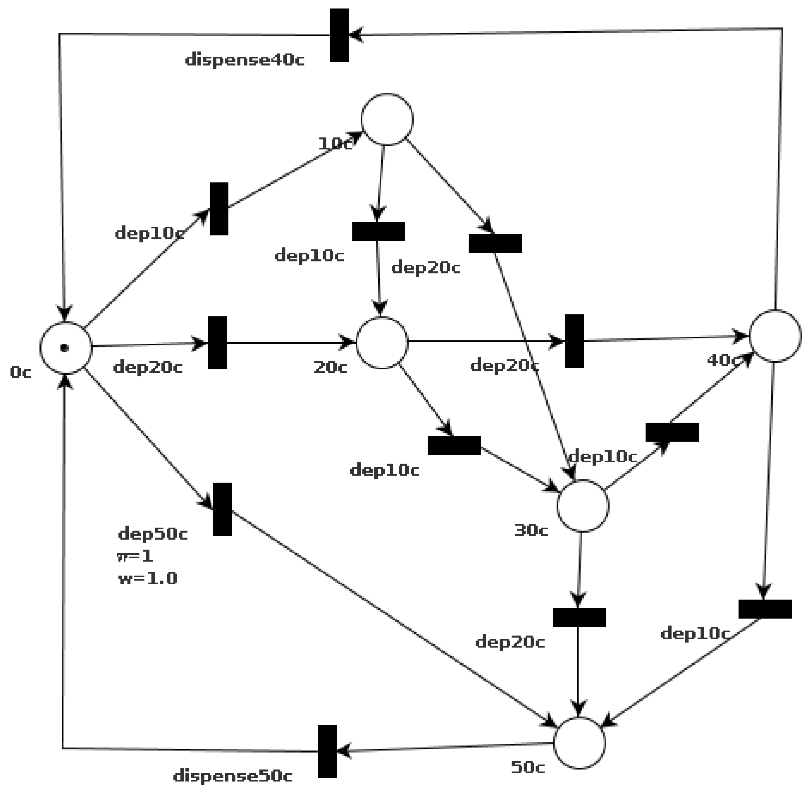


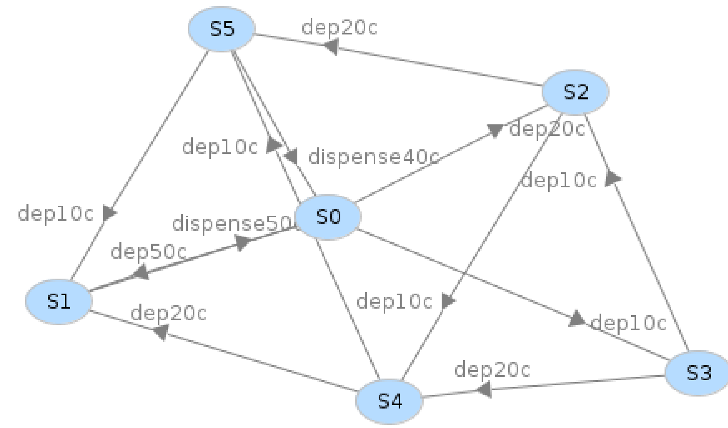
Representing a Petri net as a State Machine

Construct Reachability Graph

- Reachability Graph is State Machine
- States are tuples $(p_1, p_2, \ldots, p_n)$
- Events correspond to t_i firing
- May be infinite (both in size and in marking ω)

Finite State Automaton represented as a Petri Net





[0c, 10c, 20c, 30c, 40c, 50c]

S0 = [1, 0, 0, 0, 0, 0]
S1 = [0, 0, 0, 0, 0, 1]
S2 = [0, 0, 1, 0, 0, 0]
S3 = [0, 1, 0, 0, 0, 0]
S4 = [0, 0, 0, 1, 0, 0]
S5 = [0, 0, 0, 0, 1, 0]

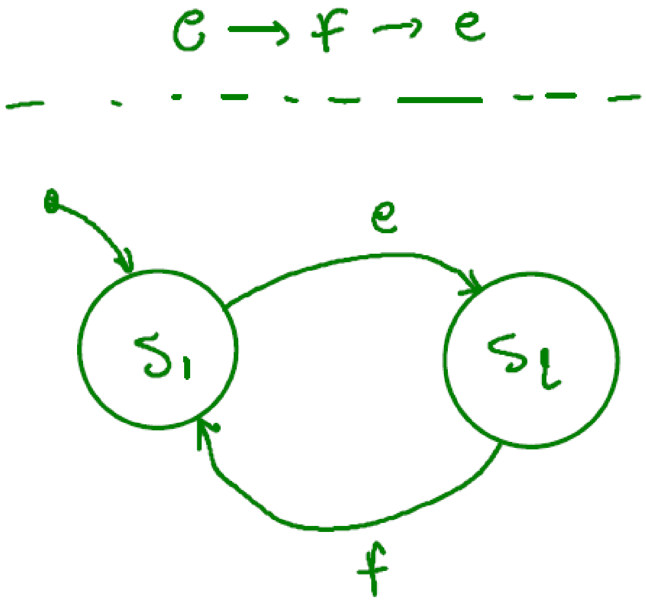
modelling the “current state” of an FSA → single token

Representing a State Machine as a Petri net

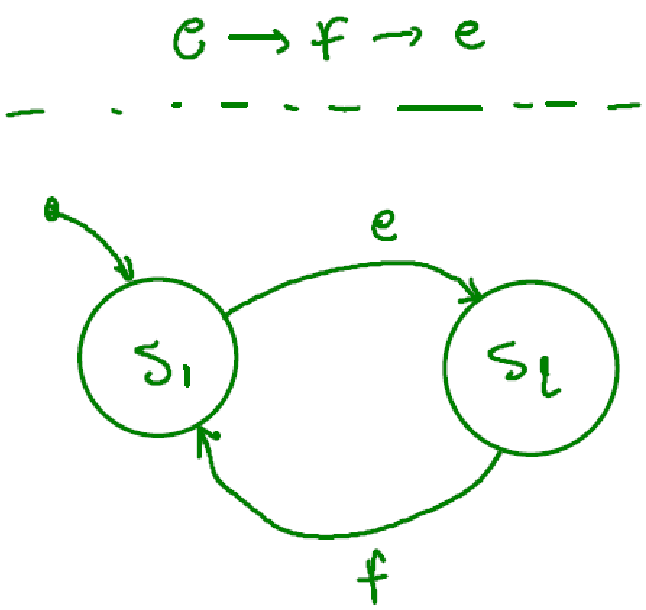
1. no output
2. with output

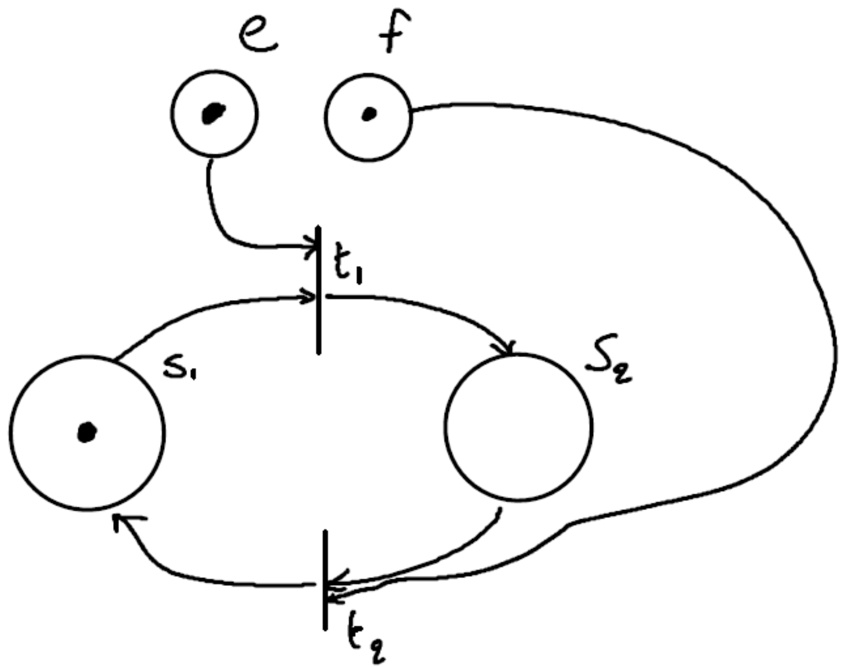
$\Rightarrow$ automatic (though inefficient) transformation

FSA without output

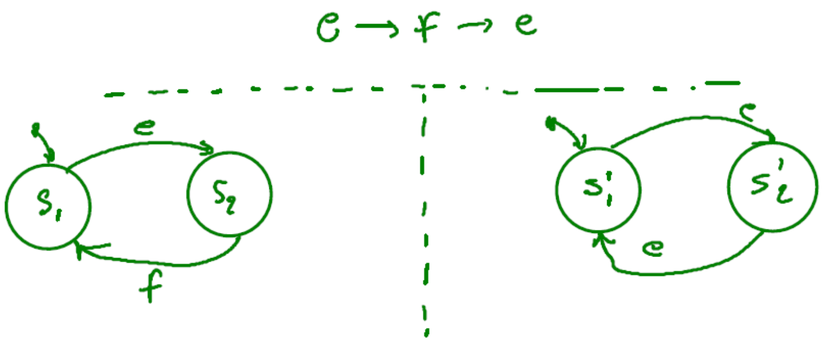


FSA without output

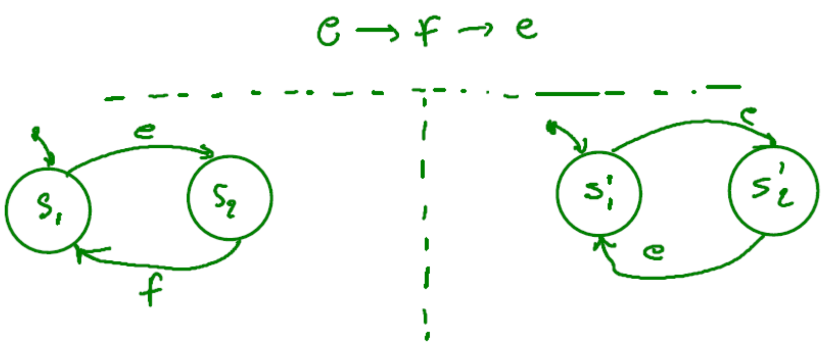


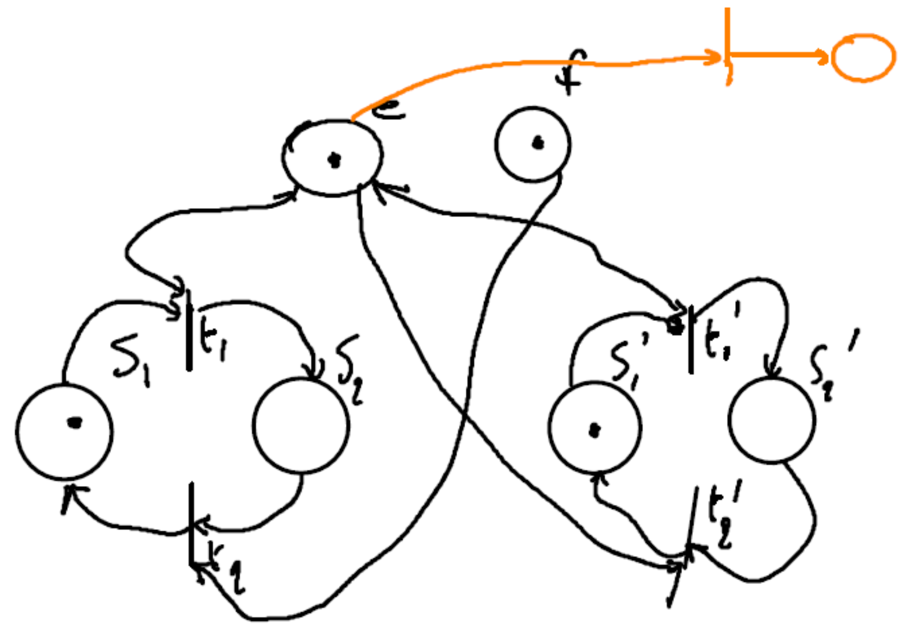


FSA without output

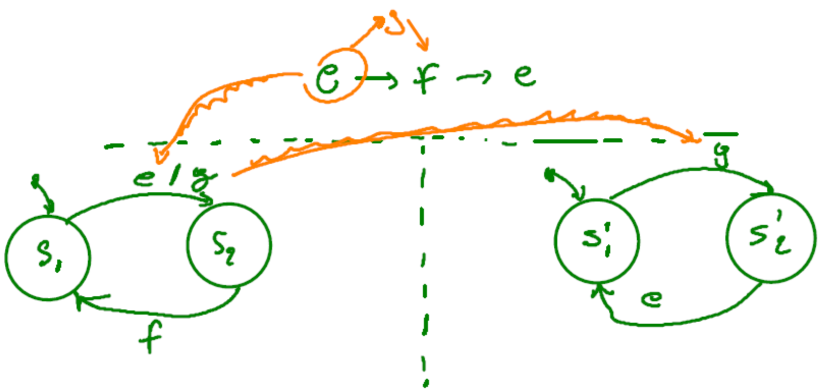


FSA without output

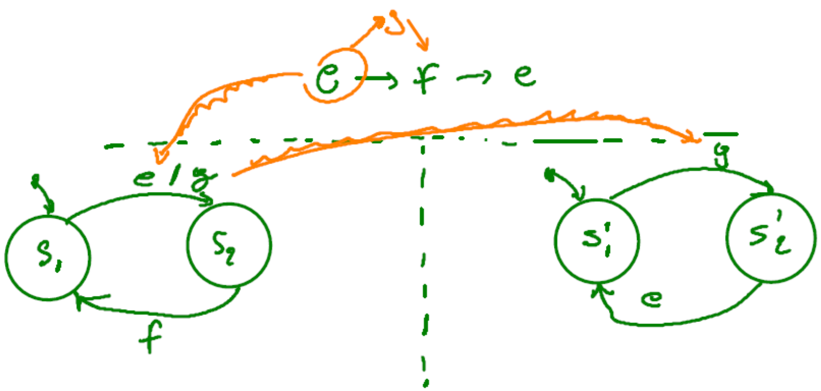


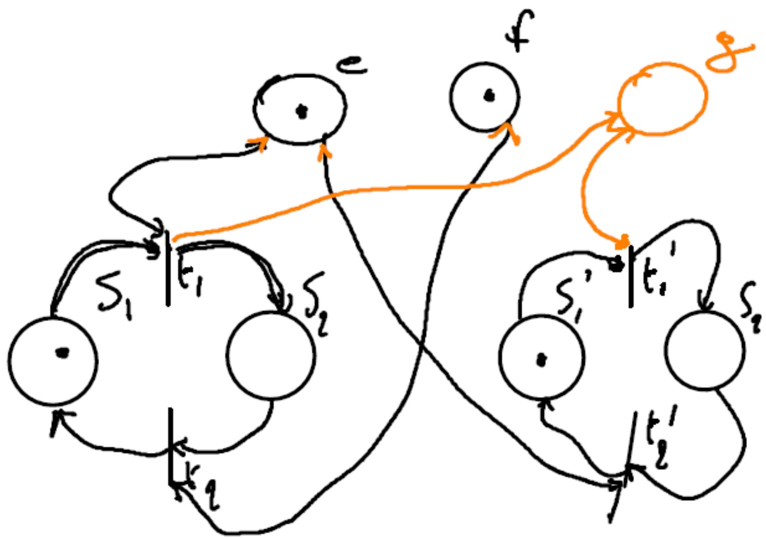


FSA with output (and communication)

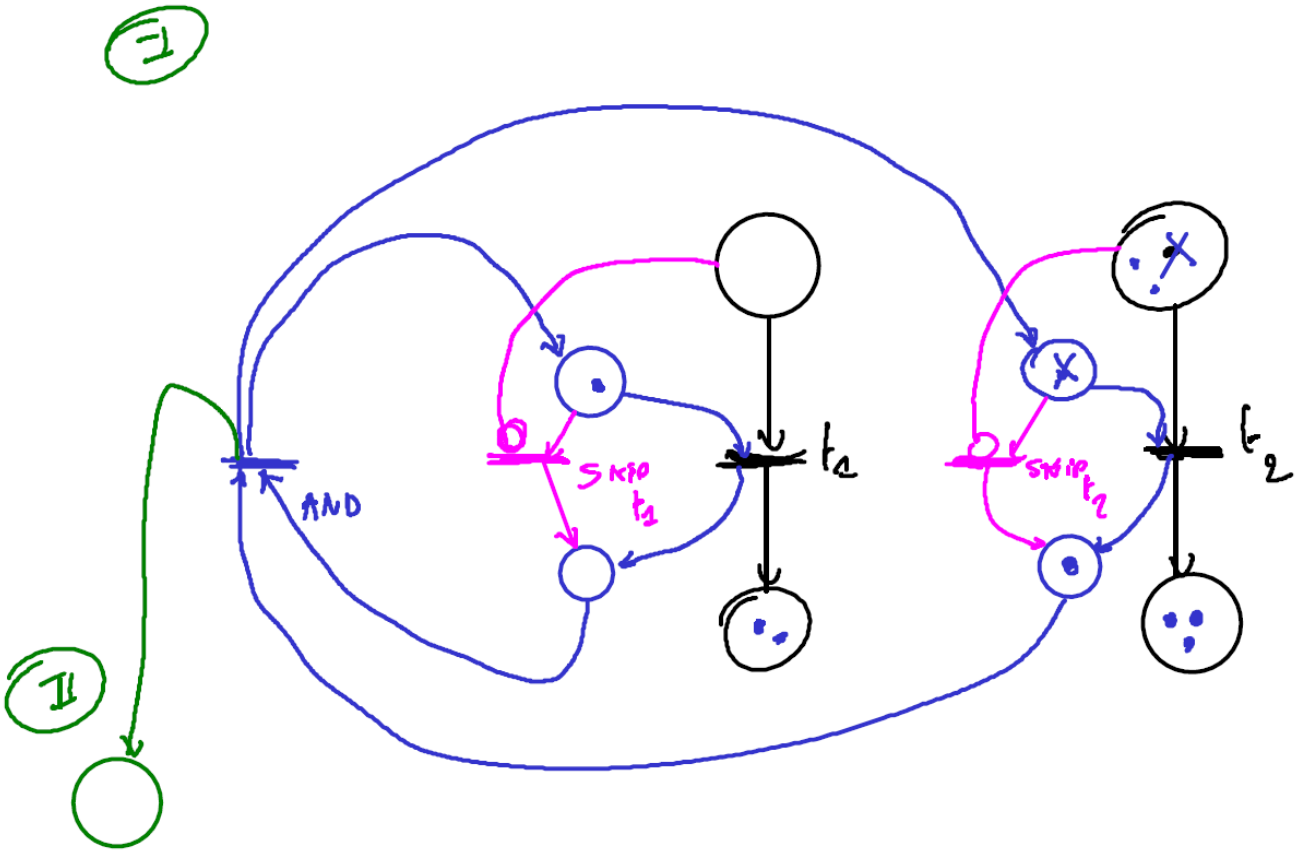


FSA with output (and communication)





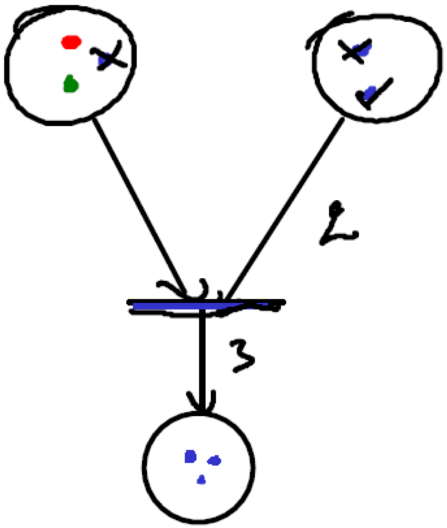
Fairness, Time … TPPN, TTPN

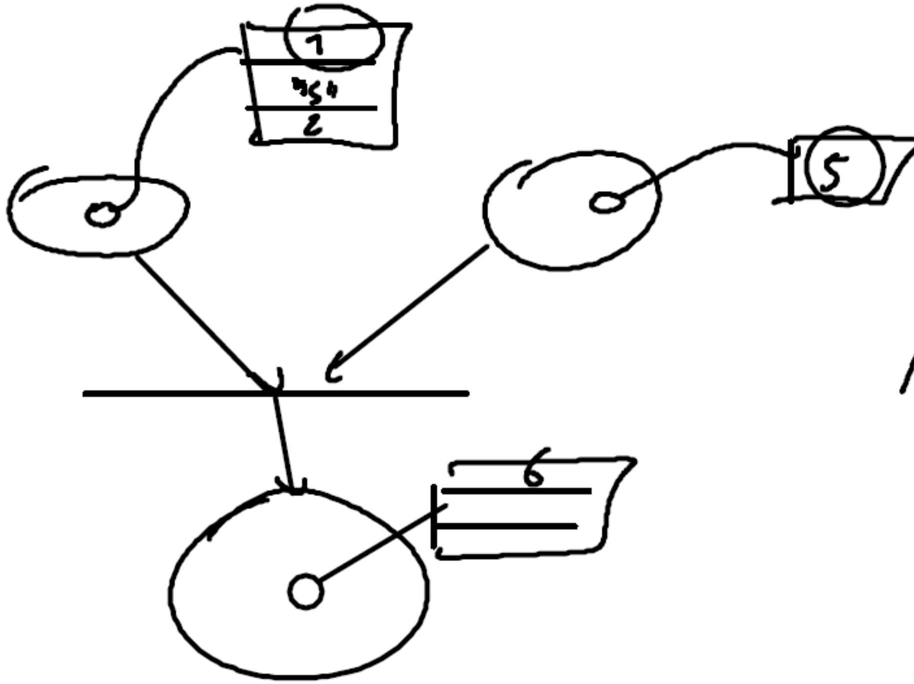


TIME SLICE

DT-END

Colour





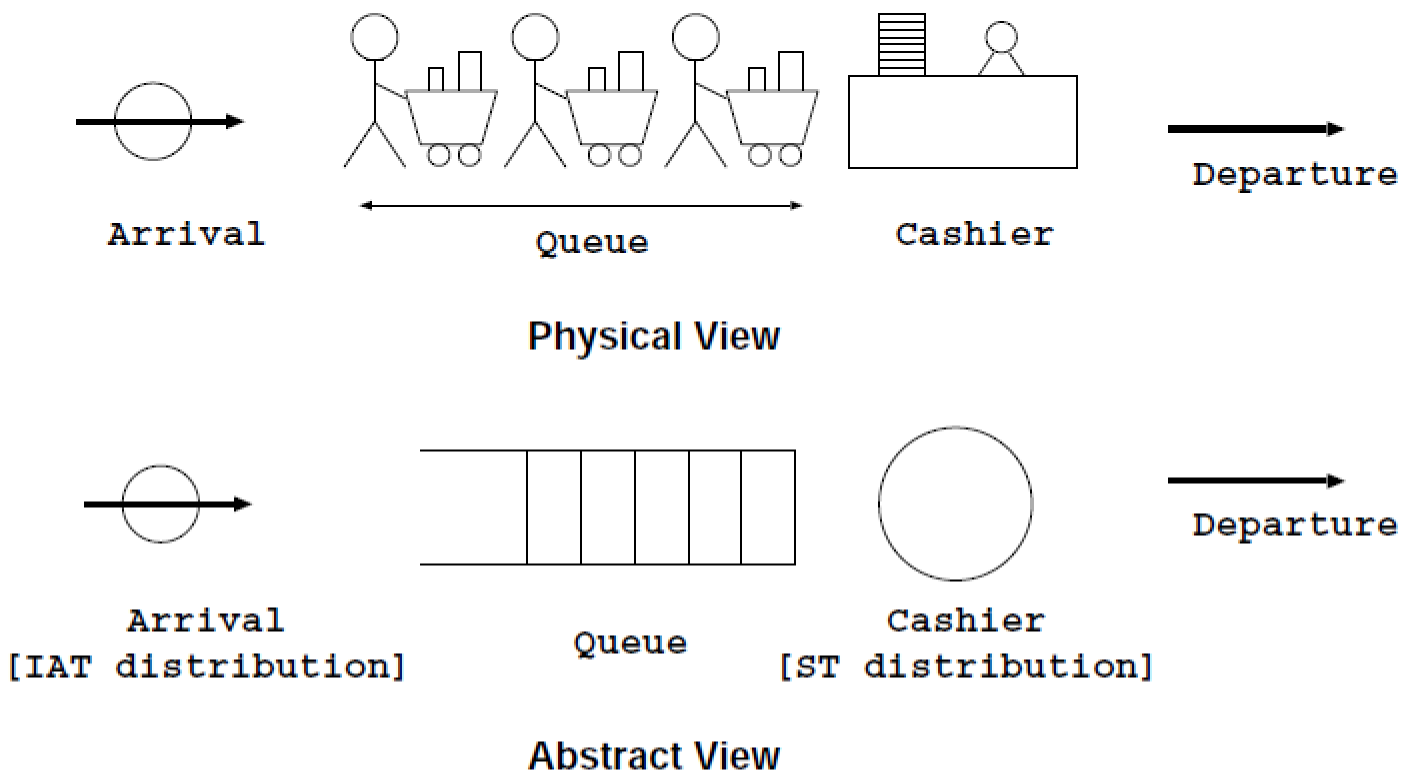
ACG

CPN

SYNTHESIS

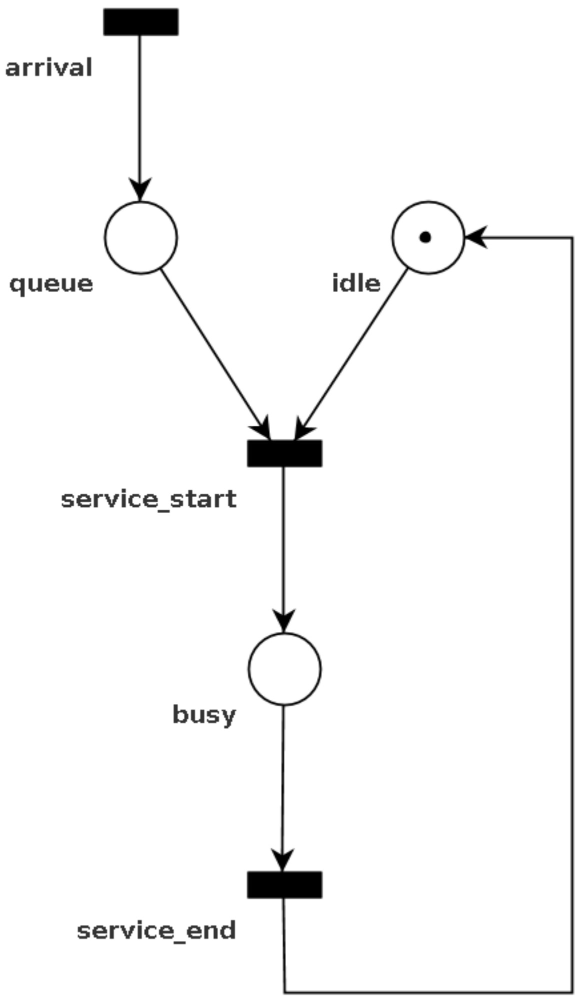
http://cpntools.org/

Petri net models for Queueing Systems

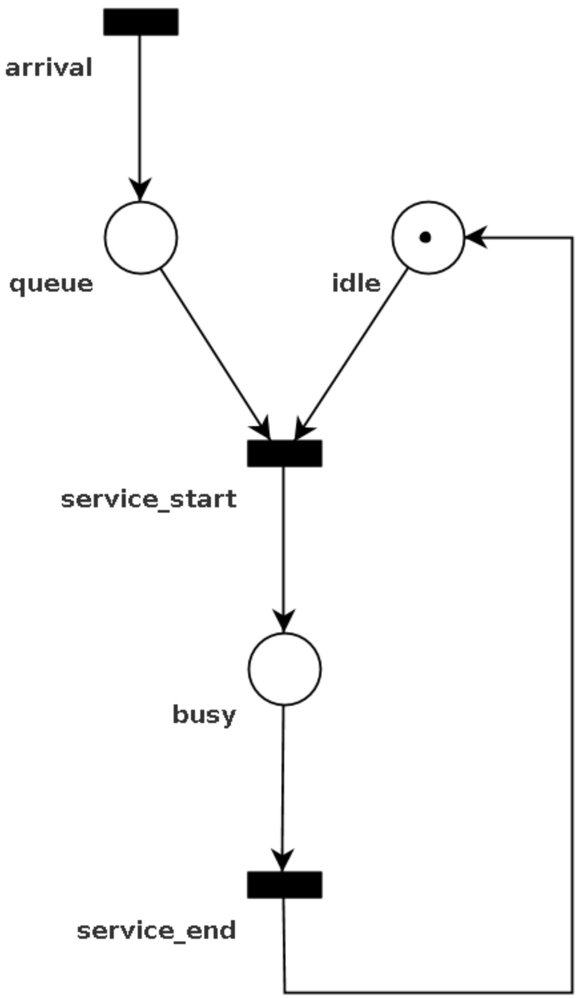


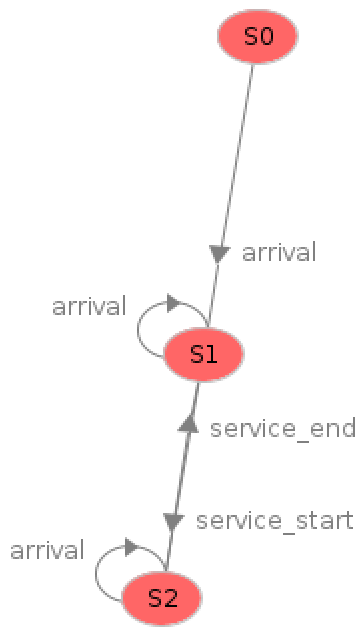
Capacity Constraints for Resource Conservation

Simple Server/Queue



Simple Server/Queue

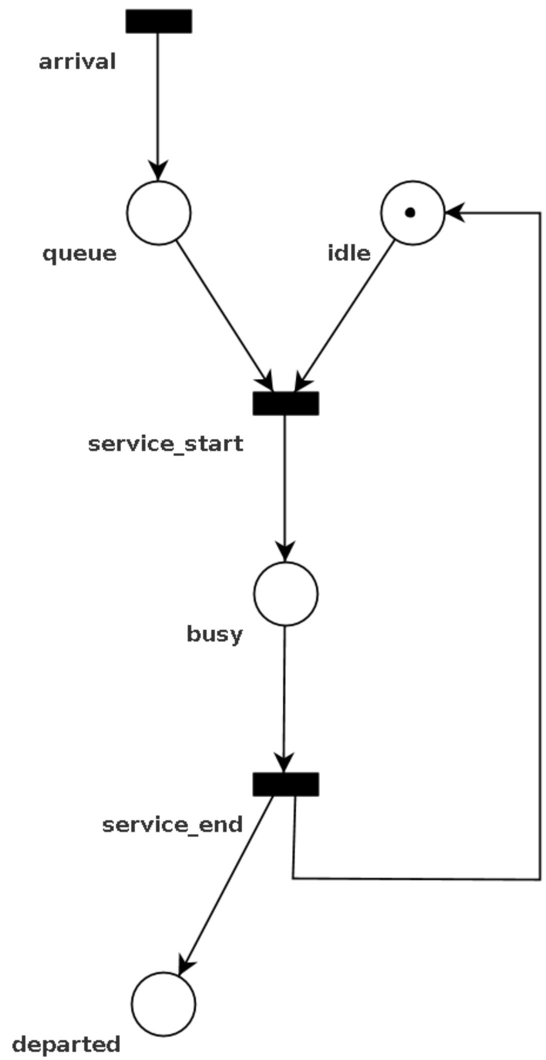




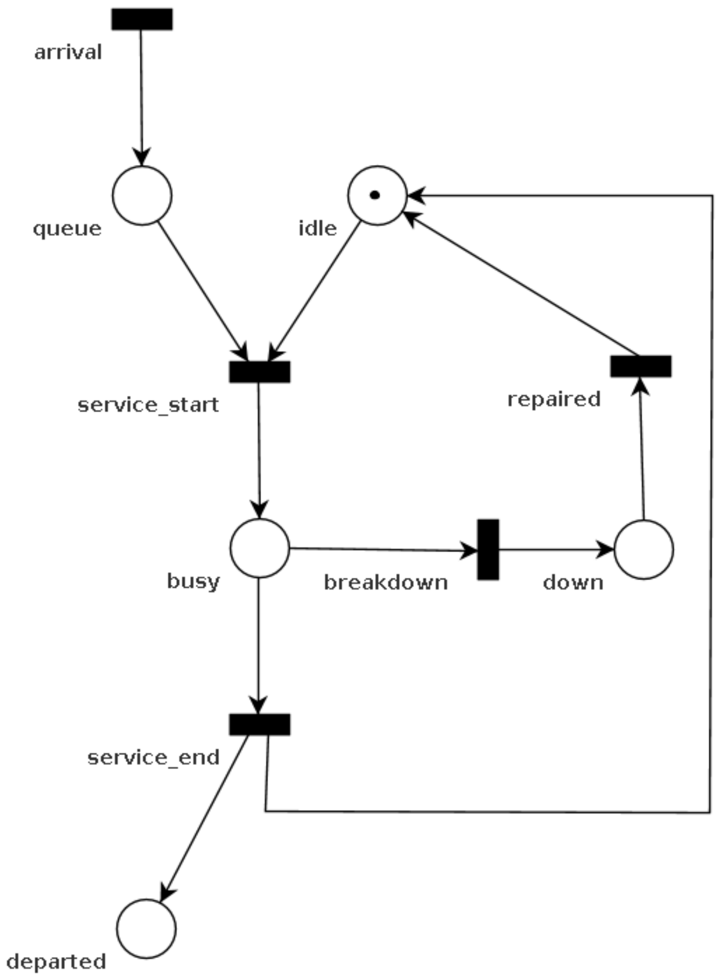
S0 = [0, 1, 0]
S1 = [ω, 1, 0]
S2 = [ω, 0, 1]

[queue, idle, busy]

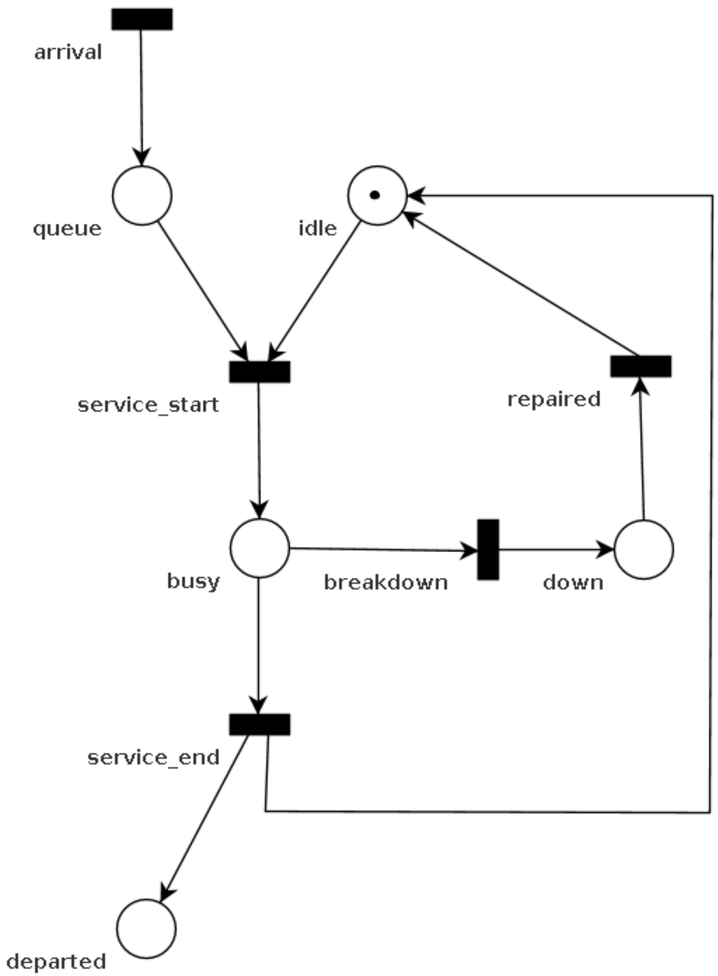
Simple Server/Queue
departure modelled explicitly

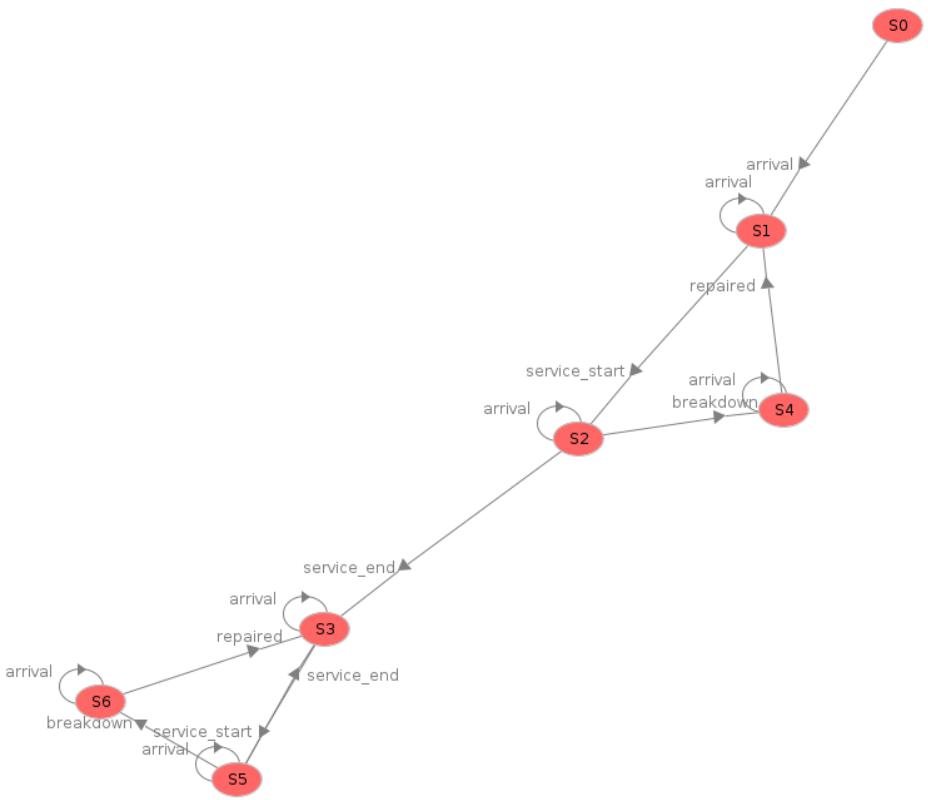


Simple Server/Queue
with server breakdown (and repair)



Simple Server/Queue
with server breakdown (and repair)





Modular Composition: Communication Protocol

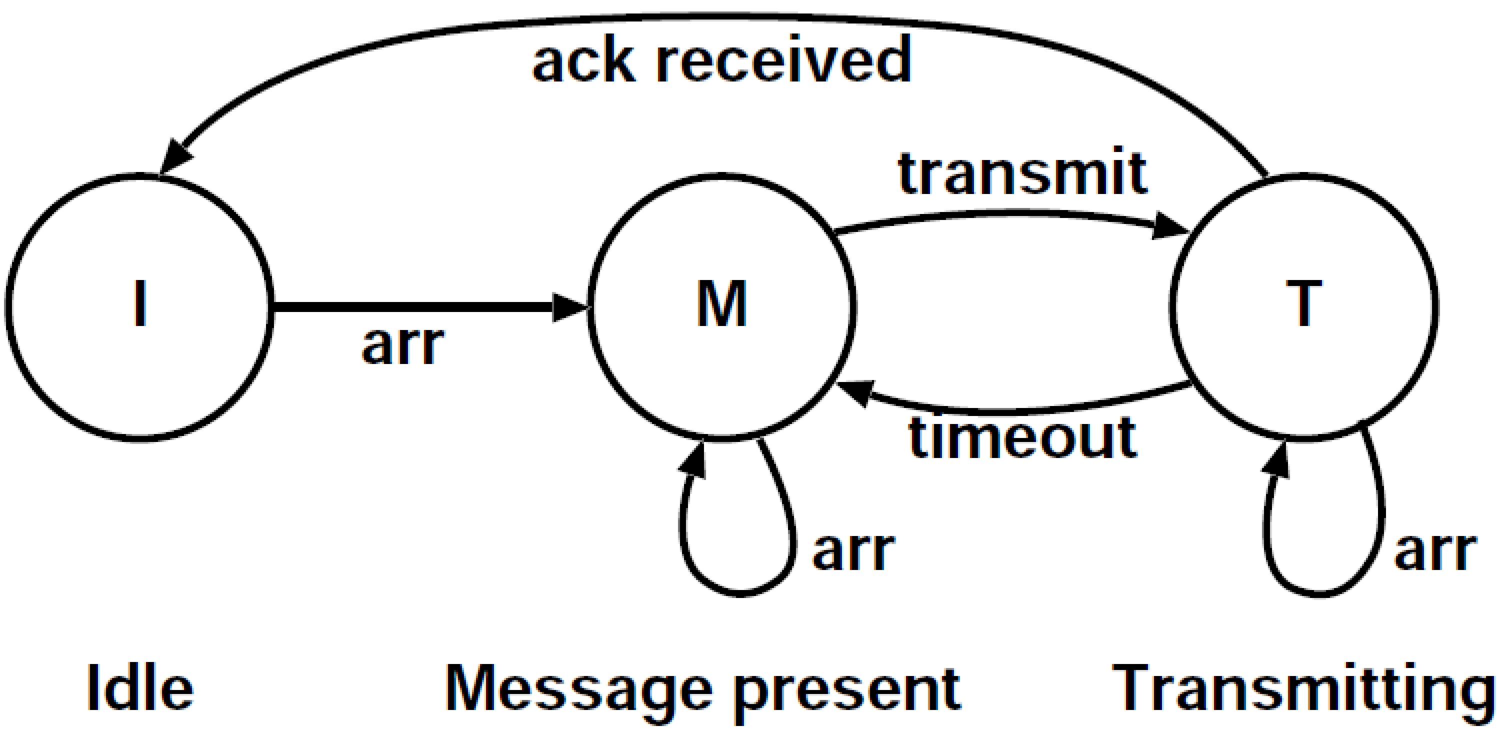
Build incrementally:

1. Single transmitter: FSA vs. Petri net
2. Two transmitters competing for channel

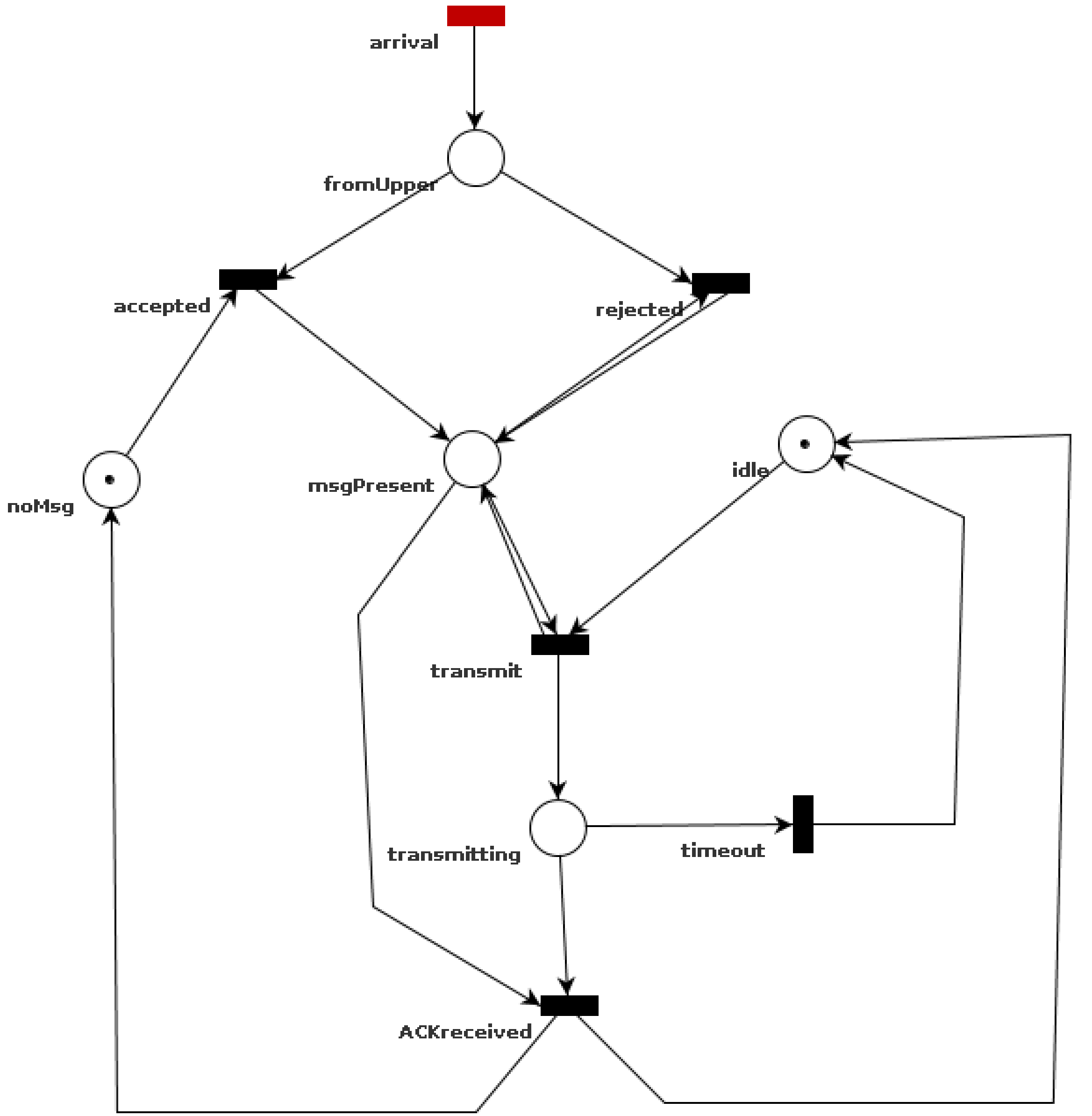
Pros/Cons of Petri net models (depends on goals !):

- Petri net is more complex than FSA for single transmitter
- More insight
- Incremental modelling
- Modular modelling
- Intuitive modelling of concurrency

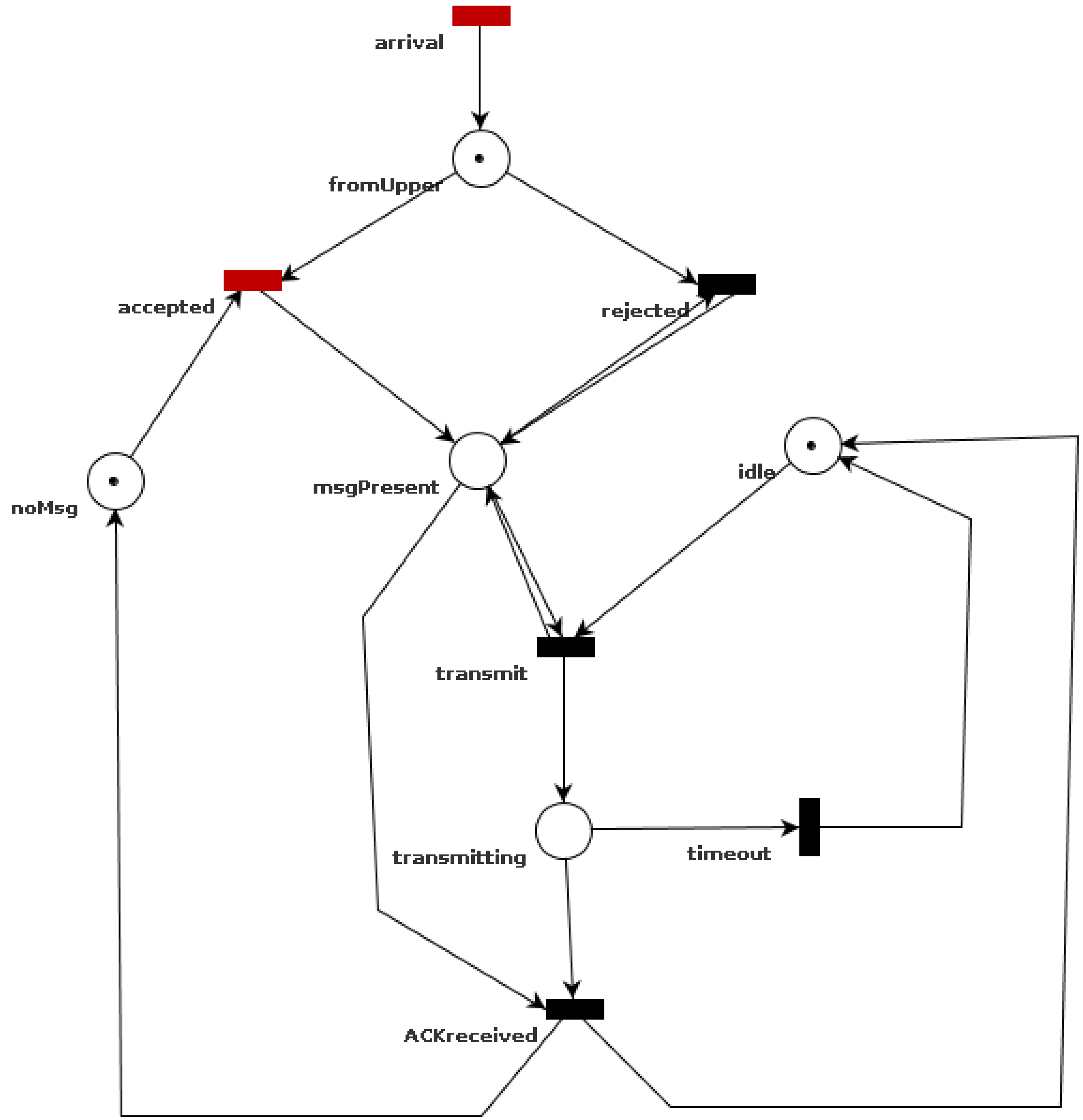
Single Transmitter FSA



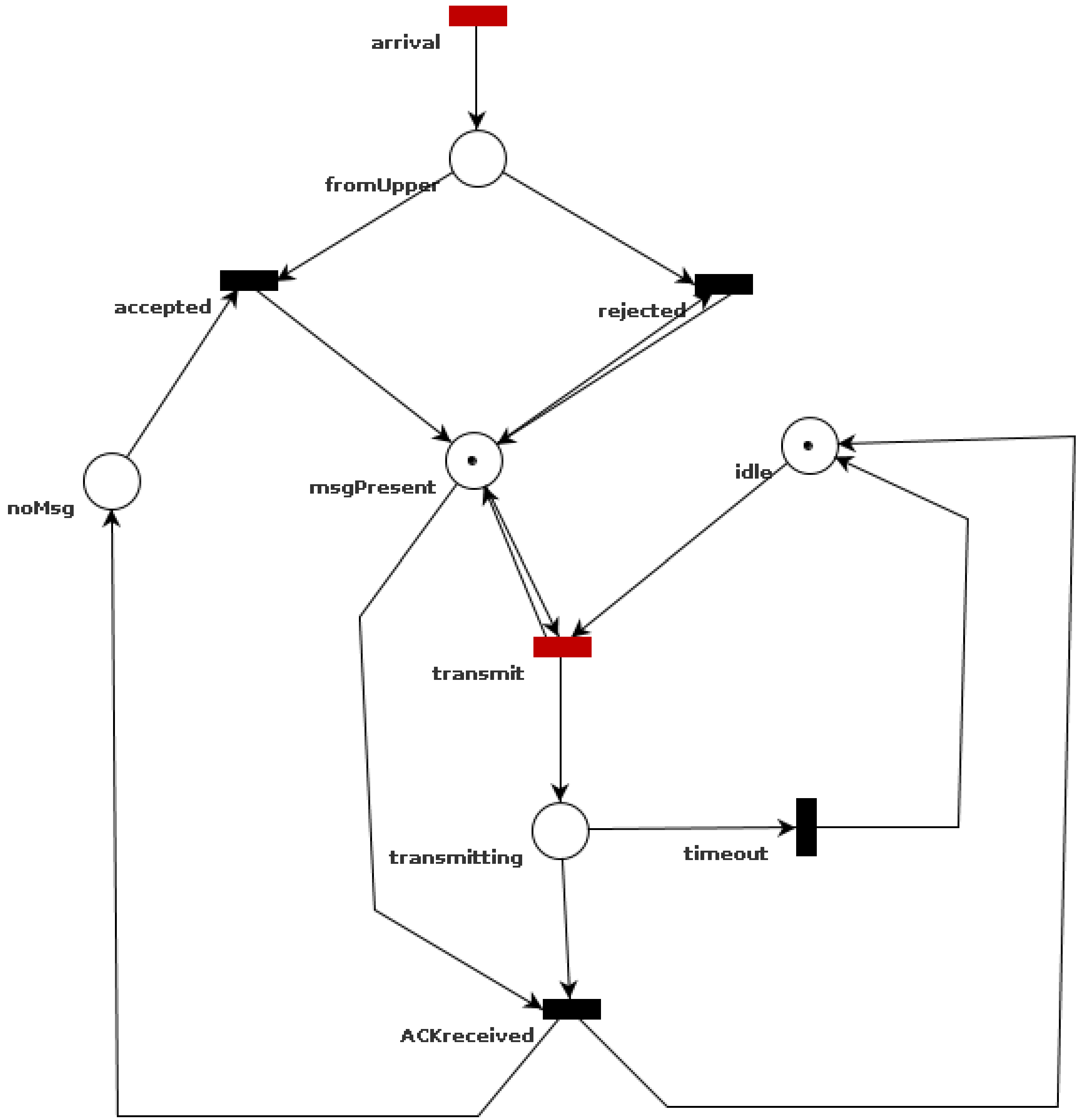
Single transmitter



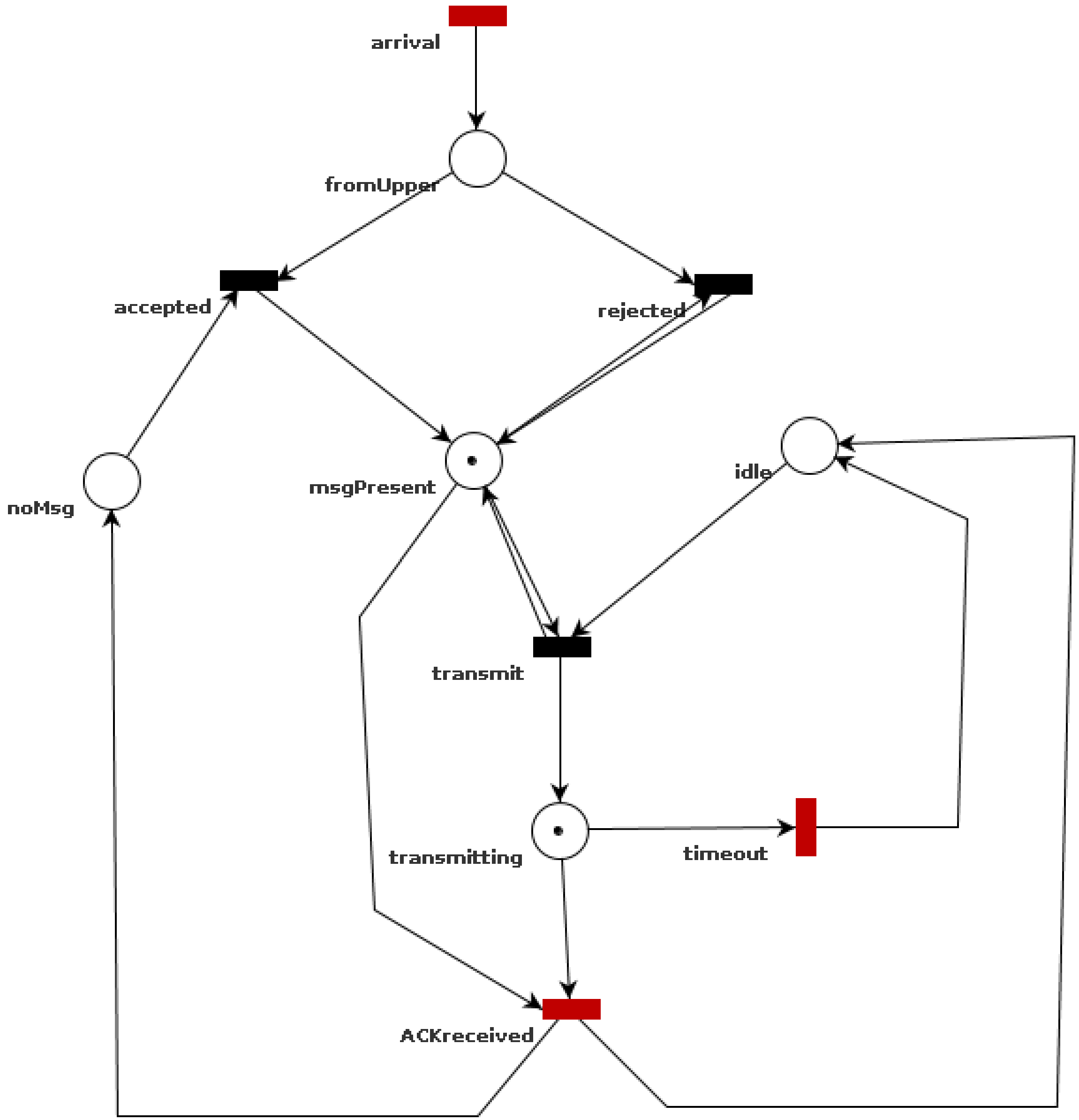
Single transmitter



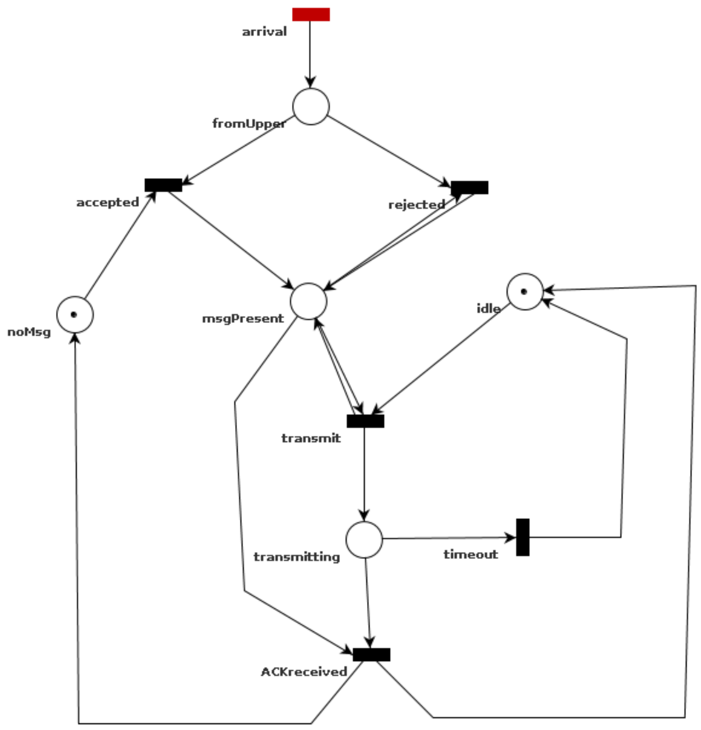
Single transmitter

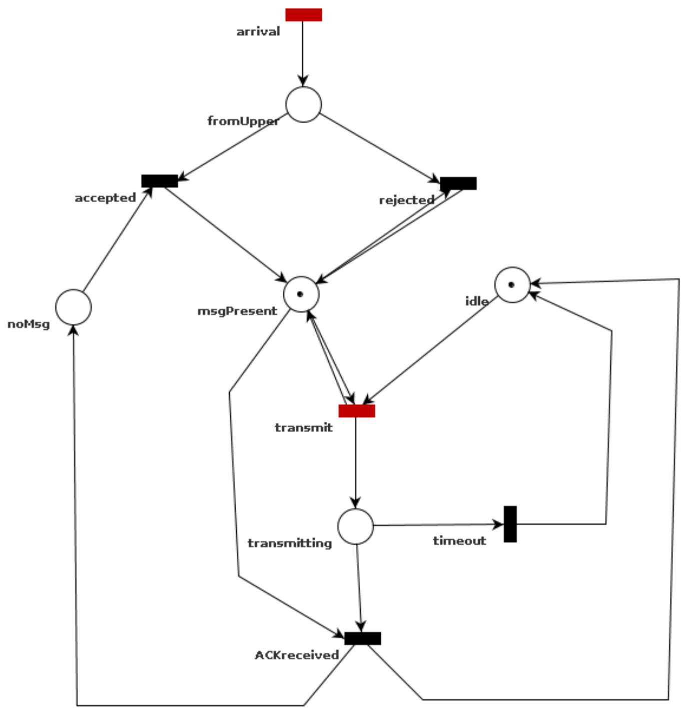


Single transmitter

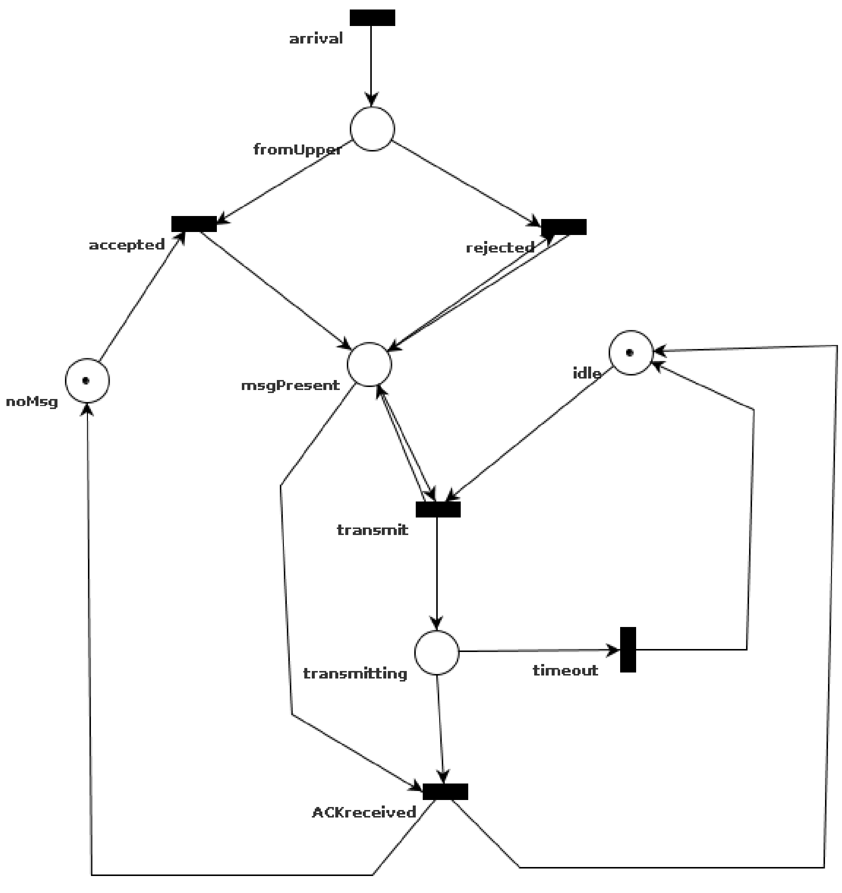


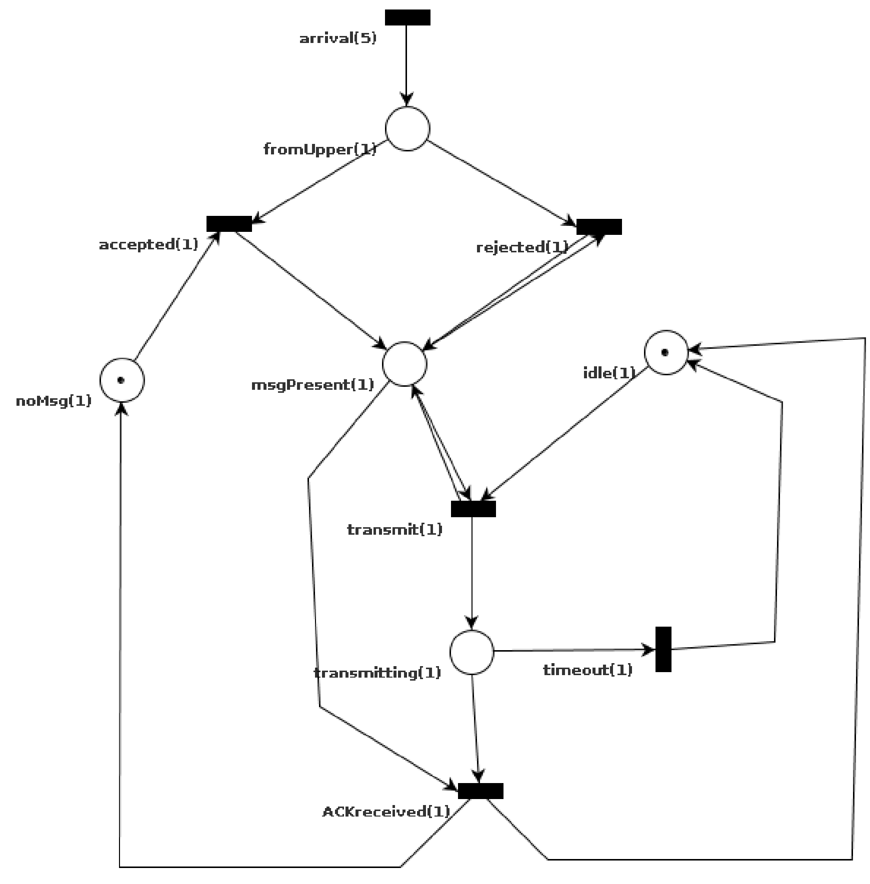
Single transmitter



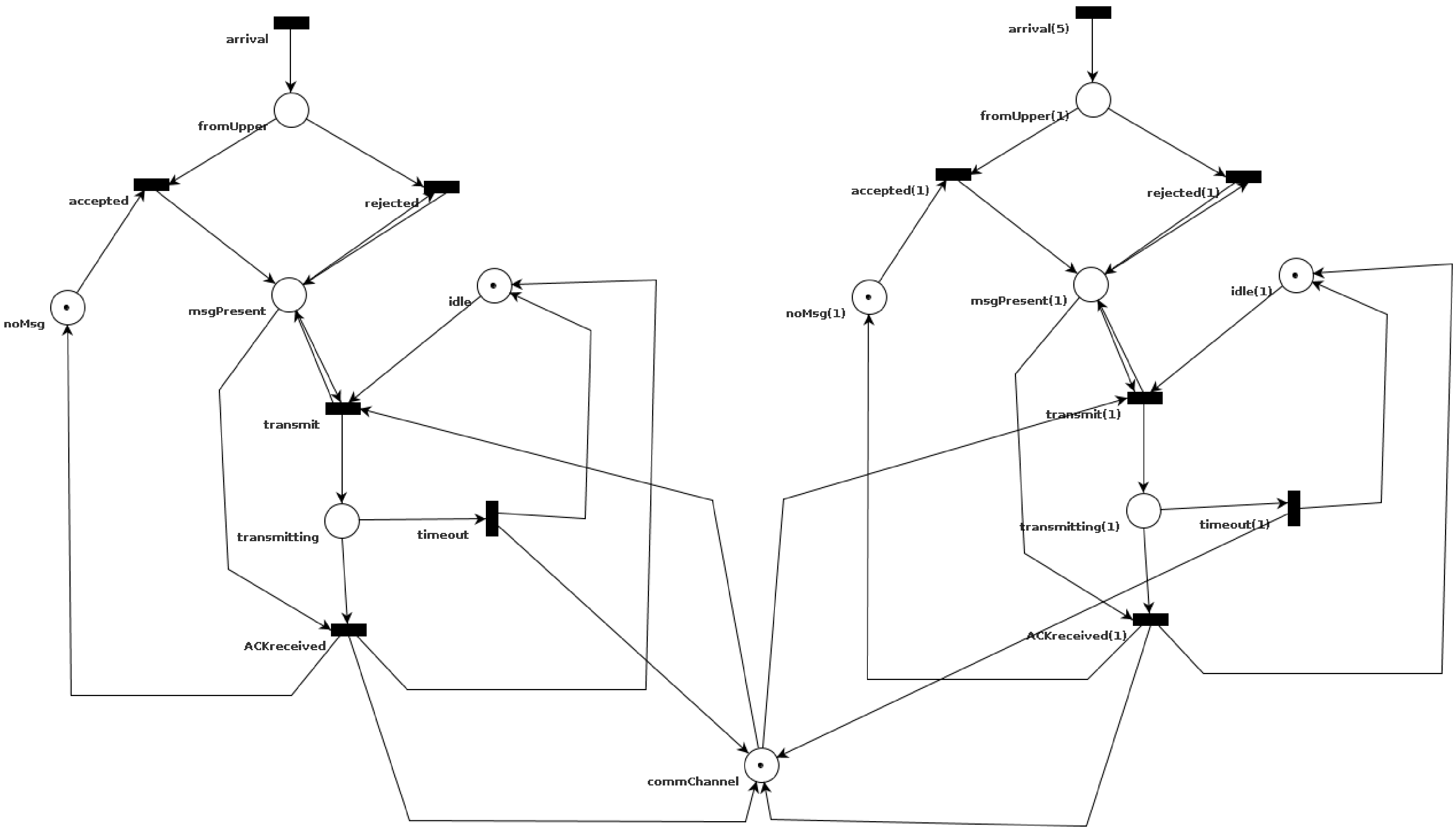


Two independent transmitters





Two transmitters competing for a single communication channel



Analysis of Petri nets

of **properties** of interest

Analysis of *logical* or *qualitative* behaviour.

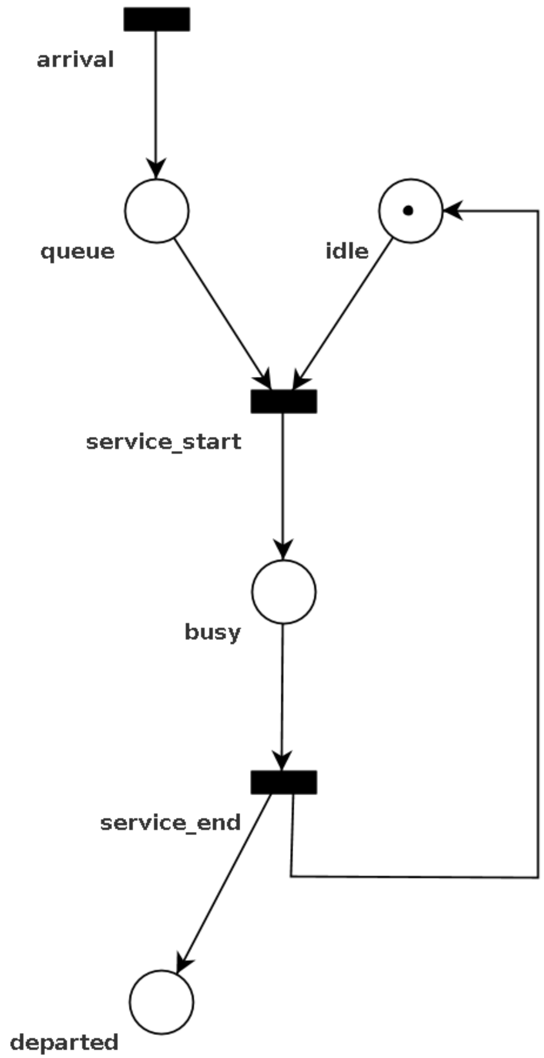
Resource sharing $\Rightarrow$ *fair* usage of resources:

- Boundedness
- Conservation
- Liveness and Deadlock
- State Reachability
- State Coverability
- Persistence
- Language Recognition

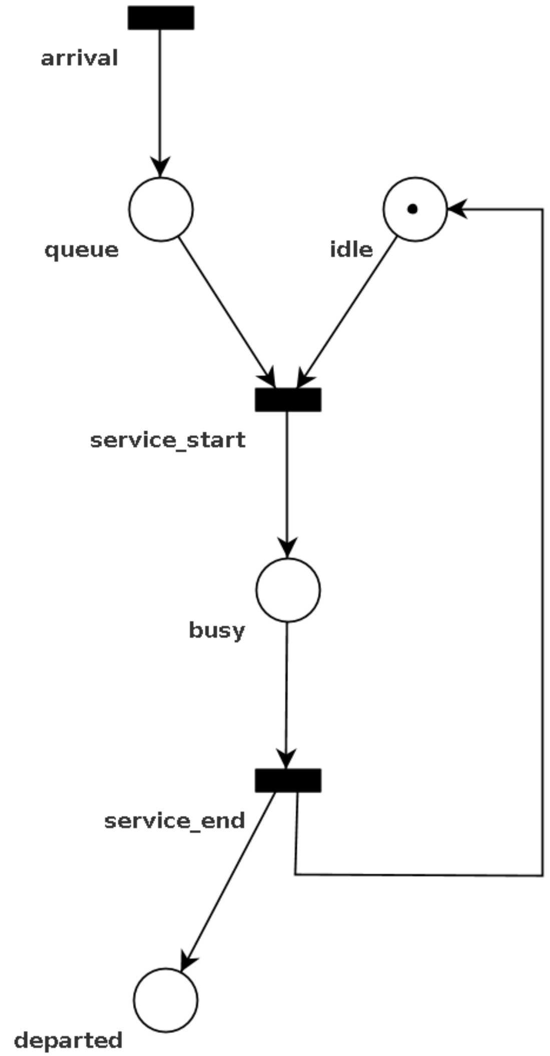
Boundedness

- Example: upper bound on number of customers in queue.
- Definition: A place $p_i \in P$ in a Petri net with initial state $\mathbf{x}_0$ is $k-$*bounded* or $k-$*safe* if
  $x(p_i) \leq k$ for all states in all possible sample paths.
- A $1-$bounded place is called *safe*.
- If a place is $k-$bounded for some k, the place is *bounded*.
- If all places are bounded, the Petri net is *bounded*.

Bounded vs. Unbounded

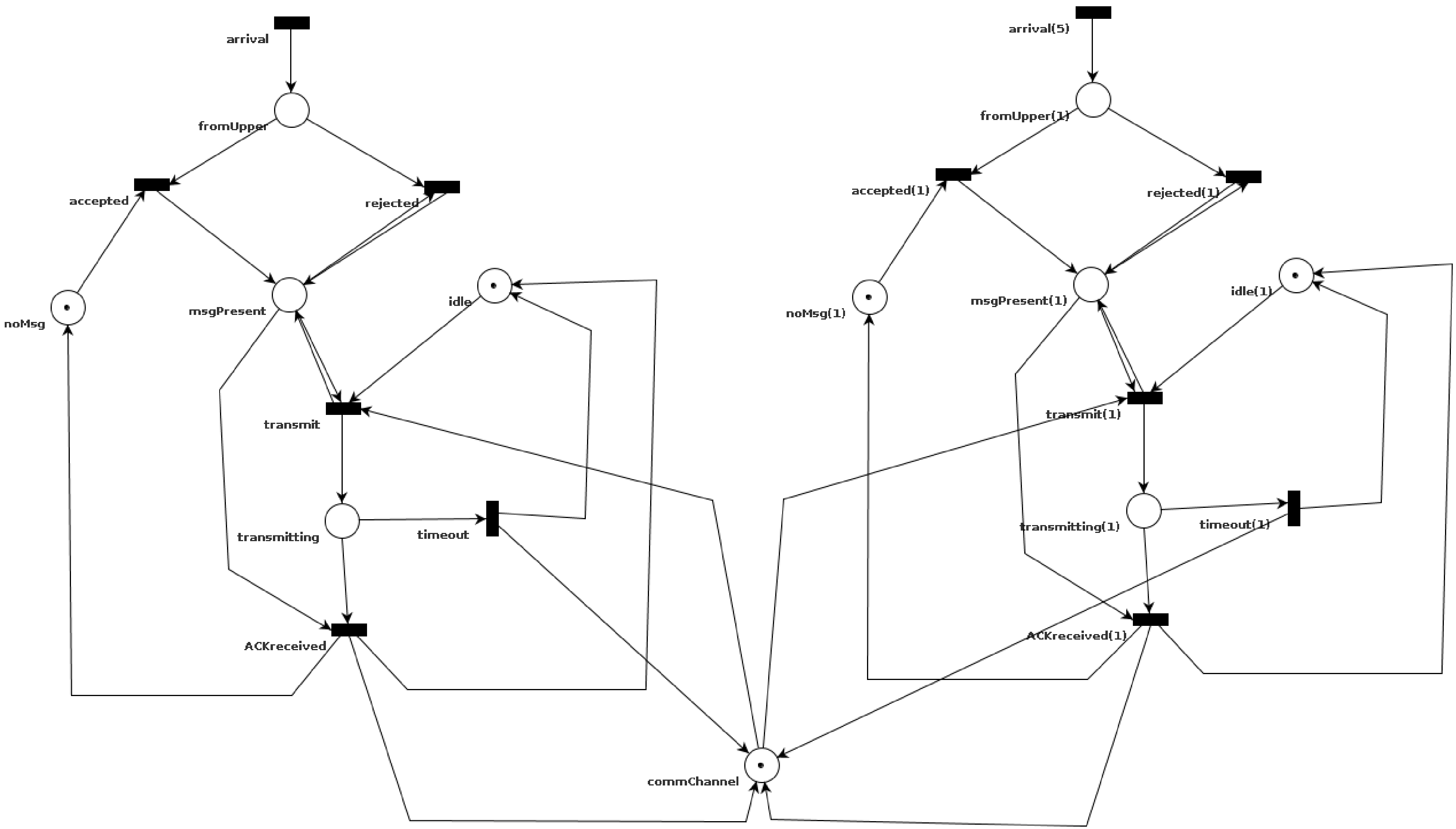


Conservation (invariants)



Sum of `busy` and `idle` marking is *constant* across *all* sample paths

Conservation (invariants): weighted sum



2 x transmitting + 1 x idle + 1x commChannel = 2

Conservation

A Petri net with initial state x_0 is *conservative with respect to* $\gamma = [\gamma_1, \gamma_2, \ldots, \gamma_n]$ if

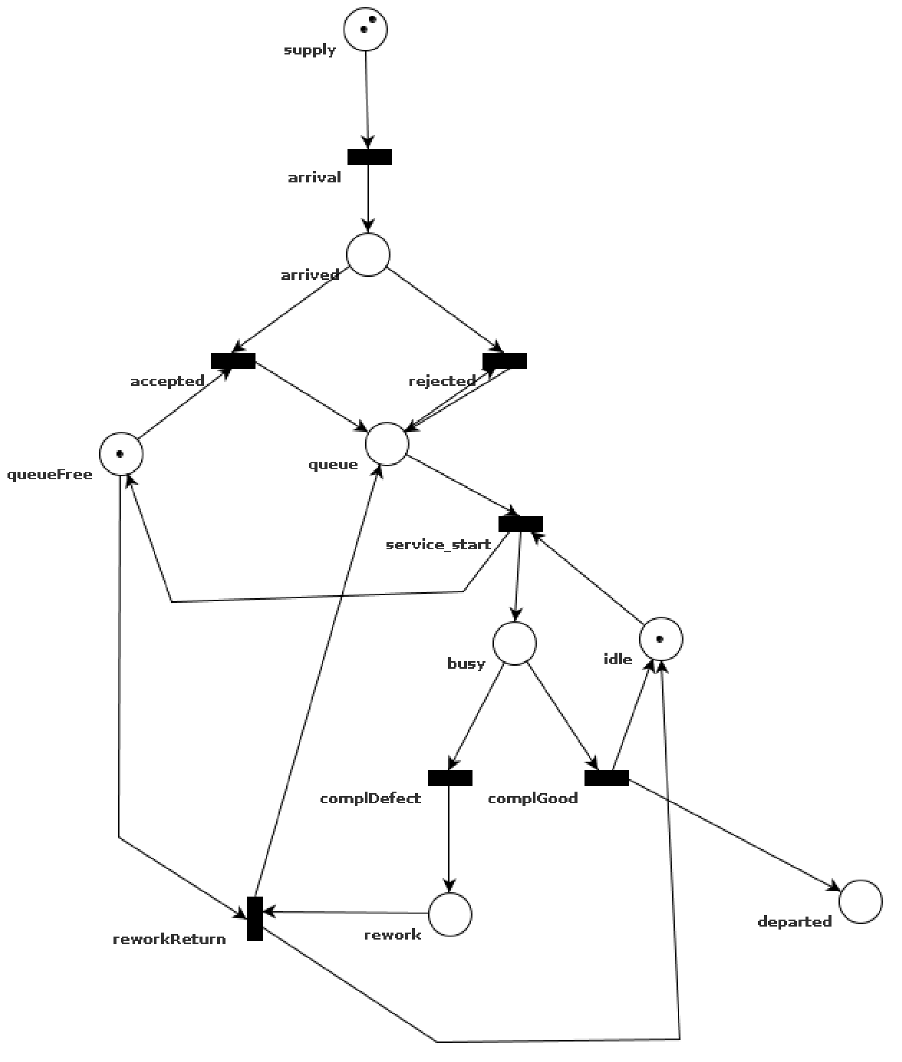
$$\Sigma_{i=1}^{n} \gamma_i x(p_i) = constant$$

for all states in all possible sample paths.

Liveness and Deadlock

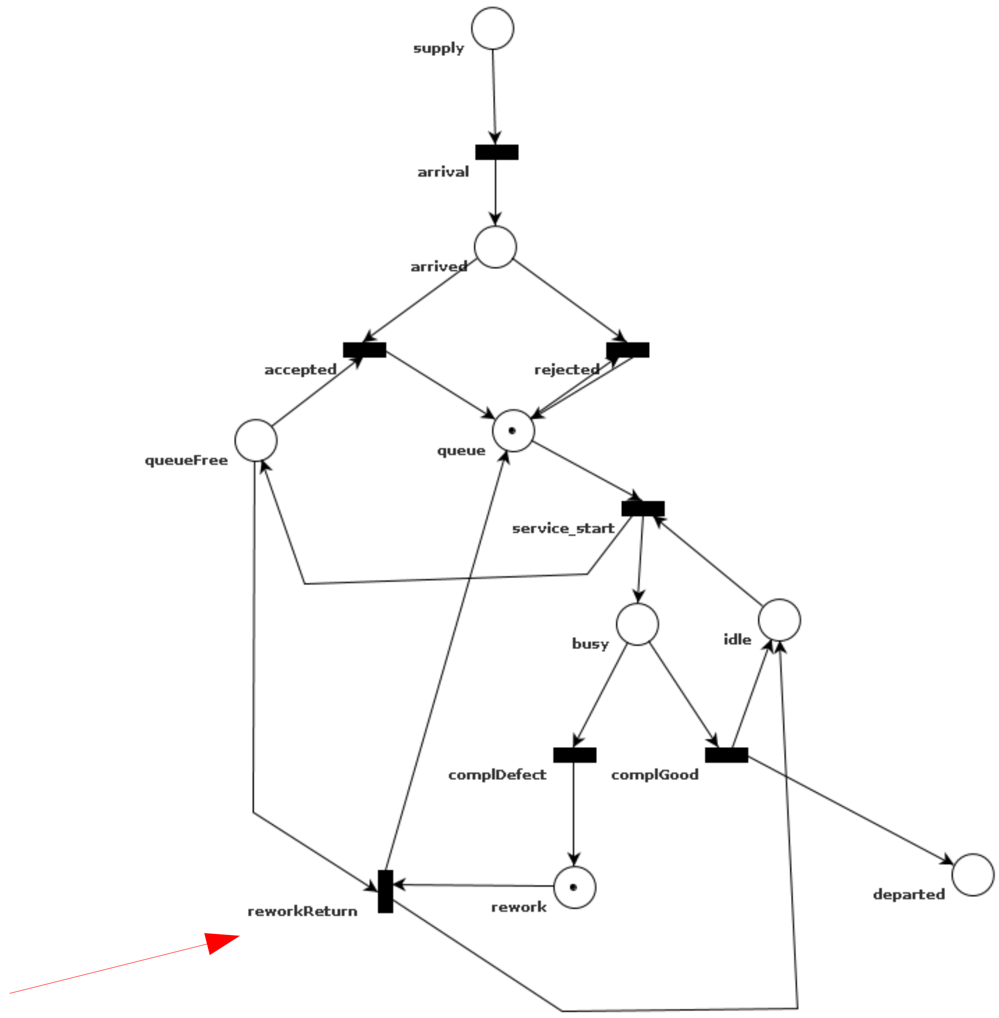
- Cyclic dependency $\Rightarrow$ wait indefinitely
- Deadlock
- Deadlock avoidance: avoid certain states in sample paths

Deadlock in queueing system with rework



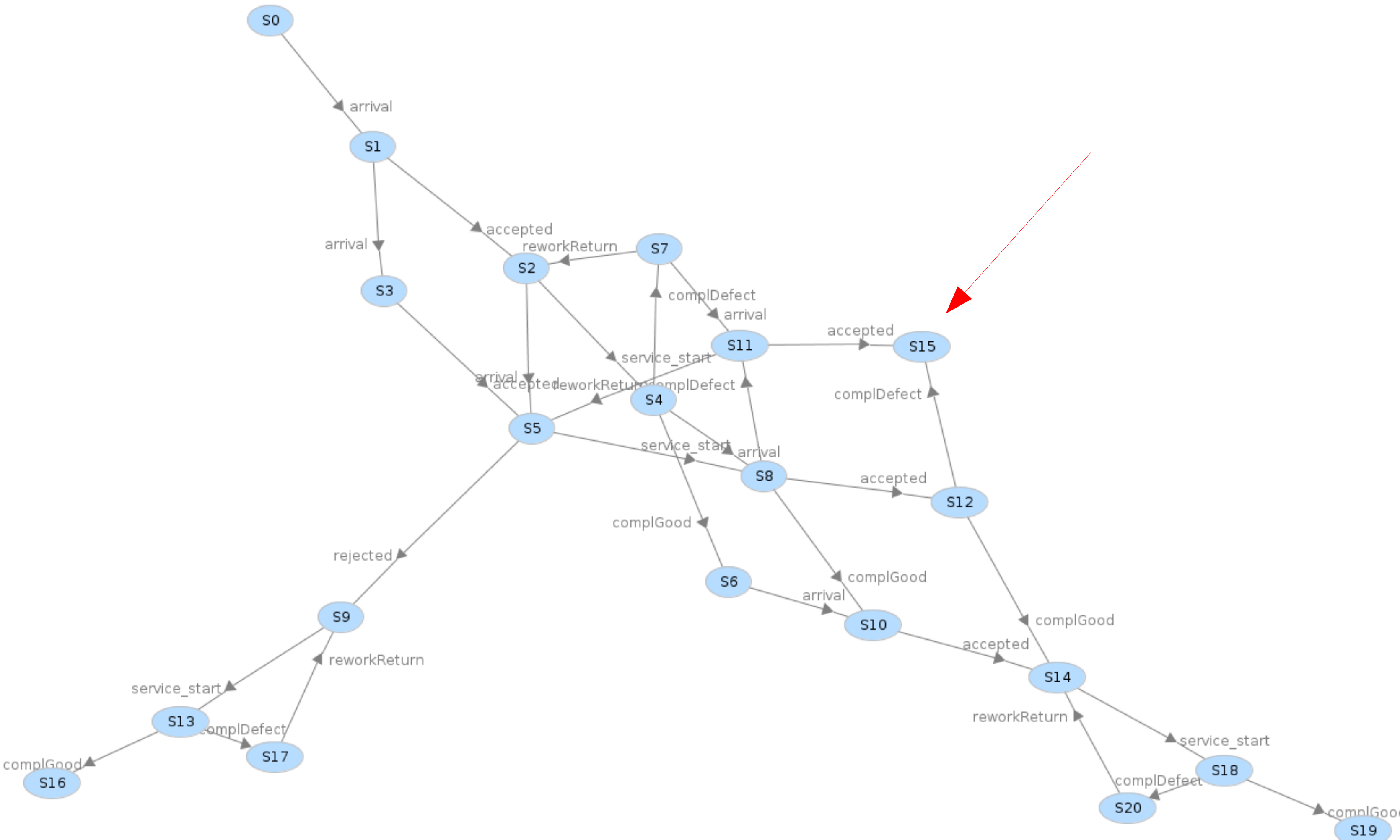
[queueFree, queue, rework] = [0, 1, 1] → deadlock

Deadlock in queueing system with rework



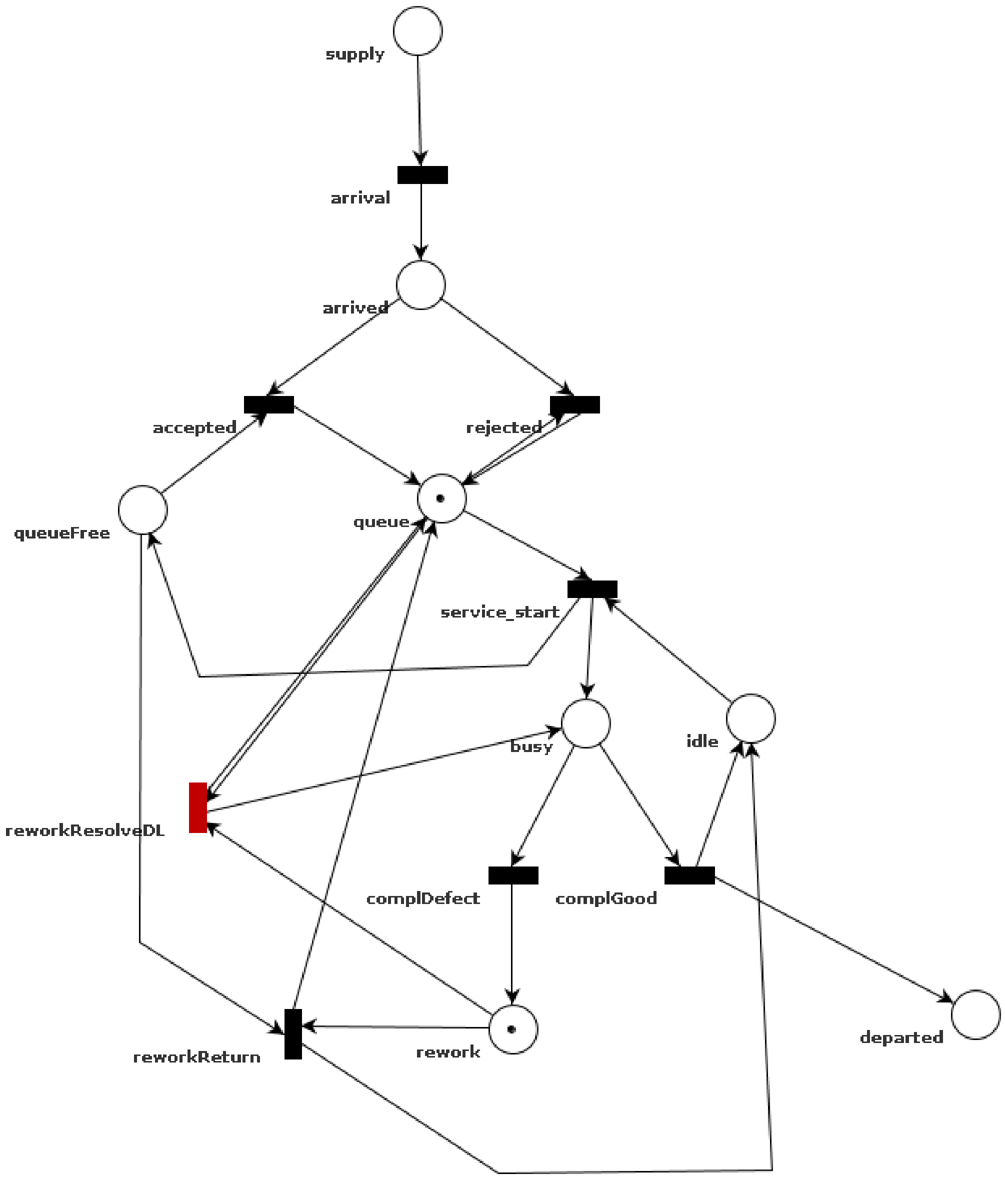
[queueFree, queue, rework] = [0, 1, 1] → deadlock

Deadlock in queueing system with rework

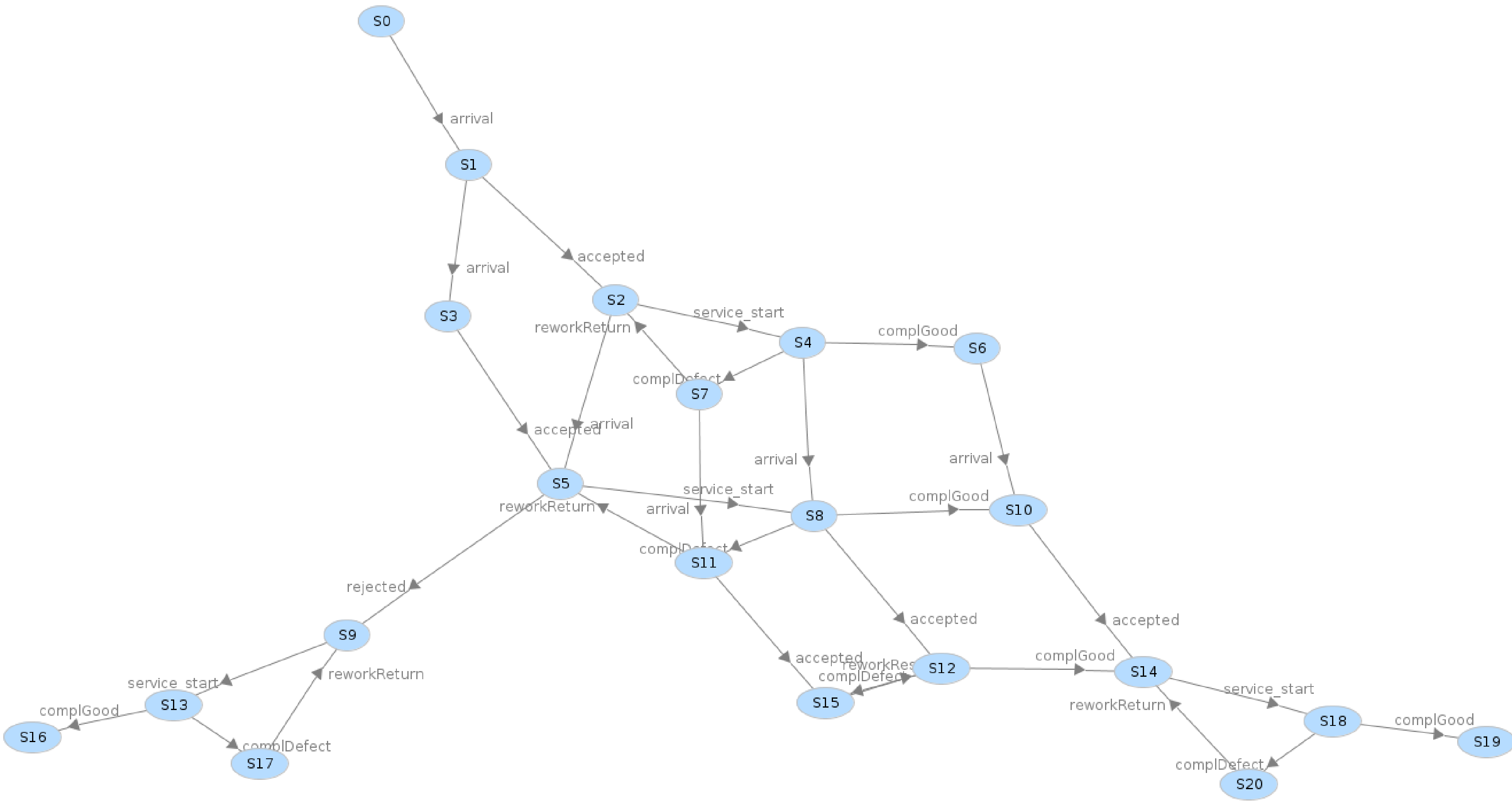


[queueFree, queue, rework] = [0, 1, 1] → deadlock

Deadlock resolved (avoided)



Deadlock resolved (avoided)

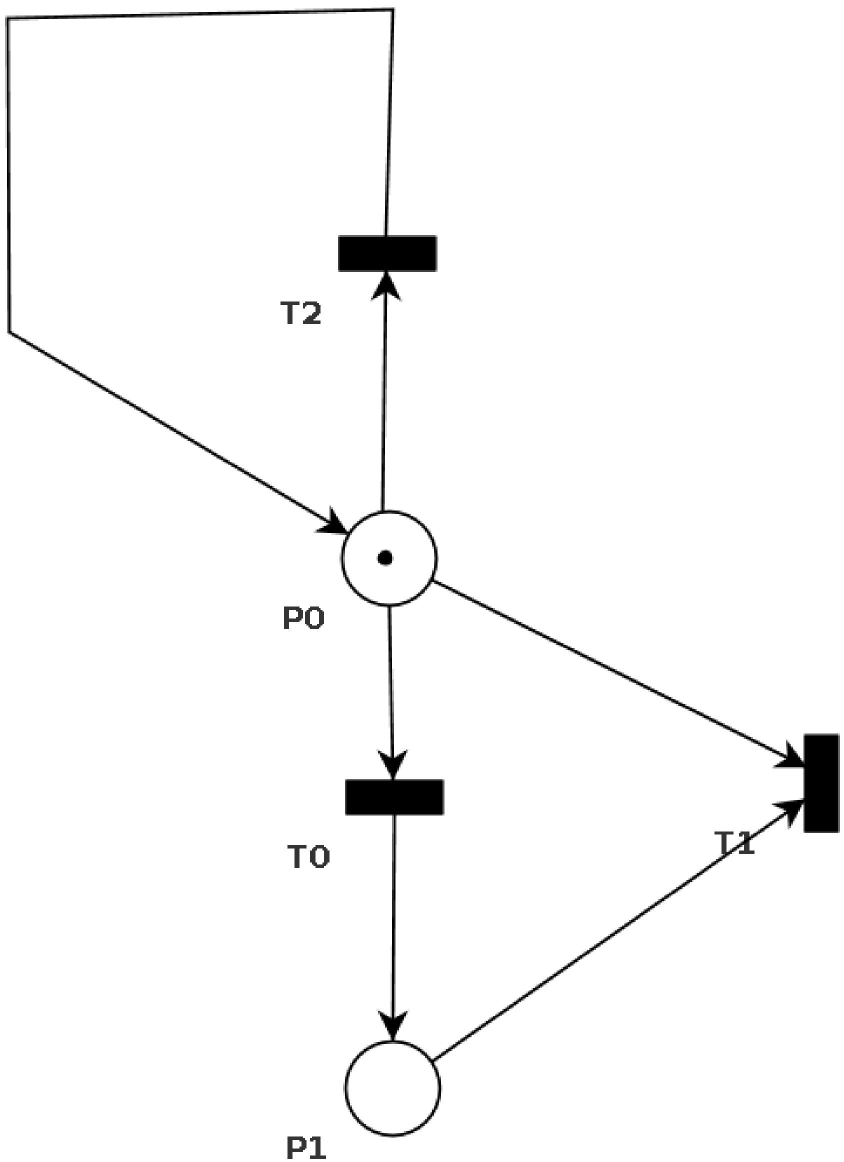


Liveness

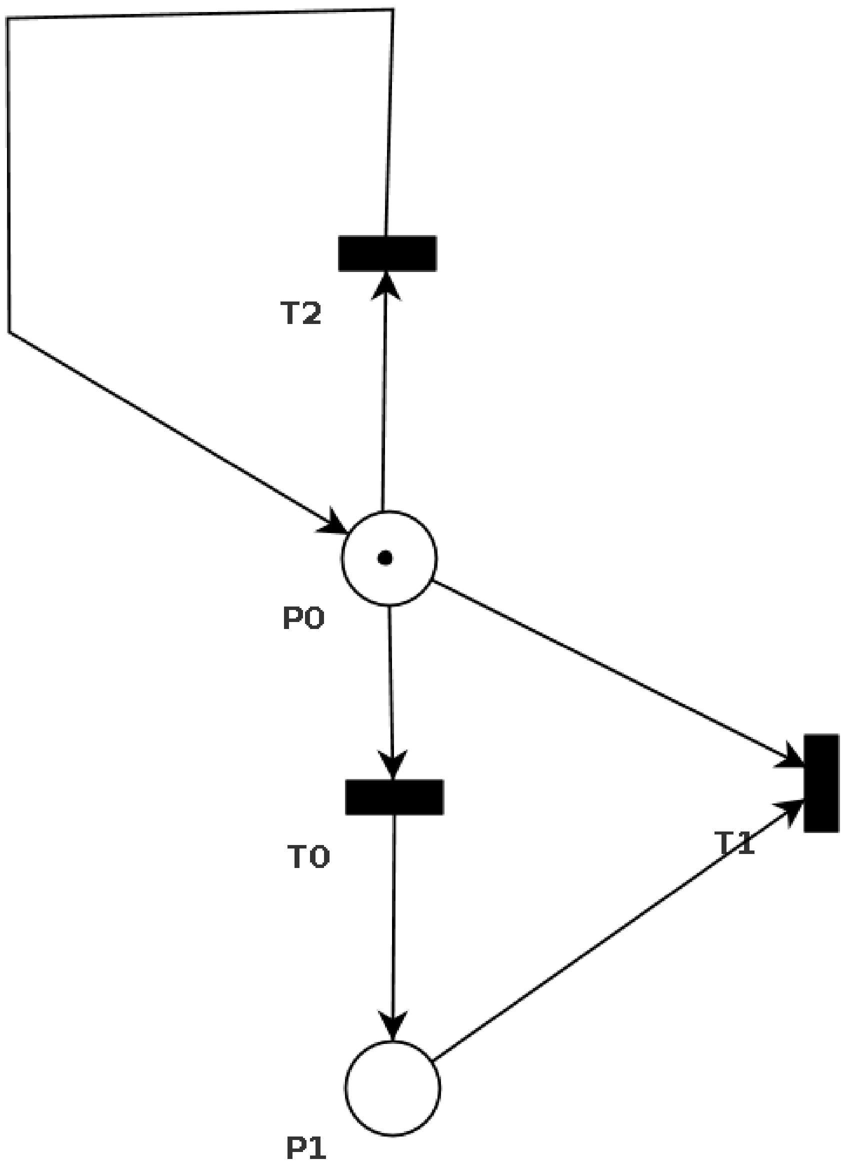
Given initial state $\mathbf{x}_0$, a transition in a Petri net is:

- L0-live (dead): if the transition can never fire.
- L1-live: if there is some firing sequence from $\mathbf{x}_0$ such that the transition can fire at least once.
- L2-live: if the transition can fire at least k times for some given positive integer k.
- L3-live: if there exists some infinite firing sequence in which the transition appears infinitely often.
- L4-live: if the transition is L1-live for every possible state reached from $\mathbf{x}_0$.

Liveness example

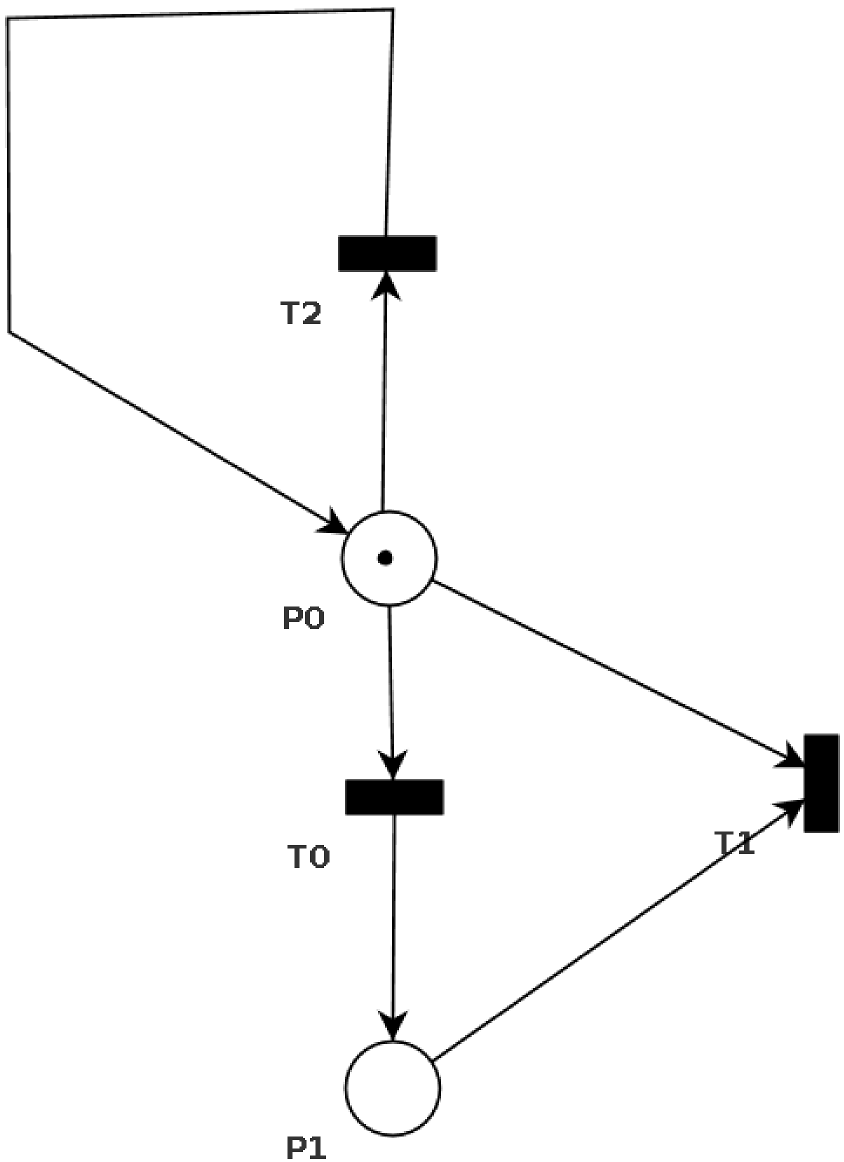


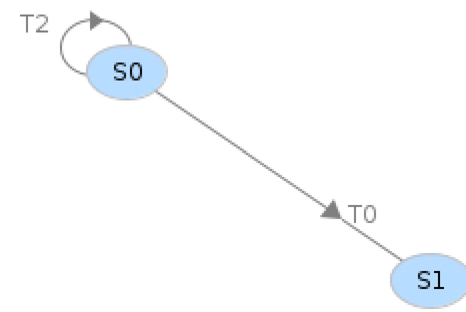
Liveness example



T0 is L1-live
T1 is dead
T2 is L3-live, not L4-live

Liveness example





S0 = [1, 0]
S1 = [0, 1]

T0 is L1-live
T1 is dead
T2 is L3-live, not L4-live

Coverability Notation

- Root node
- Terminal node
- Duplicate node

Coverability Notation

- Node *dominance*

$$\mathbf{x} = [x(p_1), x(p_2), \ldots, x(p_n)]$$

$$\mathbf{y} = [y(p_1), y(p_2), \ldots, y(p_n)]$$

$\mathbf{x} >_d \mathbf{y}$ ($\mathbf{x}$ dominates $\mathbf{y}$)if

1. $x(p_i) \geq y(p_i), \forall i \in \{1, \ldots, n\}$
2. $x(p_i) > y(p_i)$ for at least some $i \in \{1, \ldots, n\}$

- The symbol ω represents *infinity*

$$\mathbf{x} >_d \mathbf{y}$$

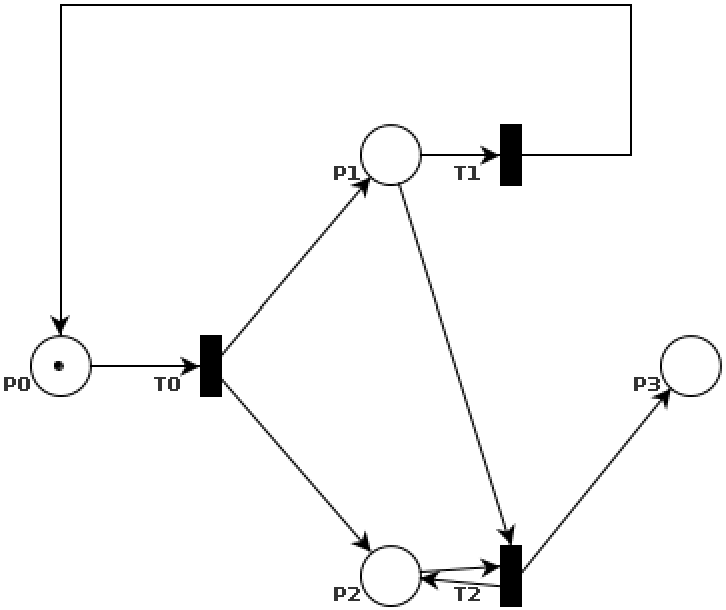
For all i such that $x(p_i) > y(p_i)$, replace $x(p_i)$ by ω

$$\omega + k = \omega = \omega - k$$

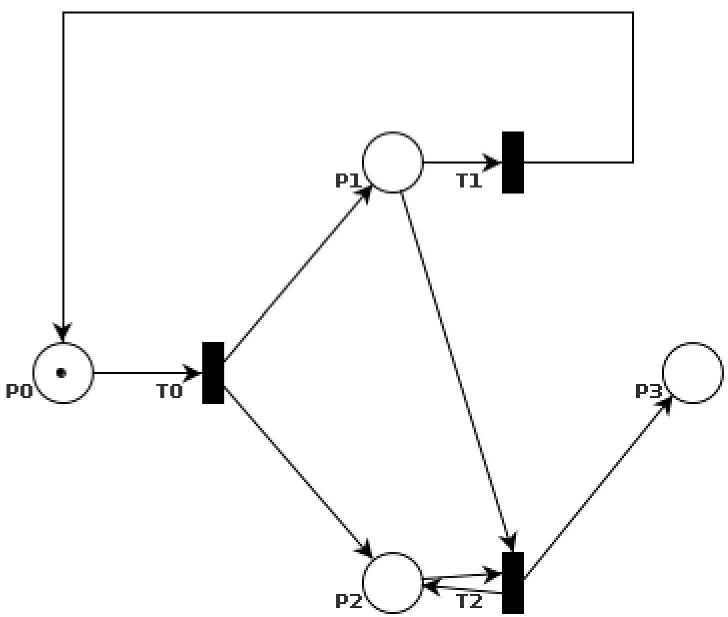
Coverability Tree Construction

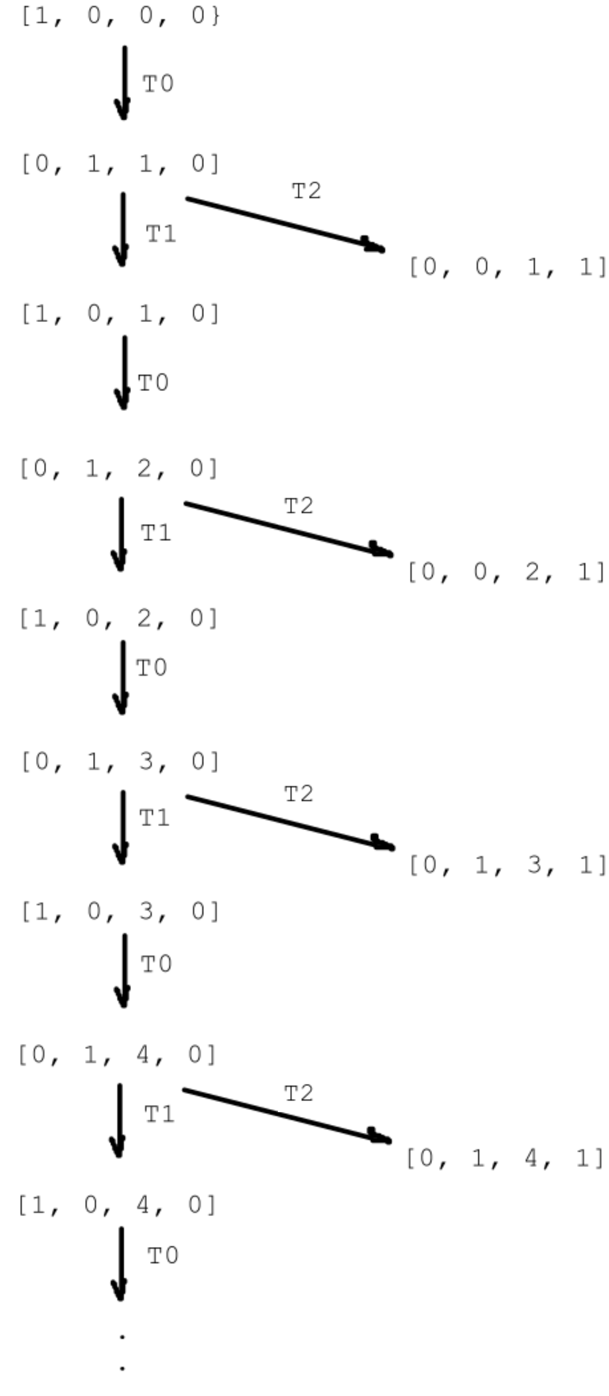
1. Initialize $\mathbf{x} = \mathbf{x}_0$ (initial state)
2. Fore each new node $\mathbf{x}$,
   evaluate the transition function $f(\mathbf{x}, t_i)$ for all $t_j \in T$:
   (a) if $f(\mathbf{x}, t_j)$ is undefined for all $t_j \in T$, then $\mathbf{x}$ is a terminal node.
   (b) if $f(\mathbf{x}, t_j)$ is defined for some $t_j \in T$,
       create a new node $\mathbf{x}' = f(\mathbf{x}, t_j)$.
       i. if $x(p_i) = \omega$ for some p_i, set $x'(p_i) = \omega$.
       ii. If there exists a node $\mathbf{y}$ in the path from root node $\mathbf{x}_0$ (included) to $\mathbf{x}$ such that $\mathbf{x}' >_d \mathbf{y}$, set $x'(p_i) = \omega$ for all p_i such that $x'(p_i) > y(p_i)$
       iii. Otherwise, set $\mathbf{x}' = f(\mathbf{x}, t_j)$.
3. Stop if all new nodes are either *terminal* or *duplicate*

Example

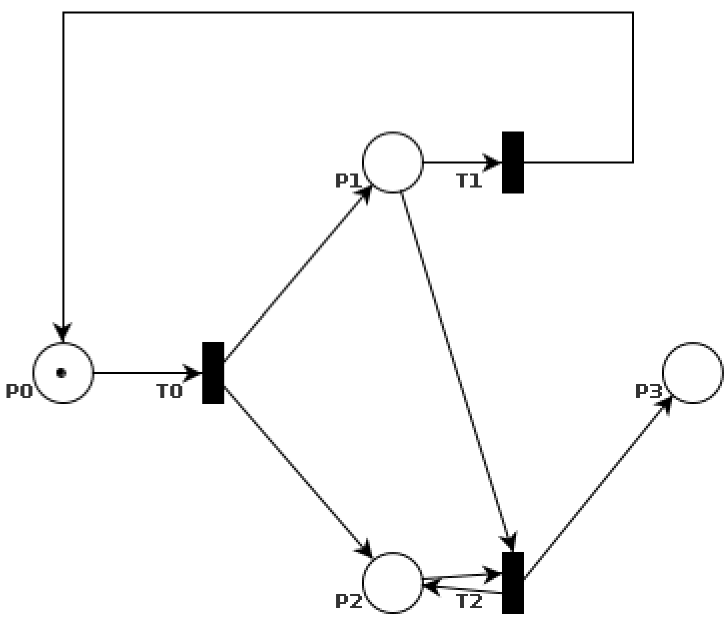


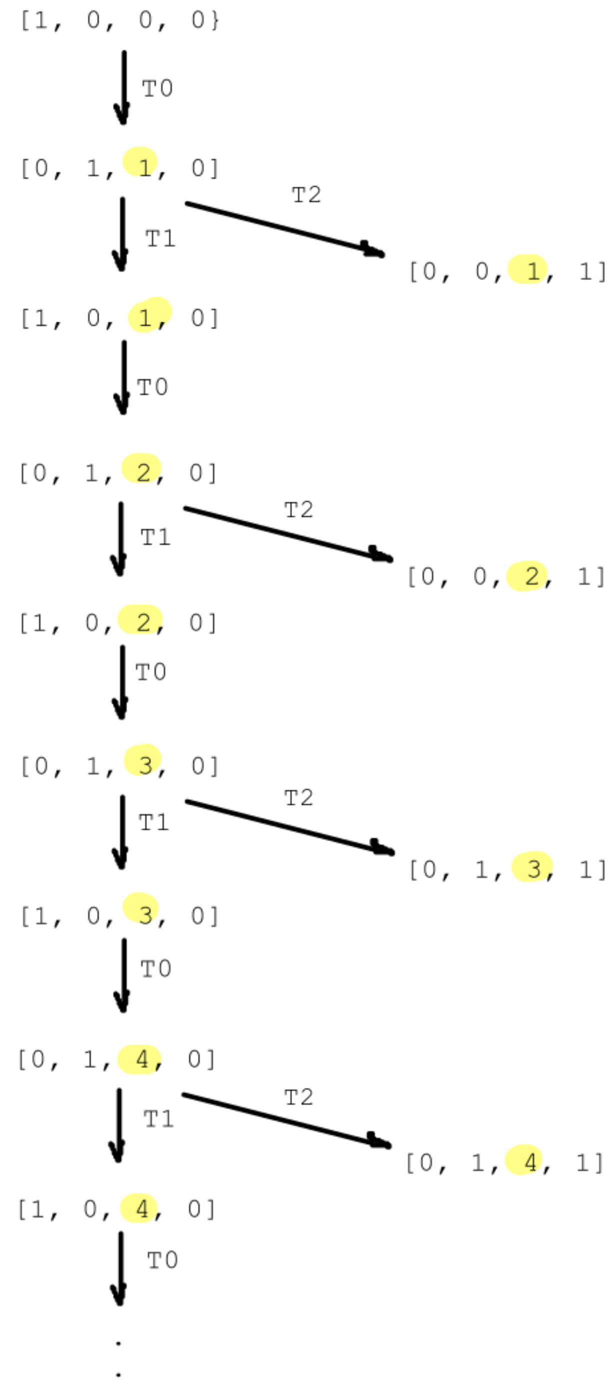
Reachability Tree (Graph)



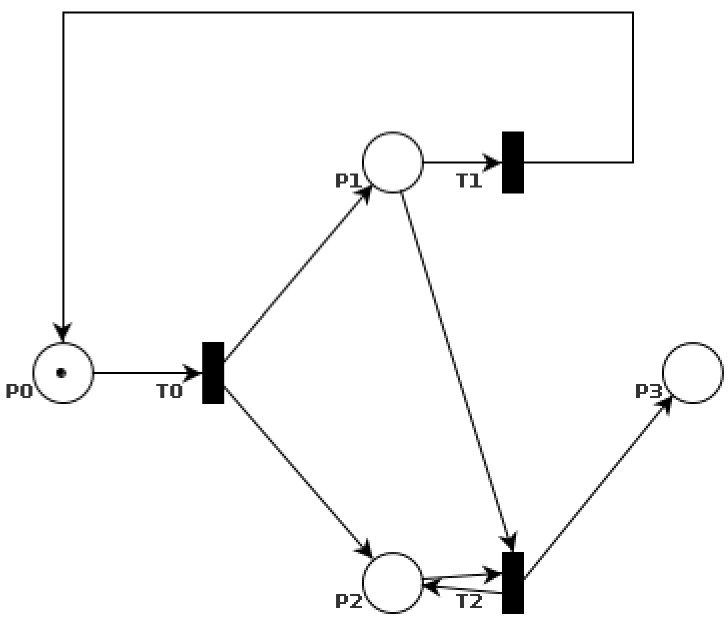


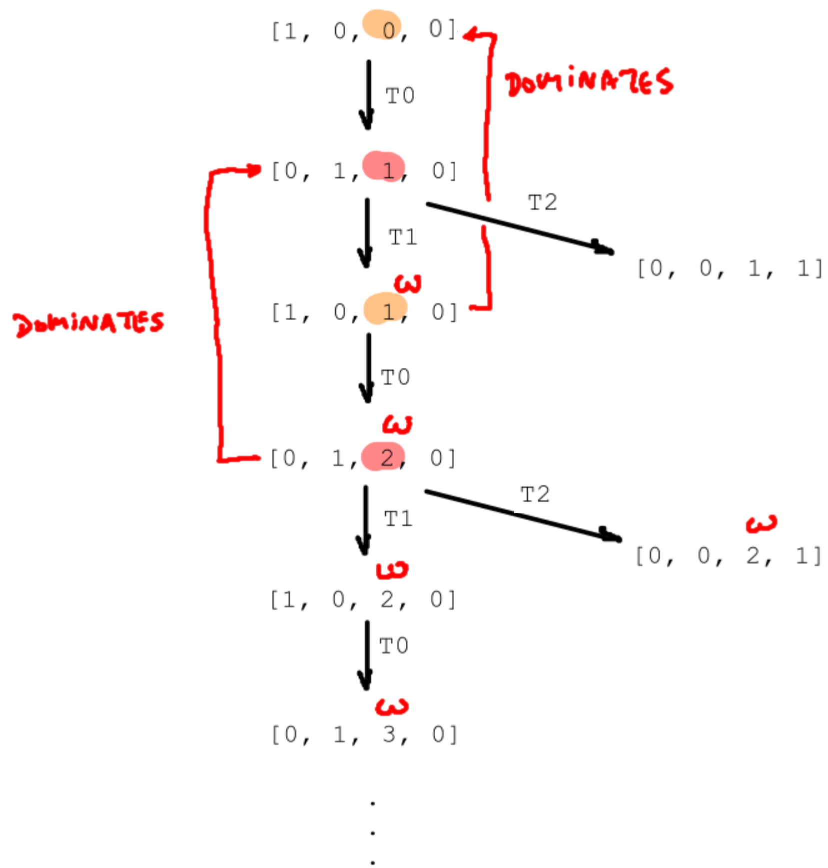
Reachability Tree (Graph)



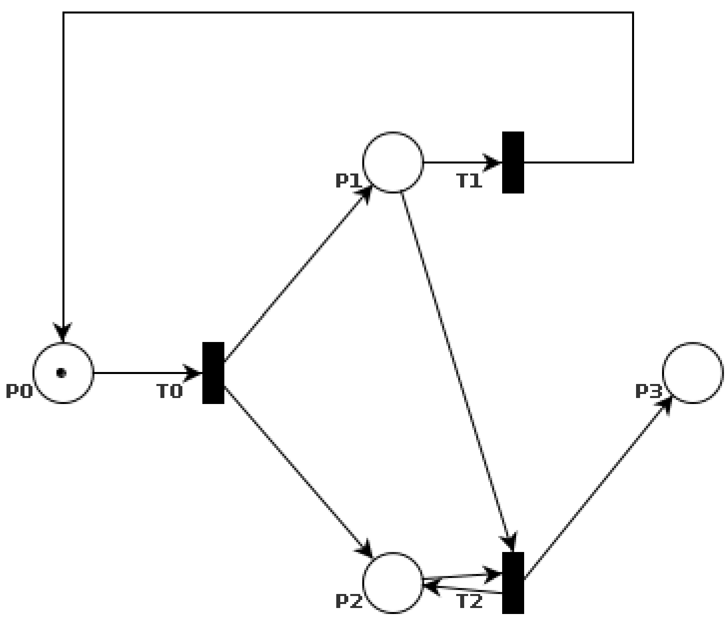


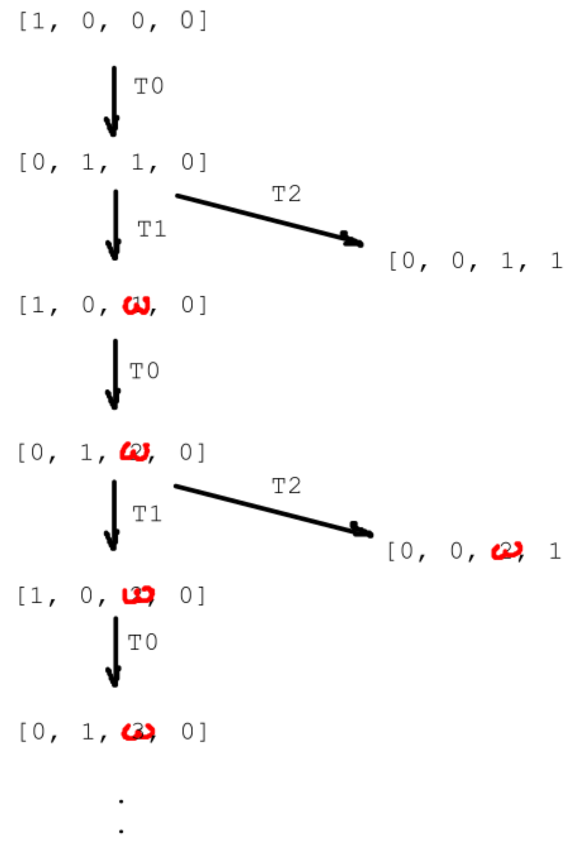
Coverability Tree



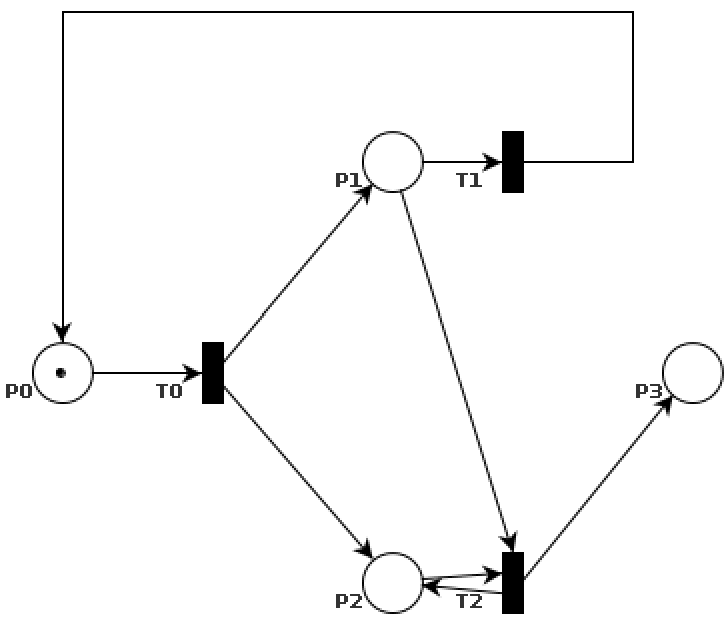


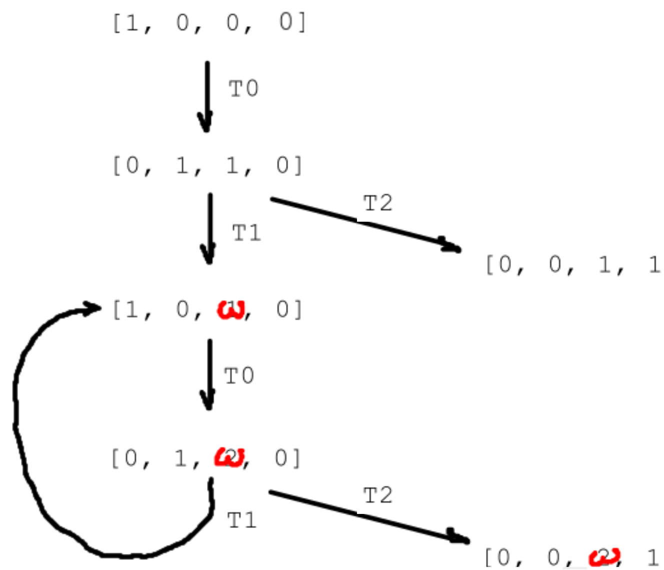
Coverability Tree

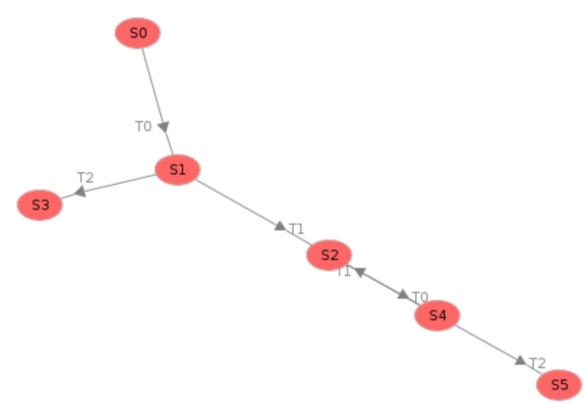




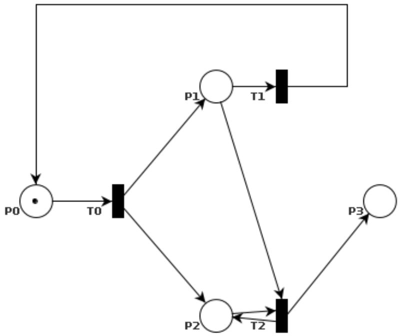
Coverability Tree (Graph)

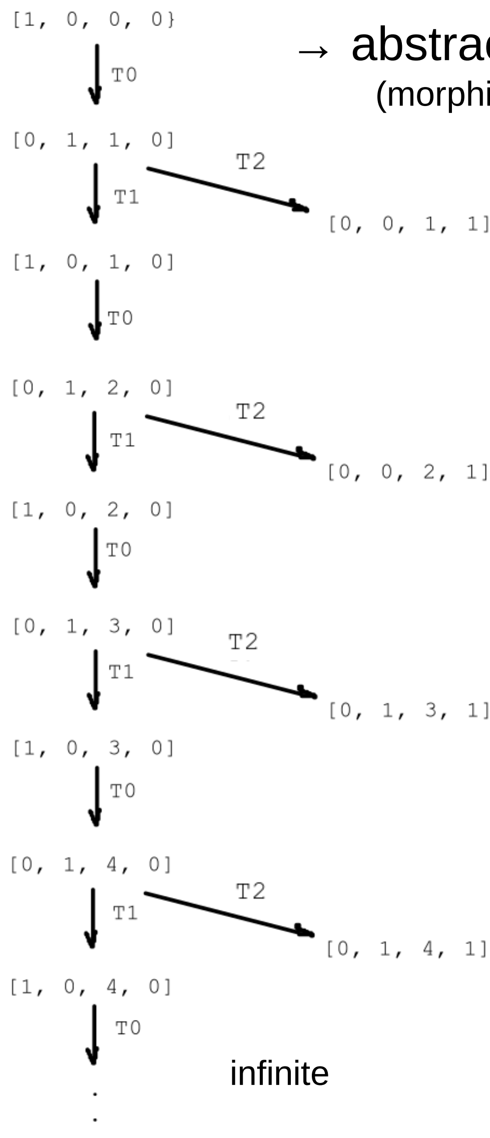






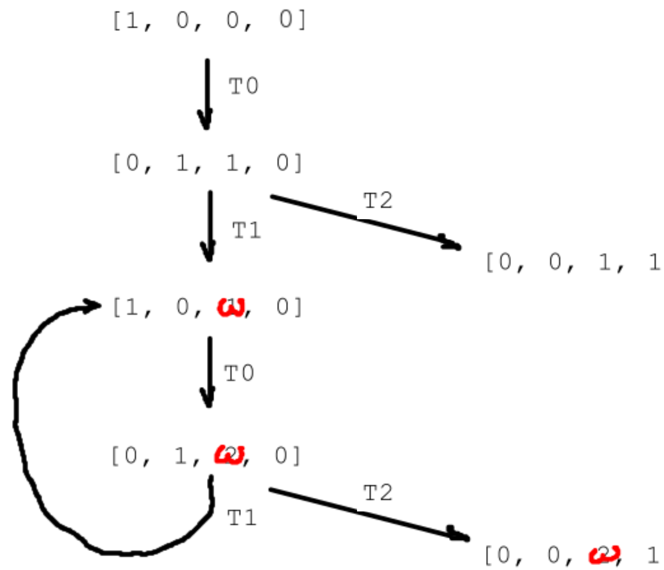
Reachability Tree/Graph vs. Coverability Tree (Graph)





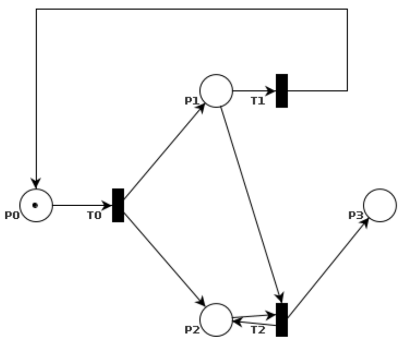
→ abstraction →
(morphism)

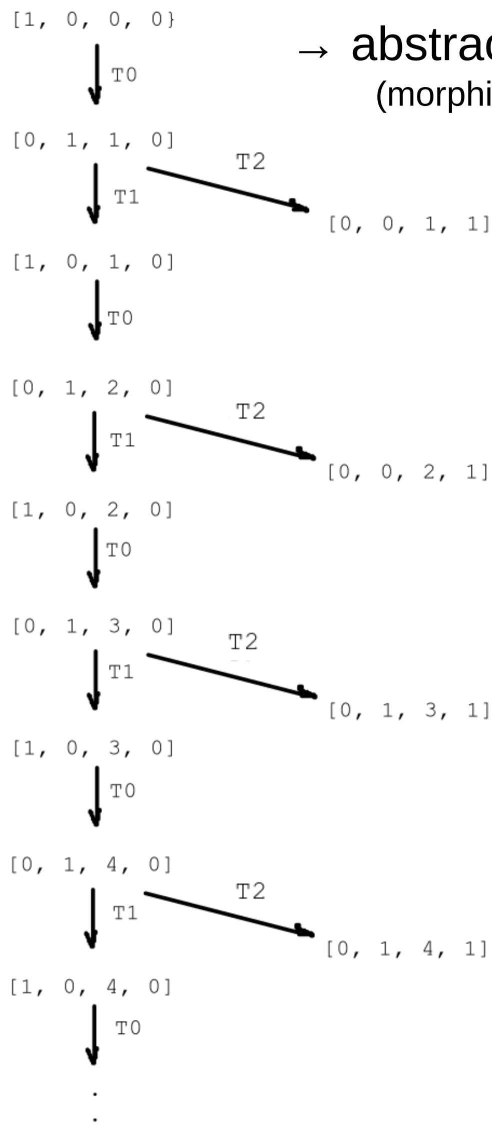
infinite



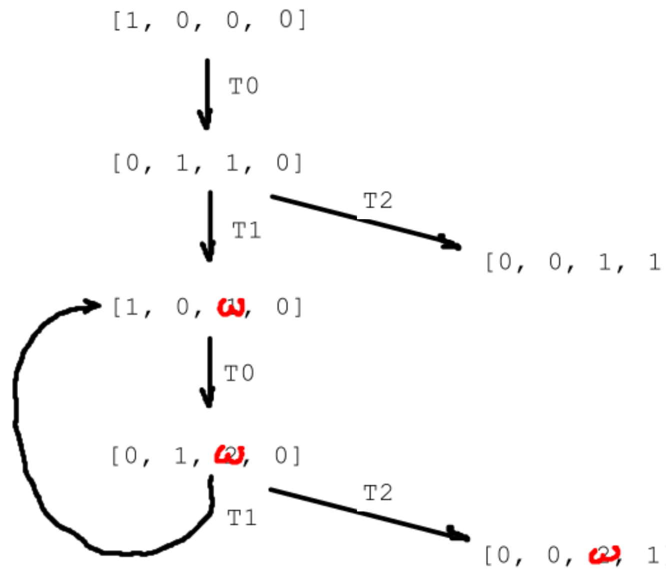
finite

Reachability Tree/Graph vs. Coverability Tree (Graph)





→ abstraction →
(morphism)



analysis property: reachable [1, 0, 3, 0]

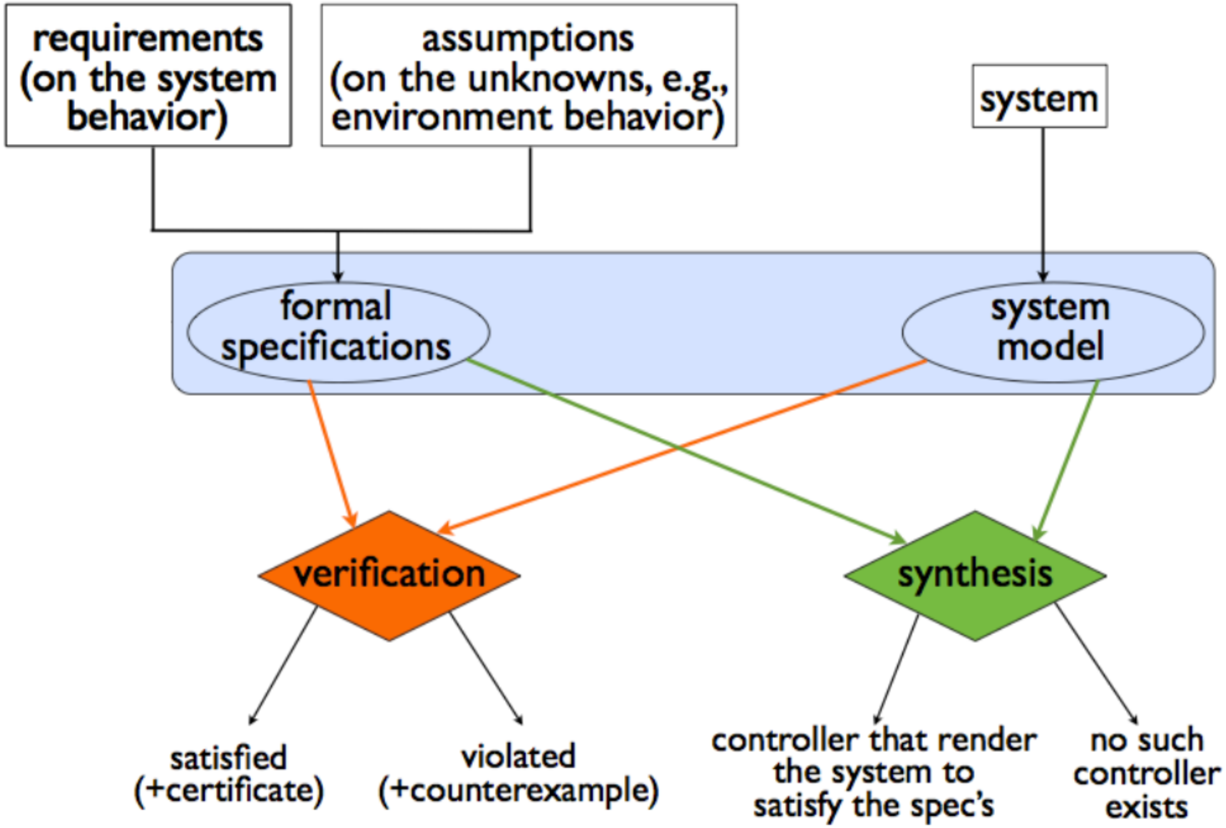
Applications of the Coverability Tree

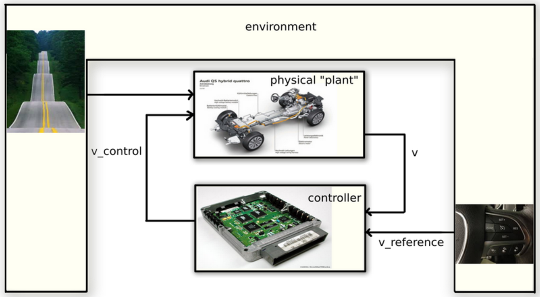
- Boundedness: ω does not appear in coverability tree
- Bounded Petri net $\Rightarrow$ reachability graph
- Conservation: $\gamma_i = 0$ for ω positions
- Inverse problem: what are γ and C ?
- Coverability: inspect coverability tree
- Limitations: deadlock detection

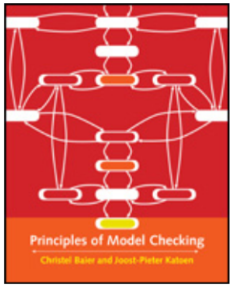
specifying and **checking properties** over **all traces**

2001WETFSP “specification and verification”

properties





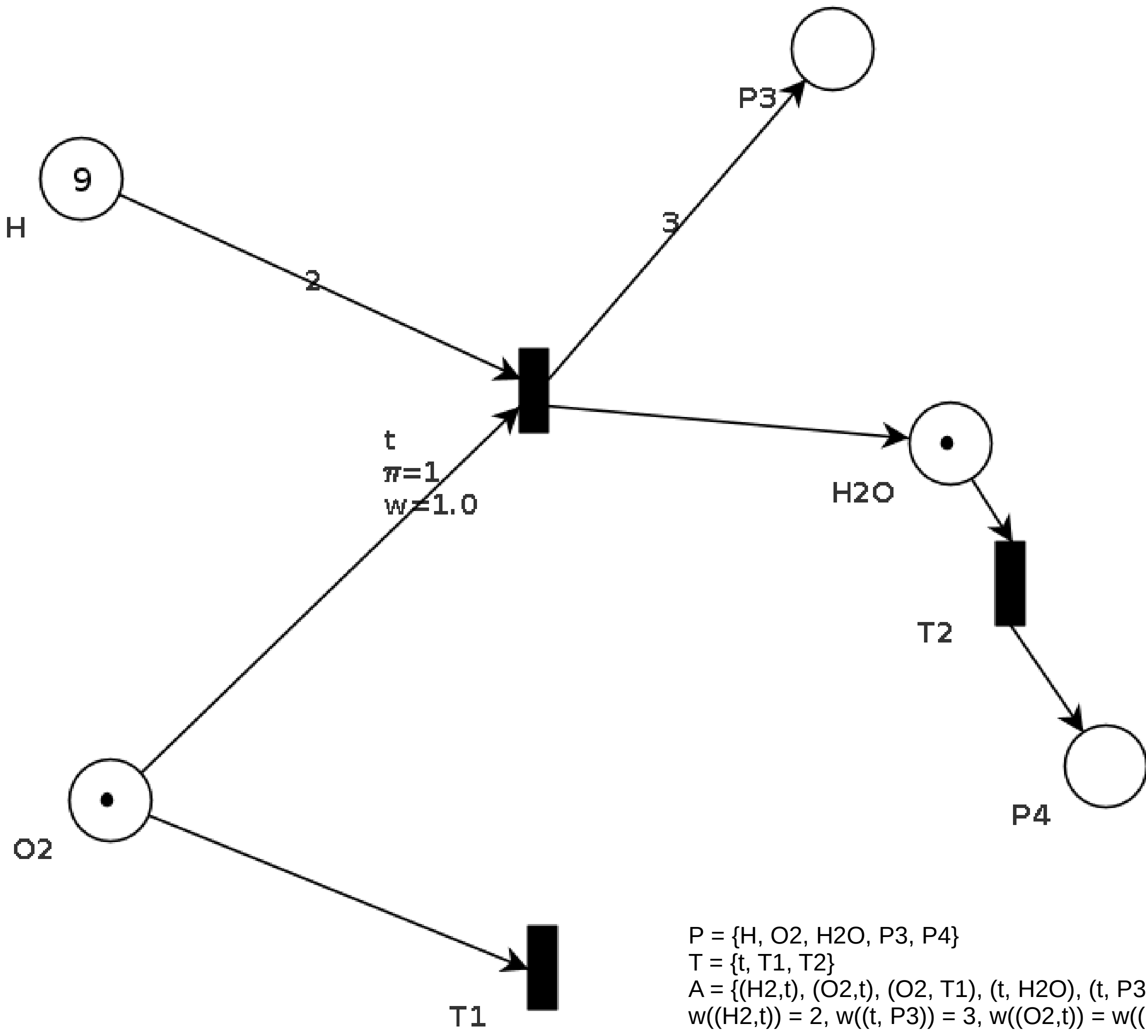


Principles of Model Checking, Christel Baier and Joost-Pieter Katoen. MIT Press, 2008.

Chapter 5

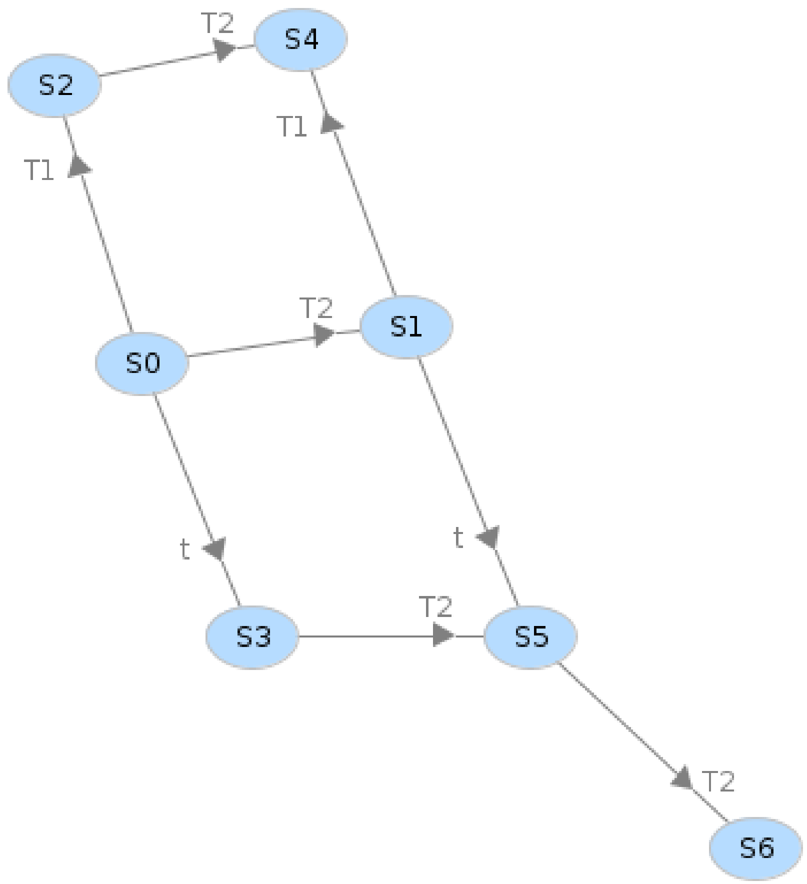
slides from Richard M. Murray @ EECI 2012

Marked Petri Net



P = {H, O2, H2O, P3, P4}
T = {t, T1, T2}
A = {(H2,t), (O2,t), (O2, T1), (t, H2O), (t, P3), (H2O, T2), (T2, P4)}
w((H2,t)) = 2, w((t, P3)) = 3, w((O2,t)) = w((O2, T1)) = w((t, H2O)) = w((H2O, T2)) = w((T2, P4)) = 1
I(t) = {H, O2}, I(T1) = {O2}, I(T2) = {H2O}
O(t) = {P3, H2O}, O(T1) = {}, O(T2) = {P4}
x = [9, 1, 1, 0, 0] corresponding to places [H, O2, H2O, P3, P4]

Marked Petri Net semantics: behaviour traces



S0 = [9, 1, 1, 0, 0]
S1 = [9, 1, 0, 0, 1]
S2 = [9, 0, 1, 0, 0]
S3 = [7, 0, 2, 3, 0]
S4 = [9, 0, 0, 0, 1]
S5 = [7, 0, 1, 3, 1]
S6 = [7, 0, 0, 3, 2]

Reachability graph =
compact notation of **all possible** “sample paths” (**behaviour traces**) =

{ `S0 -T1-> S2 -T2-> S4, S0 -T2-> S1 -T1-> S4,`
`S0 -T2-> S1 -t-> S5 -T2-> S6, S0 -t-> S3 -T2-> S5 -T2-> S6`}

Linear **<u>T</u>**<u>emporal</u> **L**ogic (LTL, Amir Pnueli in 1977): specifying properties over (behaviour) **traces**

LTL formulas:

$$\varphi ::= \text{true} \;\Big|\; a \;\Big|\; \varphi_1 \wedge \varphi_2 \;\Big|\; \neg\varphi \;\Big|\; \bigcirc\,\varphi \;\Big|\; \varphi_1 \,\mathsf{U}\, \varphi_2$$

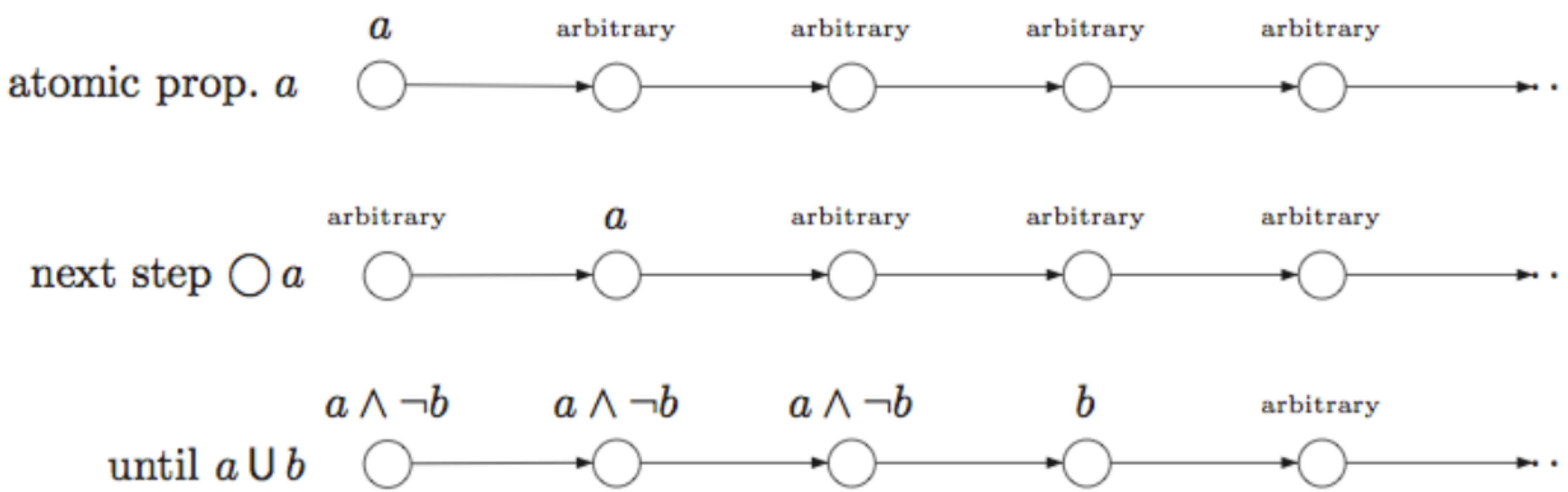
- a = atomic proposition
- $\bigcirc$ = "next": φ is true at next step
- U = "until": φ_2 is true at some point, φ_1 is true until that time

Operator precedence

- Unary bind stronger than binary
- U takes precedence over ∧, ∨ and →

(behaviour) **trace**

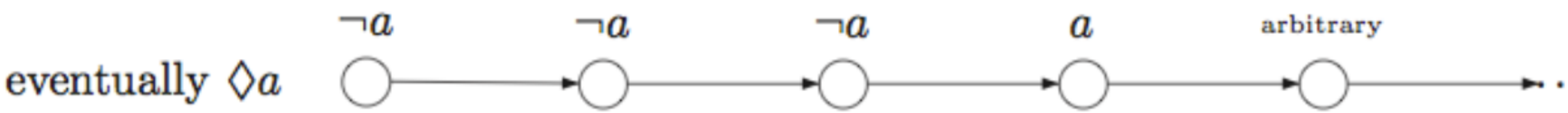
Formula evaluation: evaluate LTL propositions over a sequence of states (path):



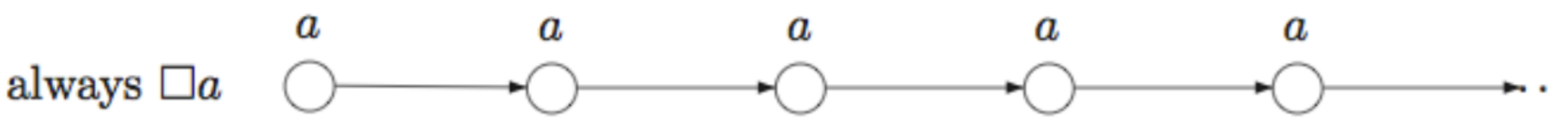
- Same notation as linear time properties: σ ⊨ φ (path "satisfies" specification)

“Primary” temporal logic operators

F	Eventually	$\Diamond\phi := \text{true}\ U\ \phi$	ϕ will become true at some point in the future
G	Always	$\Box\phi := \neg\Diamond\neg\phi$	ϕ is always true; “(never (eventually ($\neg\phi$)))”



in the **F**ullness of time



Globally

Some common composite operators

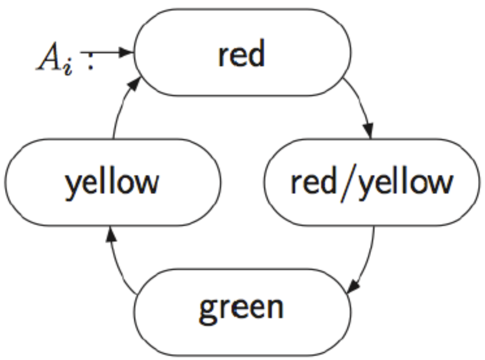
- $p \rightarrow \Diamond q$ — p implies eventually q (response)
- $p \rightarrow q\ U\ r$ — p implies q until r (precedence)
- $\Box\Diamond p$ — always eventually p (progress)
- $\Diamond\Box p$ — eventually always p (stability)
- $\Diamond p \rightarrow \Diamond q$ — eventually p implies eventually q (correlation)

Operator precedence

- Unary binds stronger than binary
- Bind from right to left:
  $\Box\Diamond p = (\Box\ (\Diamond p))$
  $p\ U\ q\ U\ r = p\ U\ (q\ U\ r)$
- U takes precedence over $\wedge$, $\vee$ and $\rightarrow$

System description

- Focus on lights in on particular direction
- Light can be any of three colors: green, yellow, red
- Atomic propositions = light color



Ordering specifications

- Liveness: “traffic light is green infinitely often”

$$\Box\Diamond \text{green}$$

- Chronological ordering: “once red, the light cannot become green immediately”

$$\Box\,(\text{red} \rightarrow \neg \bigcirc \text{green})$$

- More detailed: “once red, the light always becomes green eventually after being yellow for some time”

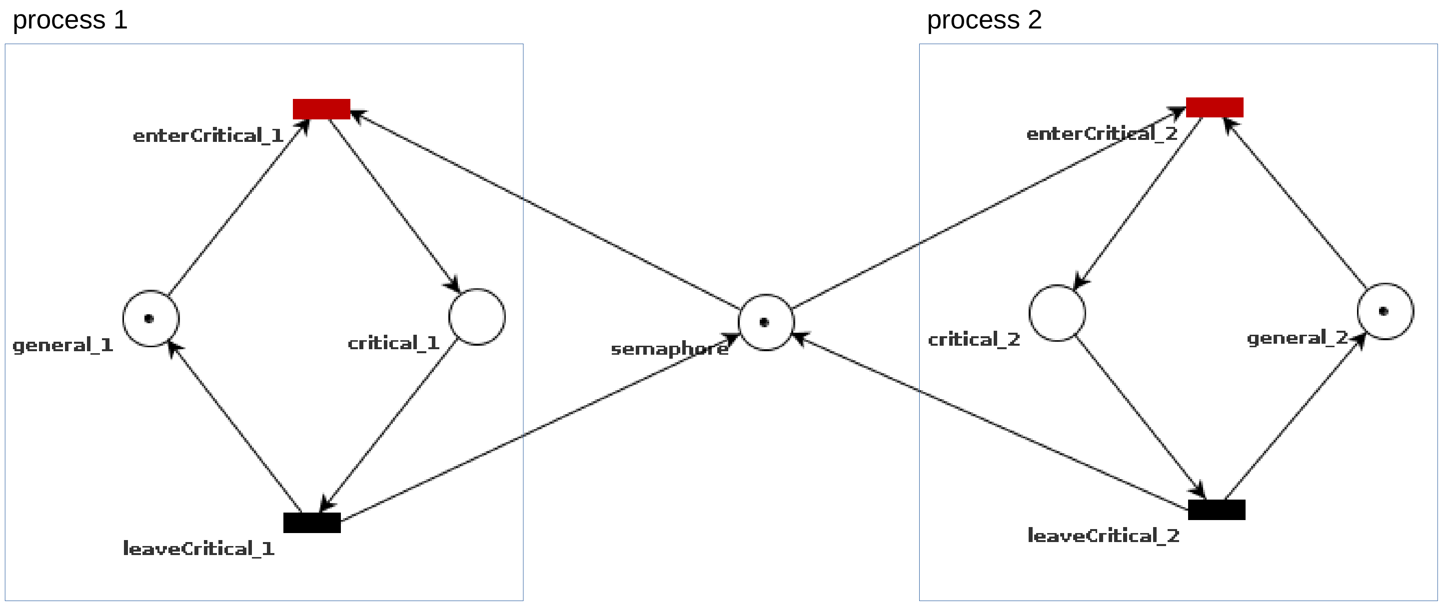
$$\Box(\text{red} \rightarrow (\Diamond\, \text{green} \wedge (\neg\, \text{green}\ U\ \text{yellow})))$$

$$\Box(\text{red} \rightarrow \bigcirc\,(\text{red}\ U\ (\text{yellow} \wedge \bigcirc\,(\text{yellow}\ U\ \text{green}))))$$

Progress property

- Every request will eventually lead to a response

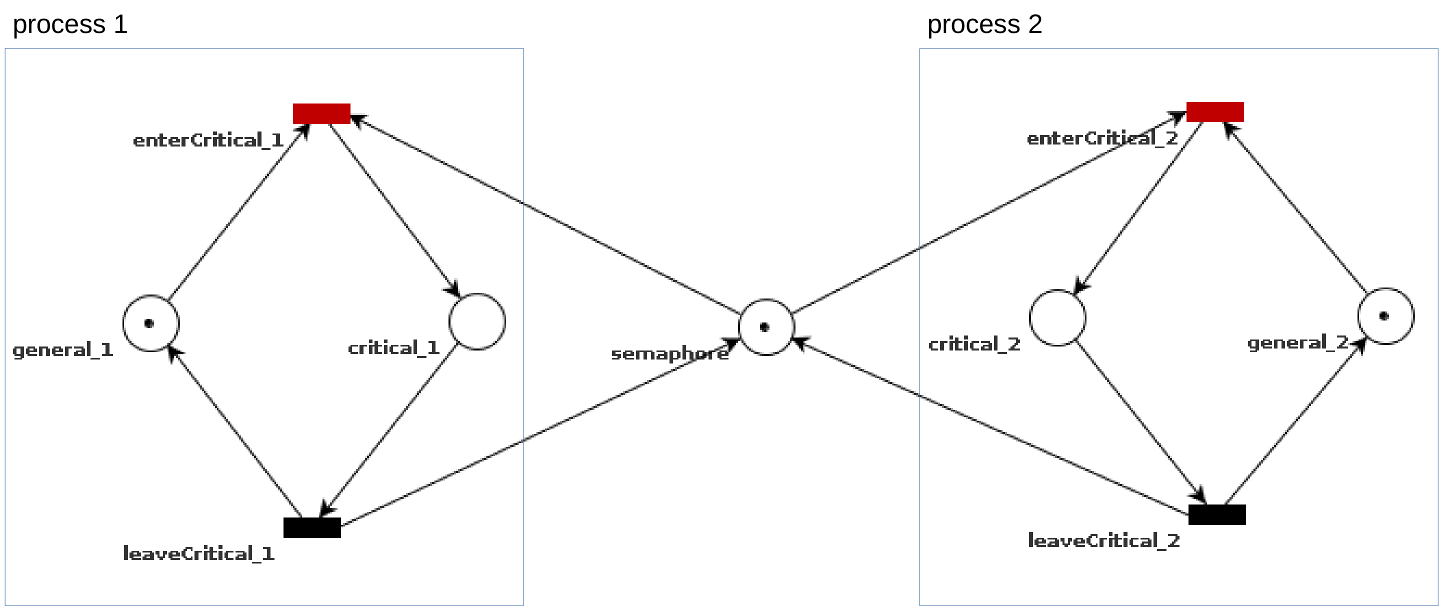
$$\Box\,(\text{request} \rightarrow \Diamond \text{response})$$



Property: Process 1 and process 1 are never both in their critical sections at the same time (mutual exclusion)

$$\Box\ \neg\big((x(critical\_1) == 1) \wedge (x(critical\_2) == 1) \big)$$

$$\Box\ \big(\neg(x(critical\_1) == 1) \vee \neg(x(critical\_2) == 1) \big)$$



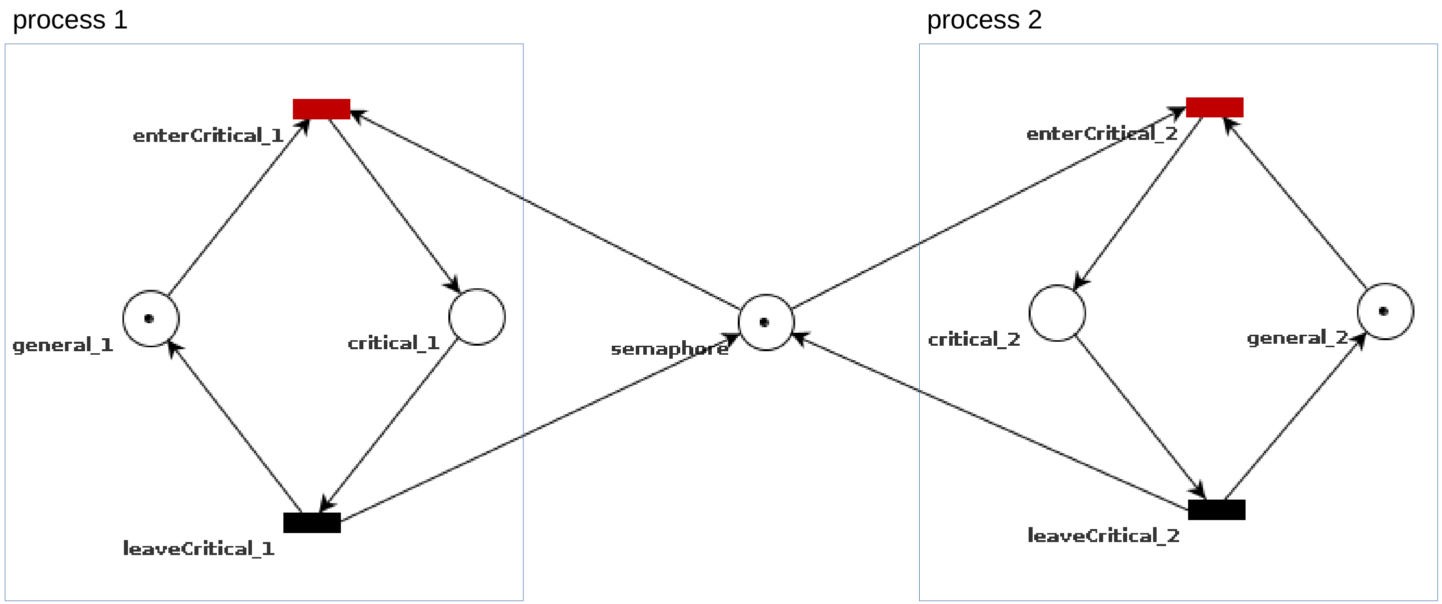
Property: Process 1 and process 1 are never both in their critical sections at the same time (mutual exclusion)

$$\Box \neg \left((x(critical\_1) == 1) \wedge (x(critical\_2) == 1) \right)$$

$$\Box \left(\neg (x(critical\_1) == 1) \vee \neg (x(critical\_2) == 1) \right)$$

forall paths (CTL)

$$\forall \; \Box \neg \left((x(critical\_1) == 1) \wedge (x(critical\_2) == 1) \right)$$



Property: Neither process monopolizes the critical section (fairness)

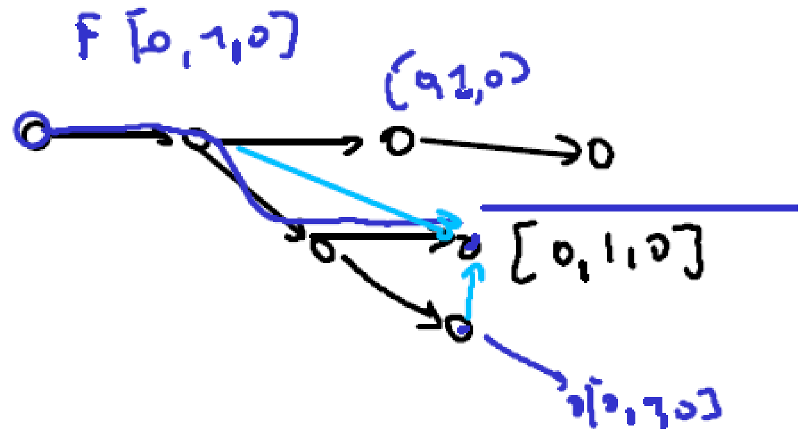
$$\Box\left((x(critical\_1) == 1) \longrightarrow \Diamond (x(critical\_2) == 1) \right)$$

$$\Box\left((x(critical\_1) == 1) \longrightarrow \Diamond \neg (x(critical\_1) == 1) \right)$$

$$\Box\left((x(critical\_2) == 1) \longrightarrow \Diamond (x(critical\_1) == 1) \right)$$

Computational **T**ree **L**ogic (CTL)

non-deterministic behaviour
leads to branches
in the behaviour trace



forall paths
(universal quantification)

∀ PATHS A F [0,1,0]

there exists a path
(existential quantification)

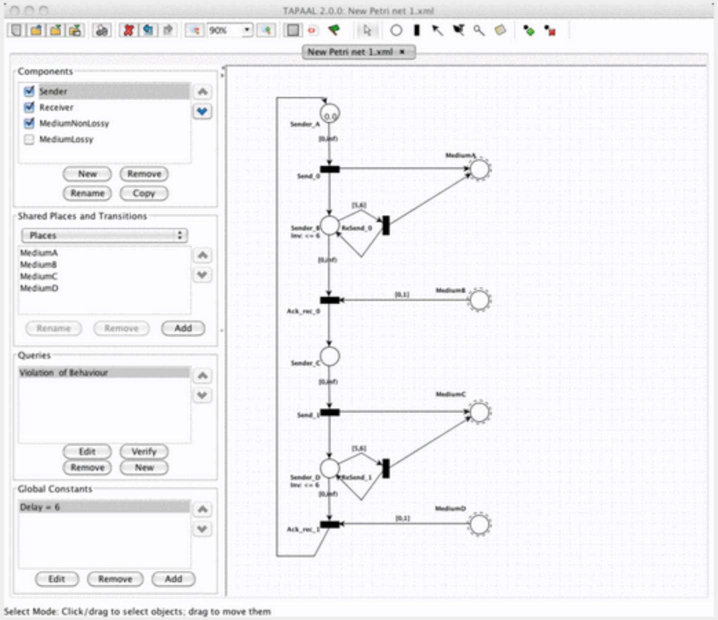
∃ A PATH E F [0,1,0]

3.6 TAPAAL

TAPAAL: Tool for Verification of Timed-Arc Petri Nets

TAPAAL is a tool for

- modelling, simulation and verification of
- **T**imed-**A**rc **P**etri nets
- developed at Department of Computer Science at **AAL**borg University in Denmark
- and available for Linux, Windows and Mac OS X platforms.



Timed-Arc Petri Net (TAPN) is a time extension of the classical Petri net model (a commonly used graphical model of distributed computations introduced by Carl Adam Petri in his disseration in 1962). The time extension we consider allows for explicit modelling of real-time, which is associated with the tokens in the net (each tokens has its own age) and arcs from places to transitions are labelled by time intervals that restrict the age of tokens that can be used in order to fire the respective transition. In TAPAAL tool a furter extension of this model with age invariants, urgent transitions, transport arcs (which are more expressive than for example previously considered read-arcs) and with inhibitor arcs is implemented.

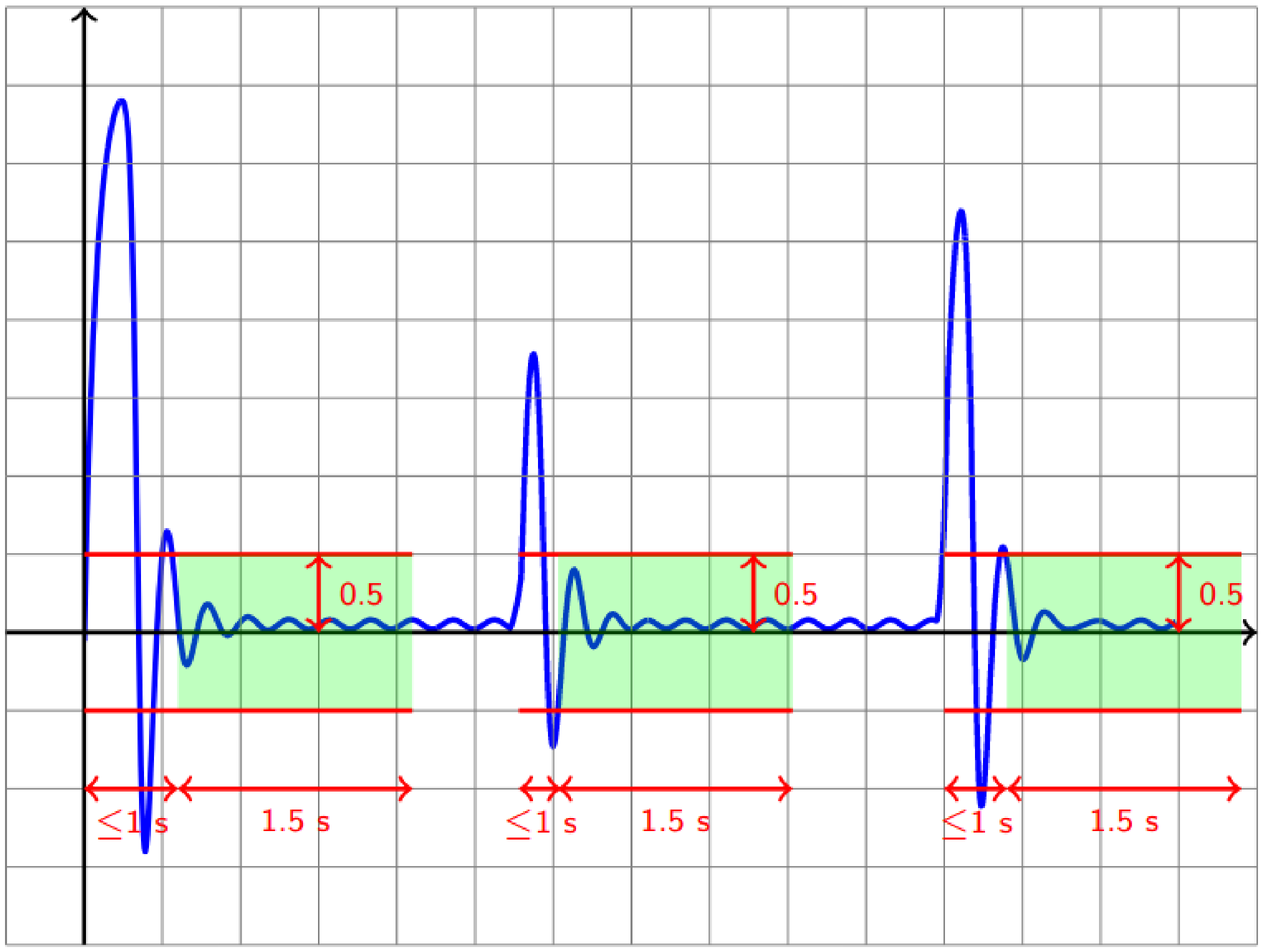
The TAPAAL tool offers a graphical editor for drawing TAPN models, simulator for experimenting with the designed nets and a verification environment that automatically answers logical queries formulated in a subset of CTL logic (essentially EF, EG, AF, AG formulae without nesting). It also allows the user to check whether a given net is k-bounded for a given number k. The newest version of TAPAAL is now equipped with three open source verification engines distributed together with TAPAAL (for continuous time semantics, discrete time semantics and a new efficient engine for the verification of untimed nets supporting both CTL and LTL logics.). It is also possible to model two-player games, both with and without time features. Optionally, the user can automatically translate TAPAAL models into UPPAAL and rely on the UPPAAL verification engine.

https://www.tapaal.net/

Signal Temporal Logic (STL)

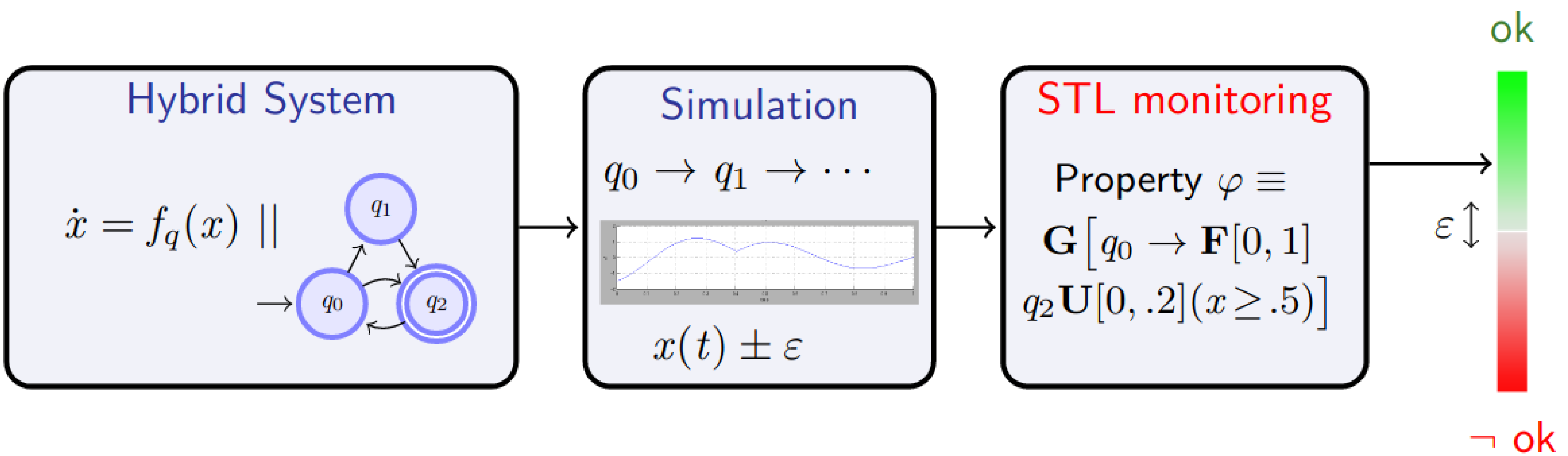
Always $|x| > 0.5 \Rightarrow$ *after 1 s,* $|x|$ *settles under 0.5 for 1.5 s*

$$\varphi := \mathsf{G}(x[t] > .5 \rightarrow \mathsf{F}_{[0,.6]} \; (\mathsf{G}_{[0,1.5]} \; x[t] < 0.5))$$



From "On Signal Temporal Logic" lecture by Alexandre Donzé. EECS114@UCB. 2013

Signal Temporal Logic (STL) for **run-time monitoring** of Hybrid Systems



From "On Signal Temporal Logic" lecture by Alexandre Donzé. EECS114@UCB. 2013

Tools:

- breach (Matlab toolbox)

  `https://github.com/decyphir/breach`

- RTAMT

  `https://github.com/nickovic/rtamt`

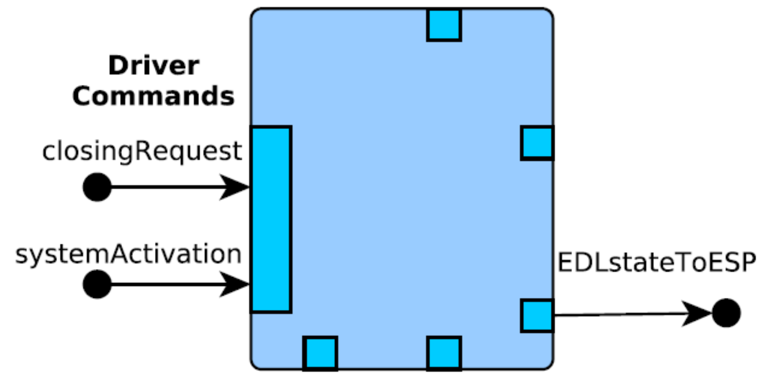
automated and Simulation based functional safety Engineering meThodology (aSET)

System Contract

```
Contract TR57{
        longname "Logic vehicle dynamics EDL state"
        [...]
        scope Globally
        pattern ResponsePattern:
            if receiveDataCAN has-occurred,
                then-in-response vehicle_EDL_State
                eventually-occurs within 50ms
        generate-STL
    }
```

+

System Architecture



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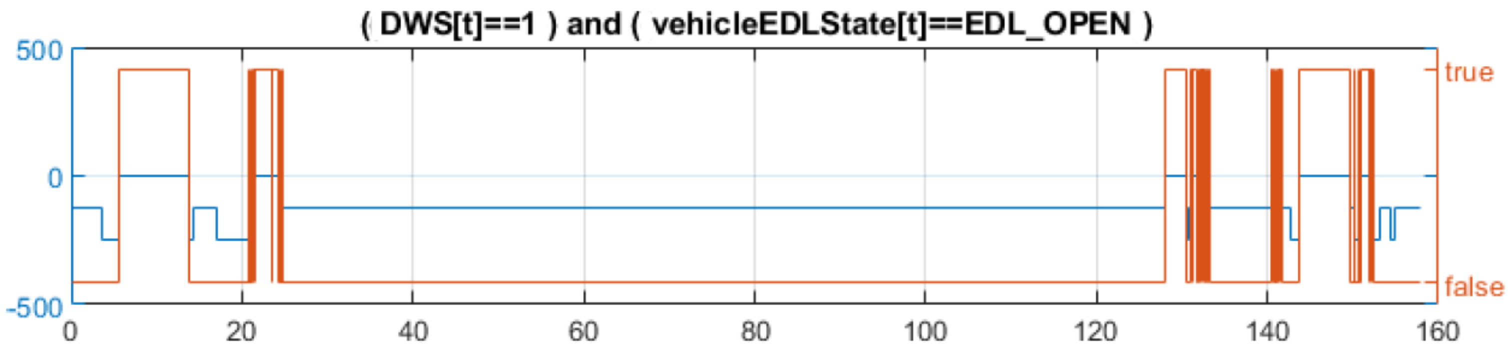
Simulation and verification of contract



Indicates whether (and when) contract is satisfied by the trace

DSL for contract specification

```
Contract FR07{
        longname "Response to driver locking command"
        description "The system must close the EDL after
                    receiving a locking command from the driver."

        statements{
            Event driver_lock :=
                Port driverCommands_closingRequest == True,
            Property close_EDL :=
                EDL_STATE in Set{State EDLphysicalSyst_CloseEDL}
        }
        scope Globally
        pattern ResponsePattern:
            if driver_lock has-occurred,then-in-response
                close_EDL eventually-occurs
        generate-STL
    }
```

Matthias Bernaerts, Bentley J. Oakes, Ken Vanherpen, Bjorn Aelvoet, Hans Vangheluwe, and Joachim Denil.
Validating industrial requirements with a contract-based approach.
Proceedings of the ACM/IEEE 22nd International Conference on Model Driven Engineering, Languages and Systems.
Companion, pages 18 – 27. IEEE, September 2019.