Causal Block Diagrams (CBDs)
a family of formalisms

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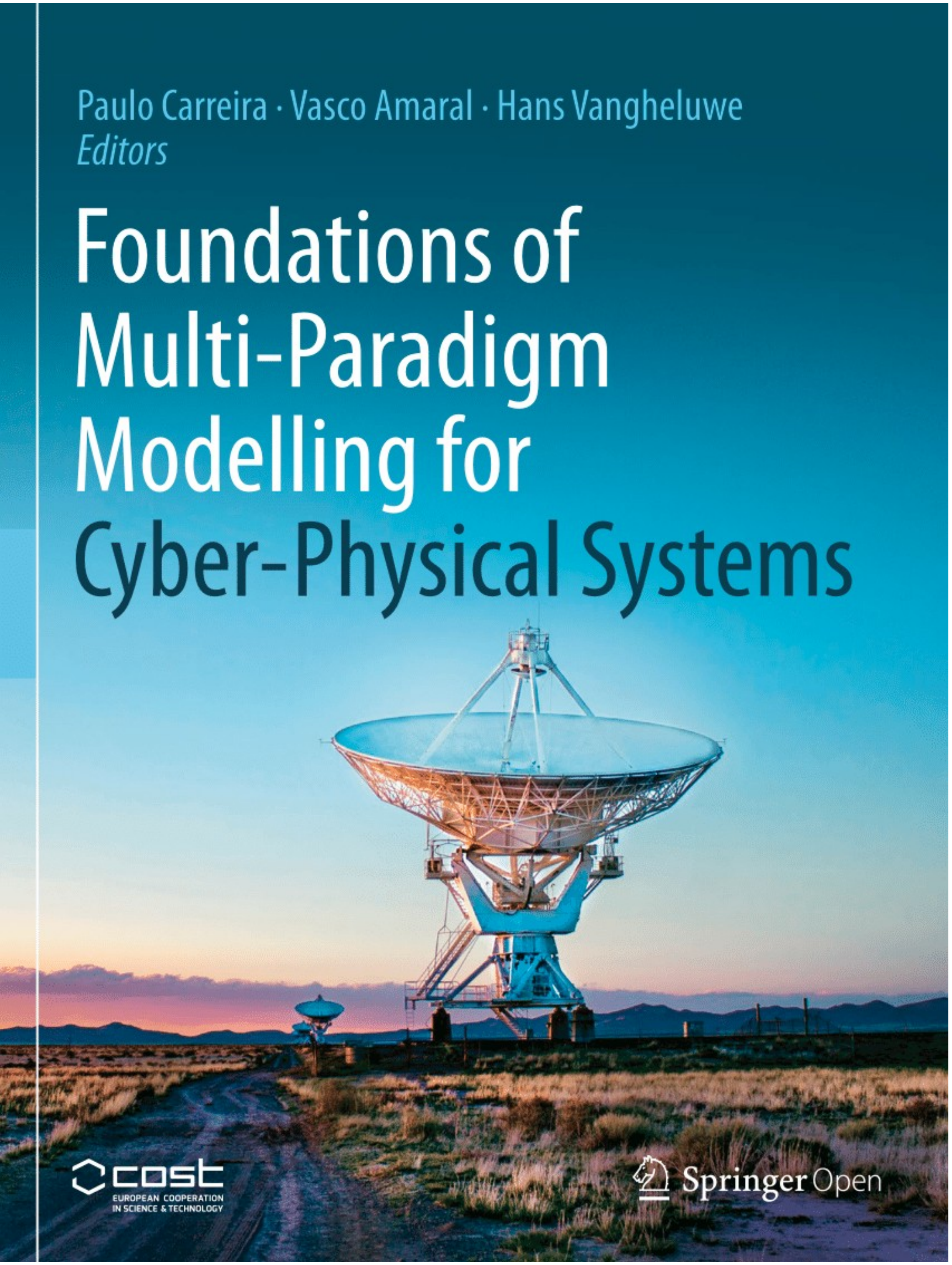
FLANDERS MAKE
MANUFACTURING INNOVATION NETWORK
MPM4CPS
cost

Physical Systems Modelling

- Problem-Specific (technological)
- Domain-Specific (e.g., translational mechanical)
- (general) Laws of Physics
- Power Flow/Bond Graphs (physical: energy/power)
- Computationally a-causal
  (Mathematical and Object-Oriented) ← **Modelica**
- Causal Block Diagrams (data flow)
- Numerical (Discrete) Approximations
- Computer Algorithmic + Numerical
  (Floating Point vs. Fixed Point)
- As-Fast-As-Possible vs. Real-time (XiL)
- Hybrid (discrete-continuous) modelling/simulation
- Hiding IP: Composition of Functional Mockup Units (FMI)
- Dynamic Structure



Gomes C., Denil J., Vangheluwe H. (2020) Causal-Block Diagrams: A Family of Languages for Causal Modelling of Cyber-Physical Systems.
In: Carreira P., Amaral V., Vangheluwe H. (eds) Foundations of Multi-Paradigm Modelling for Cyber-Physical Systems. Springer, Cham.
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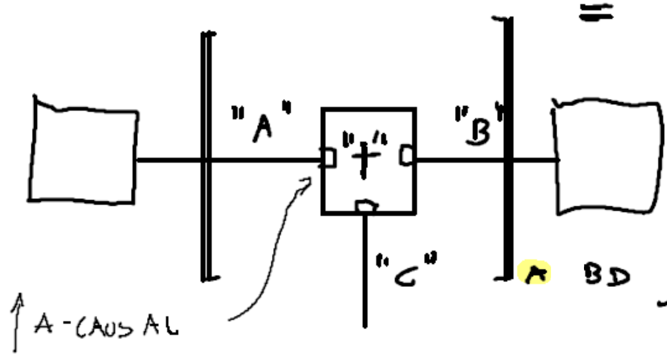
CBD CAUSAL BLOCK DIAGRAM "COMPUTATIONAL CAUSALITY"

ABD A-CAUSAL BLOCK DIAGRAM CAUSE ⟶ CONSEQUENCE



=

$mvA + mvB + mvC = \phi$

$(mvA, mvB, mvC) \in \mathbb{R}^3$

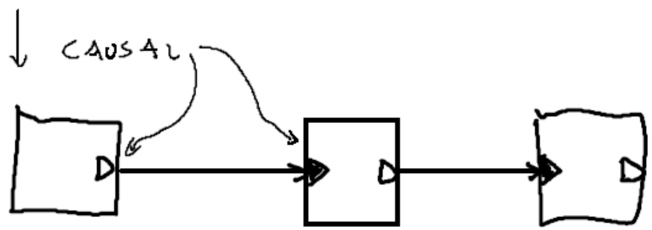
A-CAUSAL

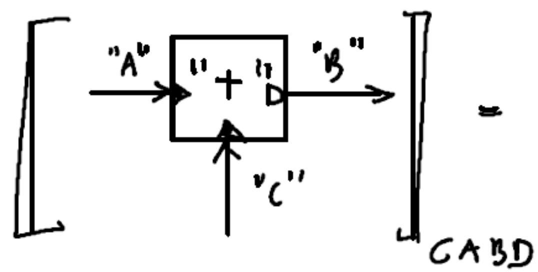
$+ : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$

MATH

"WHAT"

CAUSALITY ASSIGNMENT





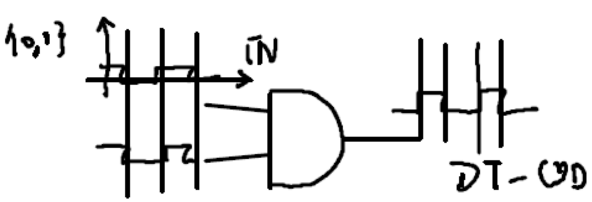
=

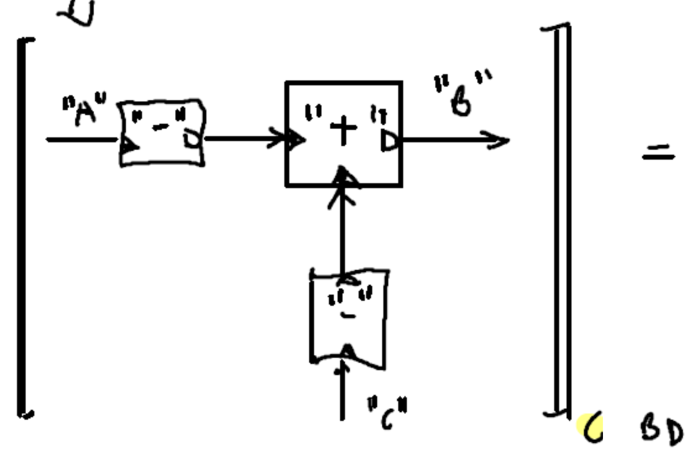
vA, vB, vC : float

vB := vA + vC

CODE

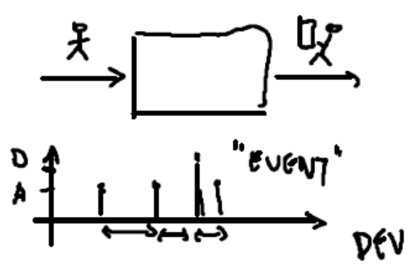
"HOW"



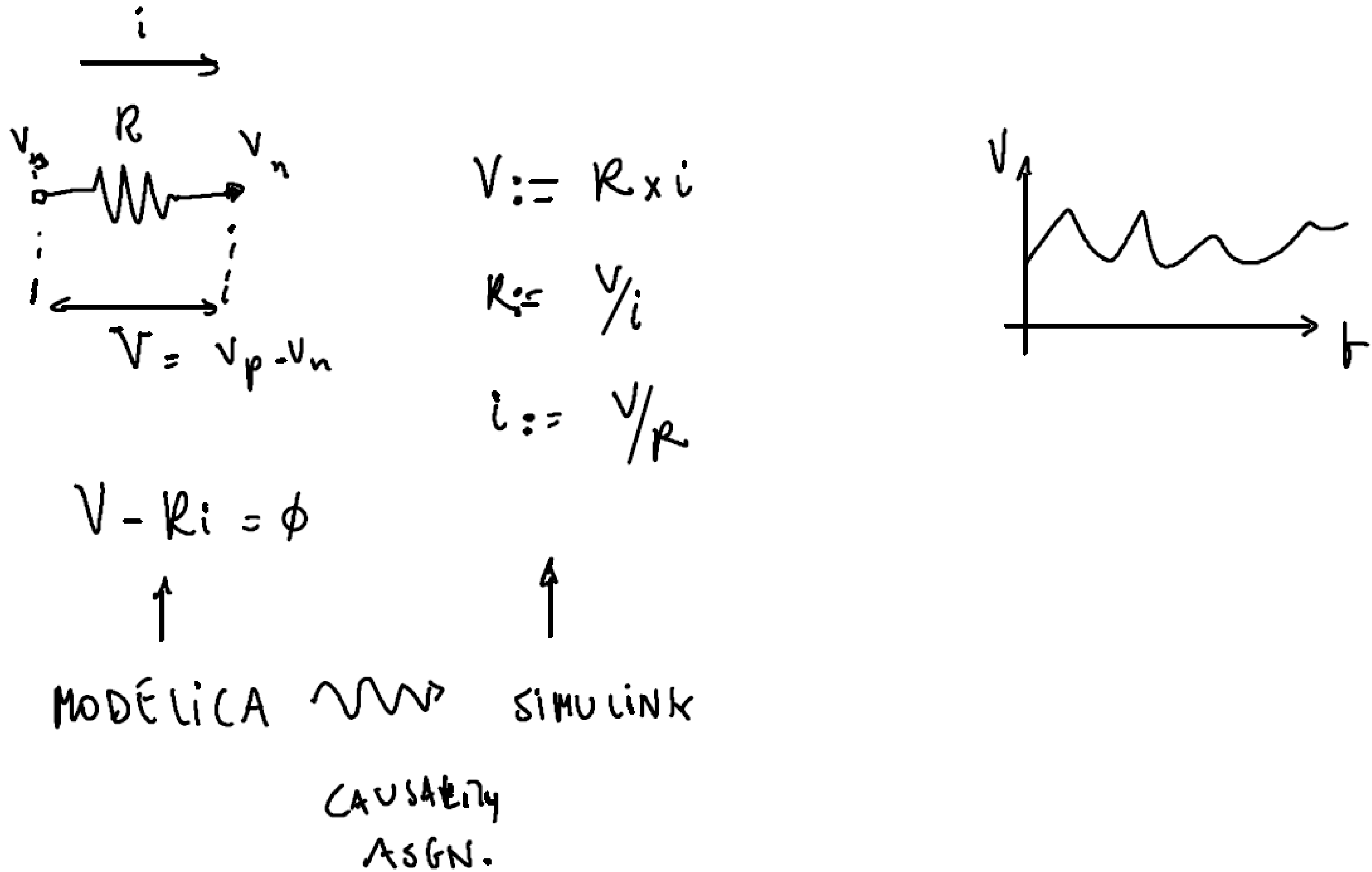


=

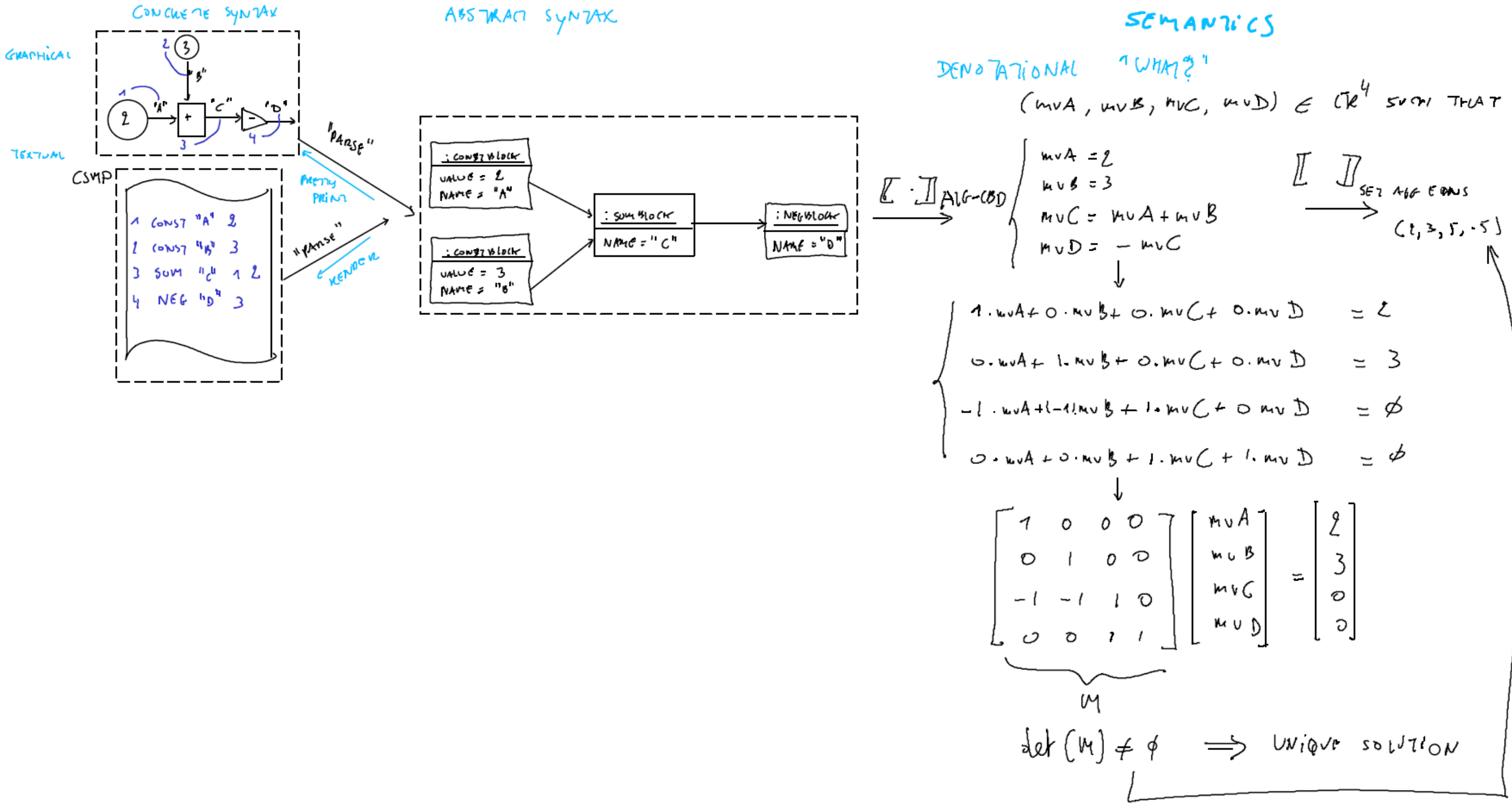
vB := - vA - vC



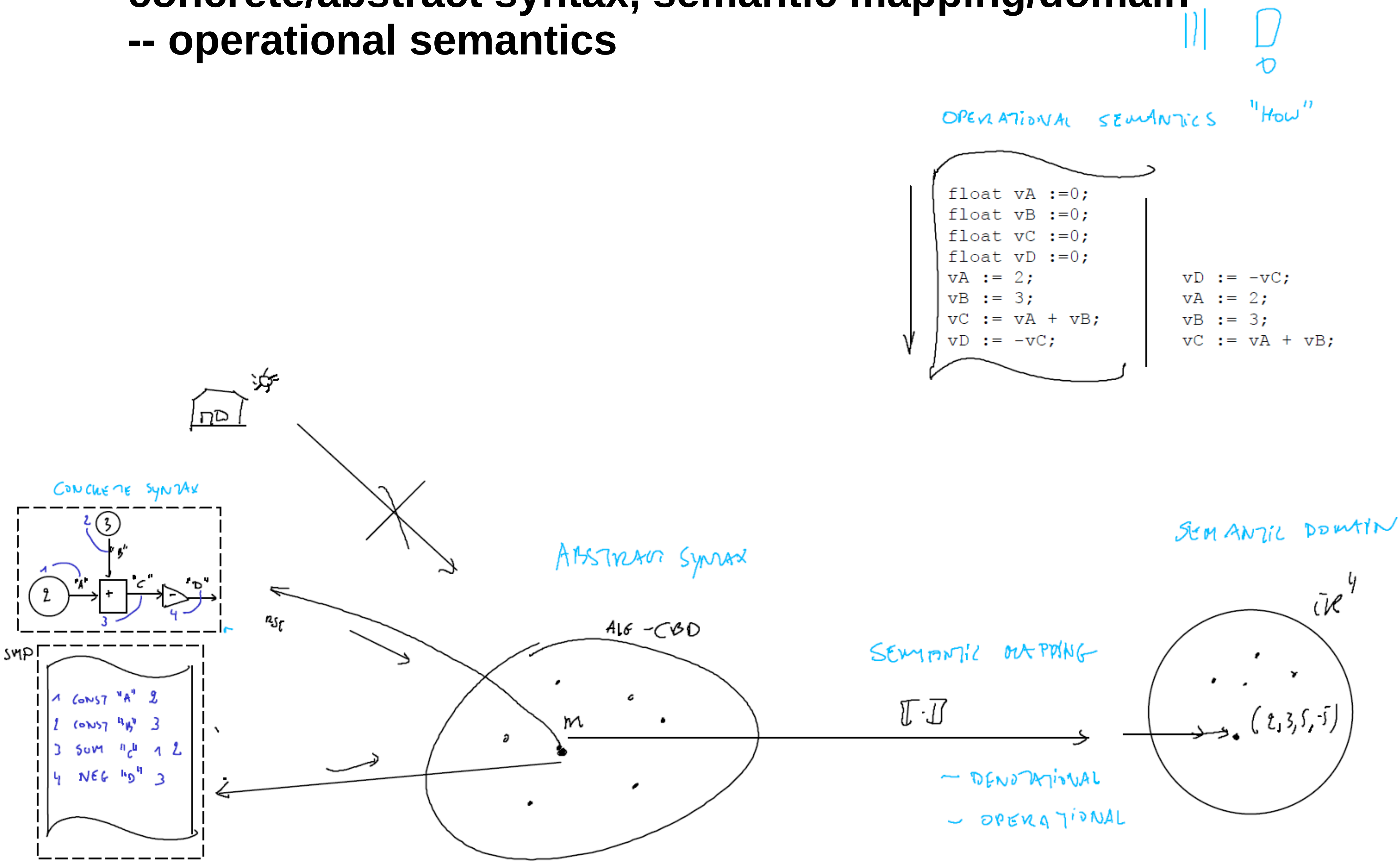
(computationally) a-causal vs. causal



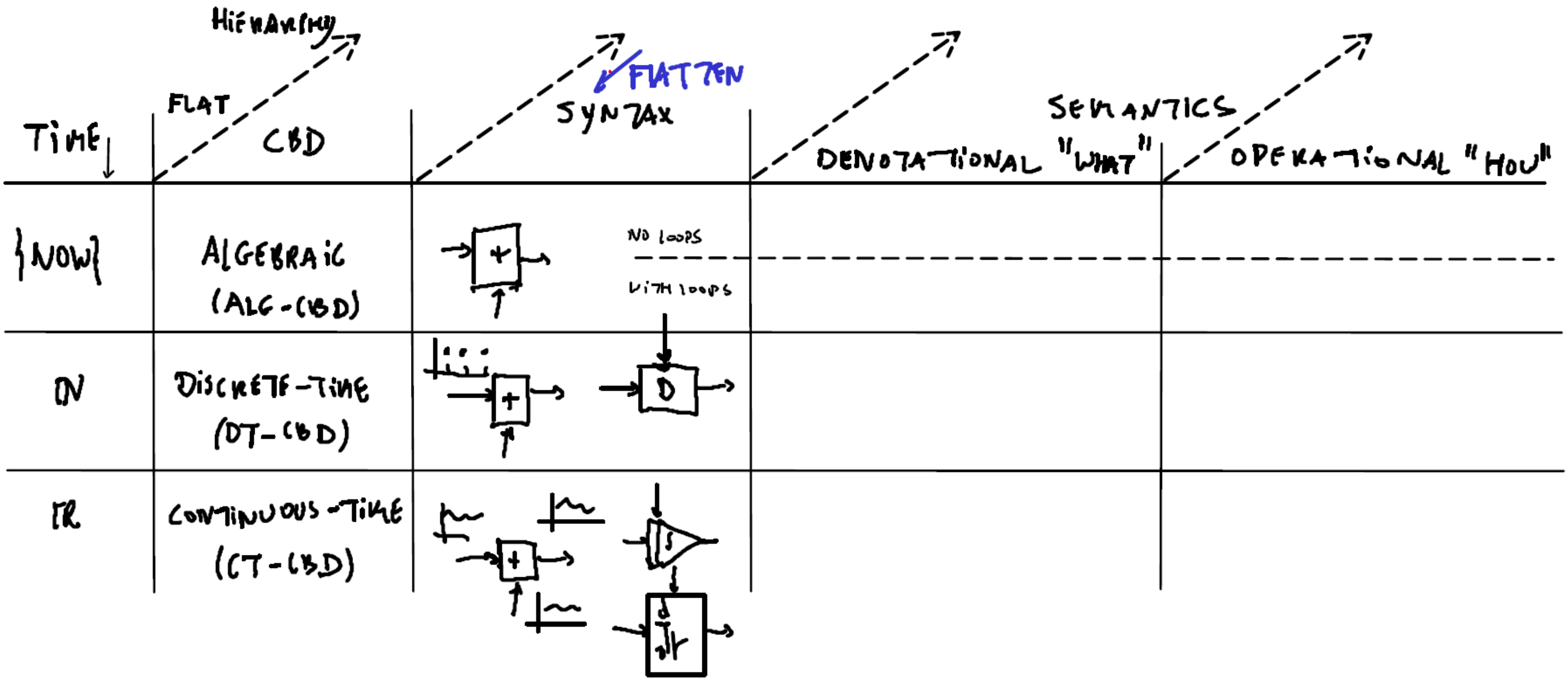
concrete/abstract syntax, semantic mapping/domain -- denotational semantics



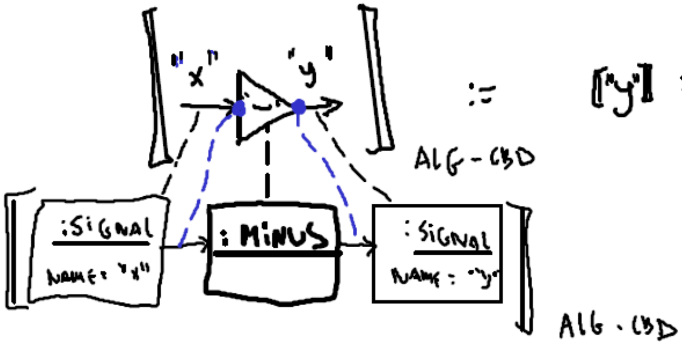
concrete/abstract syntax, semantic mapping/domain -- operational semantics



a family of Causal Block Diagram (CBD) formalisms



denotational semantics of individual blocks

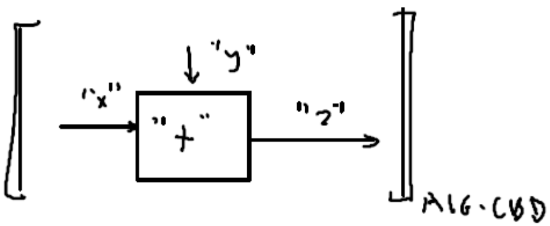


$:=$ $[\![\text{"y"}]\!] = -[\![\text{"x"}]\!]$

$mvy := -\, mvx$

$mvx, mvy \in \mathbb{R}$

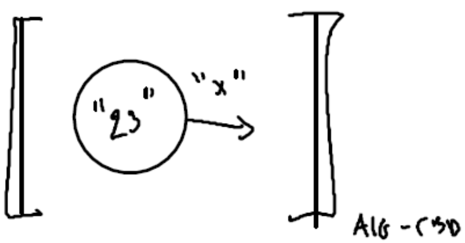
$- : \mathbb{R} \to \mathbb{R}$



$:=$ $[\![\text{"z"}]\!] = [\![\text{"x"}]\!] + [\![\text{"y"}]\!]$

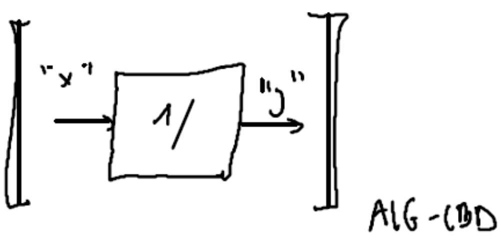
$mvz = mvx + mvy$ $\quad mvx, mvy, mvz \in \mathbb{R}$

$+ : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$



$:=$ $[\![\text{"x"}]\!] = [\![\text{"23"}]\!]$

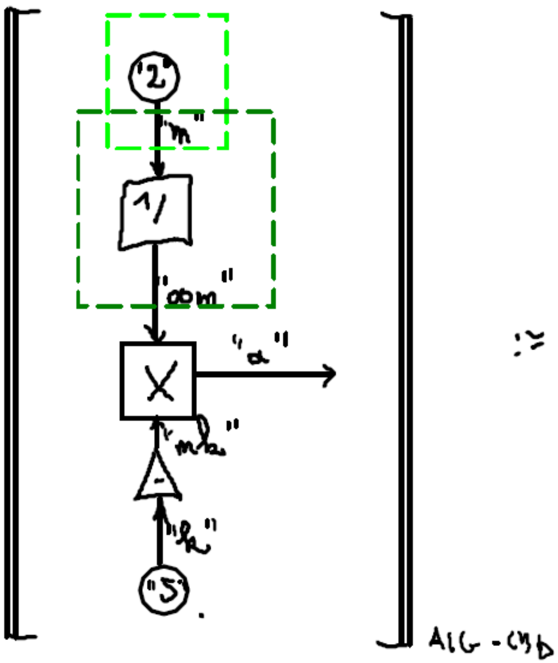
$mvx = 23$ $\quad mvx \in \mathbb{R}$



$:=$ $[\![\text{"y"}]\!] = 1/[\![\text{"x"}]\!]$

$mvy = 1/mvx$ $\quad mvx \in \mathbb{R}\setminus\{0\},\ mvy \in \mathbb{R}$ $\quad mvx, mvy \in \mathbb{R} \cup \{except\}$

denotational semantics of “composition”



:=

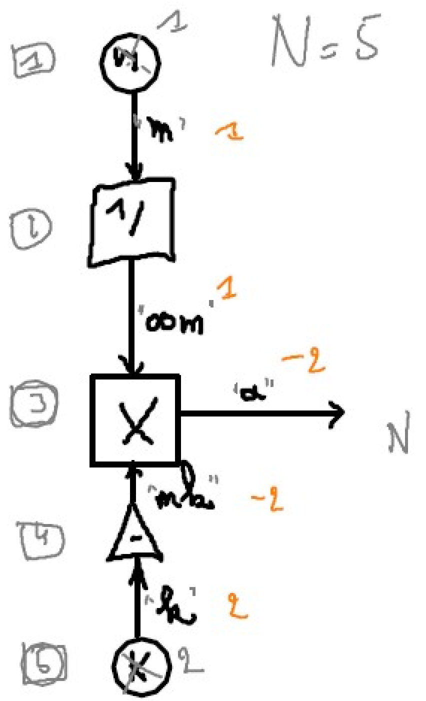
mvm, mvoom, mvm, mva, mvmk, mvk $\in \mathbb{R}$

SUCH THAT mvm $\neq \phi$

$$\begin{cases} mvm = 2 \\ mvoom = 1/mvm \\ mva = mvoom \times mvmk \\ mvmk = -mvk \\ mvk = 5 \end{cases}$$

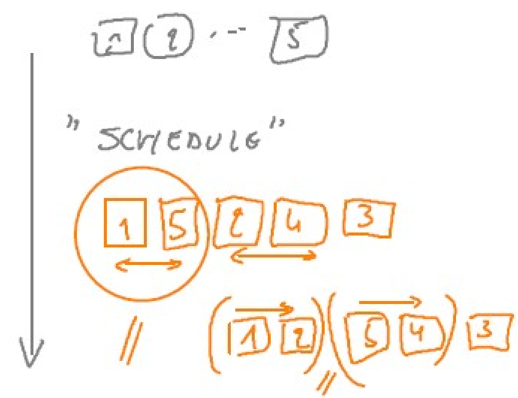
$= (2, 0.5, 2.5, -5, 5) \in \mathbb{R}^5$

operational semantics naive/optimal approach



N = 5

	vm	voom	vα	vmk	vk
	Uk	Uk	Uk	Uk	Uk
1	1	Uk	Uk	Uk	Uk
2	1	1	Uk	Uk	Uk
3	1	1	Uk	Uk	Uk
4	1	1	Uk	Uk	Uk
5	1	1	Uk	Uk	2
1	1	1	Uk	Uk	2
2	1	1	Uk	Uk	2
3	1	1	Uk	Uk	2
4	1	1	Uk	-2	2
5	1	1	Uk	-2	2
1	1	1	Uk	-2	2
2	1	1	Uk	-2	2
3	1	1	-2	-2	2
4					
5					



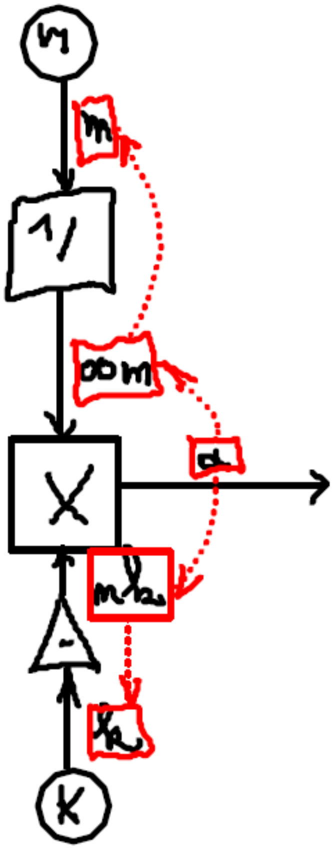
WORST-CASE

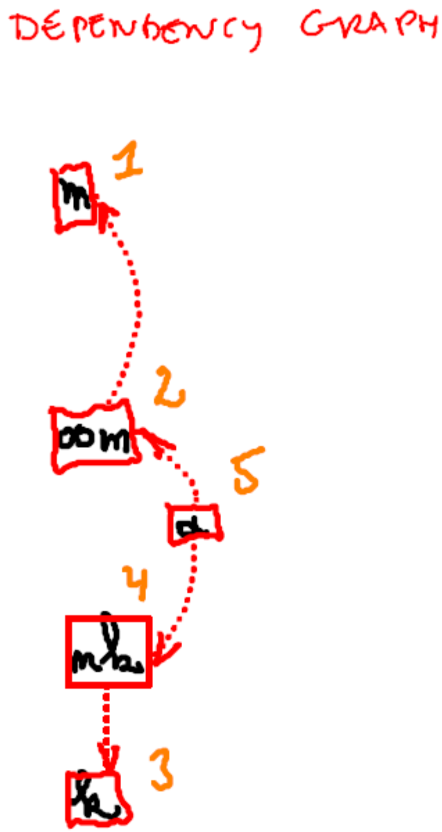
\# ITER $\mathcal{O}(N)$ $\mathcal{O}(1)$

\# COMP $\mathcal{O}(N^2)$ $\mathcal{O}(N)$

DISTRIBUTED

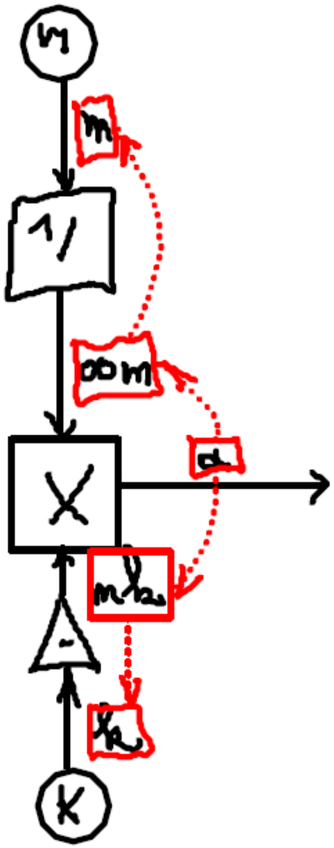
operational semantics
scheduling

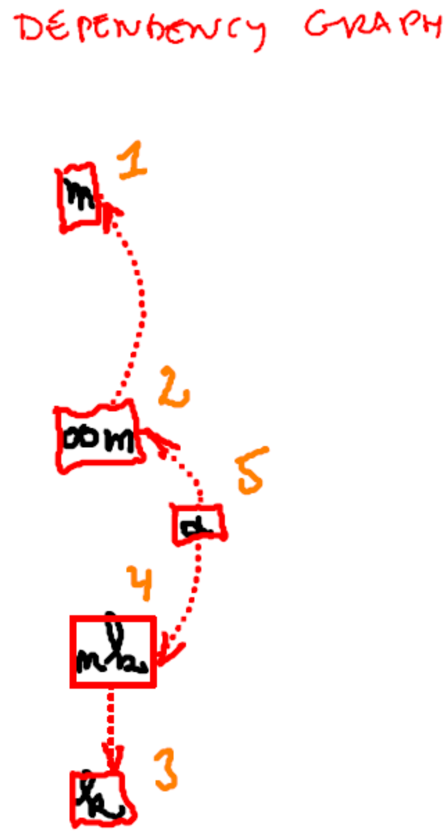




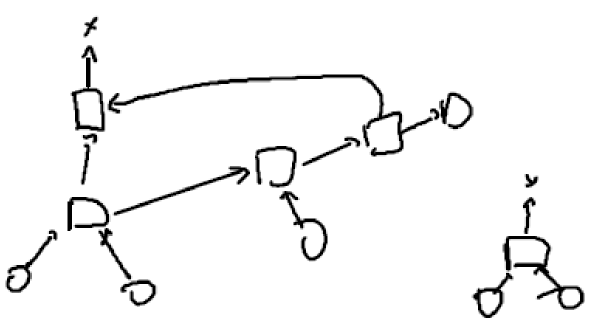
schedule = [m, oom, k, mk, a]

operational semantics scheduling





schedule = [m, oom, k, mk, a]



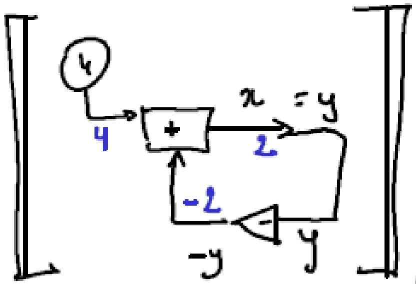
```
operational semantics

depGraph = buildDepGraph(CBD)
schedule = topologicalSort(depGraph)

for block in schedule:
  block.compute()
```

$\Theta(N)$

$\Theta(N)$

$\Theta(N)$

operational semantics
cyclic dependency
aka algebraic loop



ALG-CDD $:=$

$$\begin{cases} x + y = 4 \\ x - y = \emptyset \end{cases} \qquad \begin{aligned} x &= 4 - y \\ y &= x \end{aligned}$$

SET LINEAR EQNS.

2 UNKNOWNS

2 EQNS

$$\begin{cases} 1 \times x + 1 \times y = 4 \\ 1 \times x - 1 \times y = \emptyset \end{cases}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\det \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = -2 \neq \emptyset$$

$$x = \frac{\begin{vmatrix} 4 & 1 \\ \emptyset & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-4}{-2} = 2$$

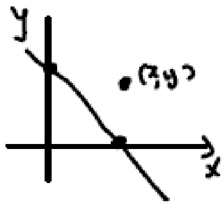
$$y = \frac{\begin{vmatrix} 1 & 4 \\ 1 & \emptyset \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-4}{-2} = 2$$

under-determined set of equations

→ **solution is sub-space**

$$\begin{cases} 2x + 2y = 4 \\ x + y = 2 \end{cases}$$

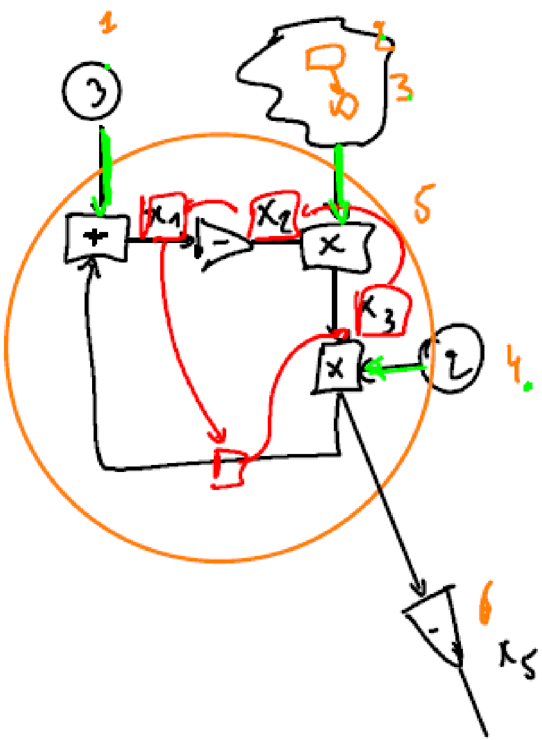
$$y = 2 - x$$



$$\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = \phi$$

“solving” a linear loop



$$\begin{cases} k_{11} x_1 + k_{21} x_2 + \dots + k_{n1} x_n = k_1 \\ k_{12} x_1 + k_{22} x_2 + \dots + k_{n2} x_n = k_2 \\ \vdots \\ k_{1n} x_1 + k_{2n} x_2 + \dots + k_{nn} x_n = k_n \end{cases}$$

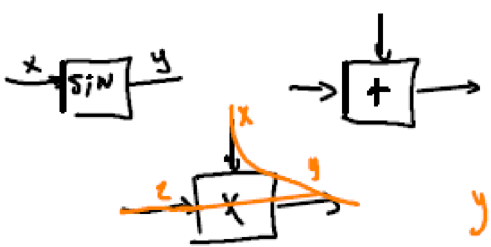
$$\underbrace{\begin{bmatrix} k_{11} & \cdots & \\ \vdots & & \vdots \\ & \cdots & k_{nn} \end{bmatrix}}_{K} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \underbrace{\begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}}_{k}$$

unique? $\det(K) \neq 0$

$(x_1, \dots, x_n)$

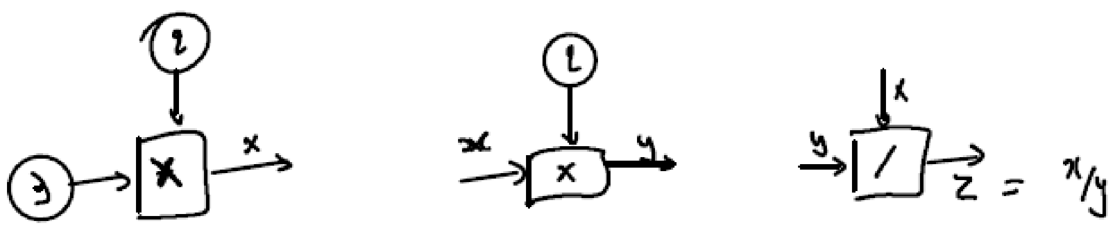
Gaussian elim $O(n^3)$

Blocks in "linear loop"

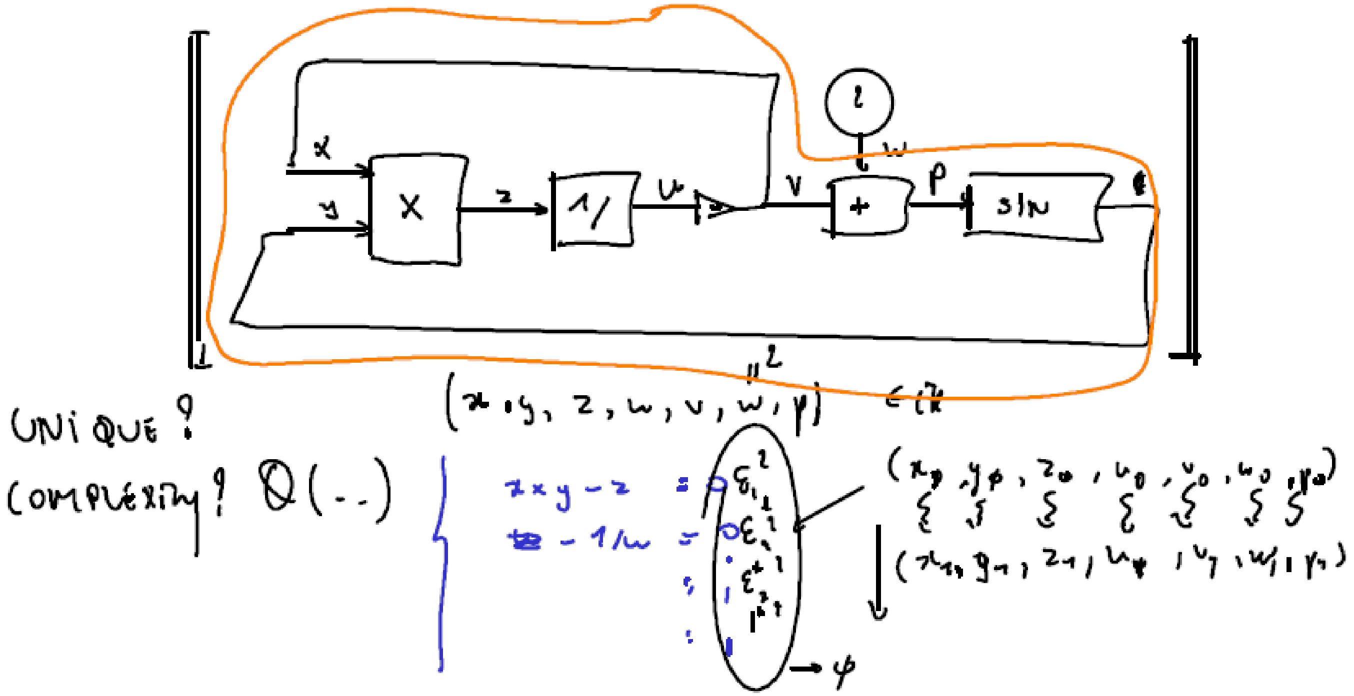


$y = x \times z$

$xy - x \times z = 0$

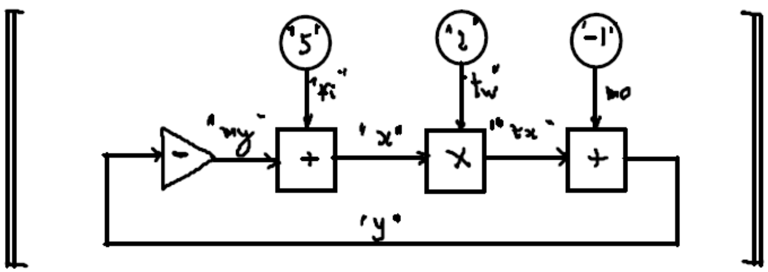


non-linear loop



how to find an algebraic loop?

"algebraic loop"



:=

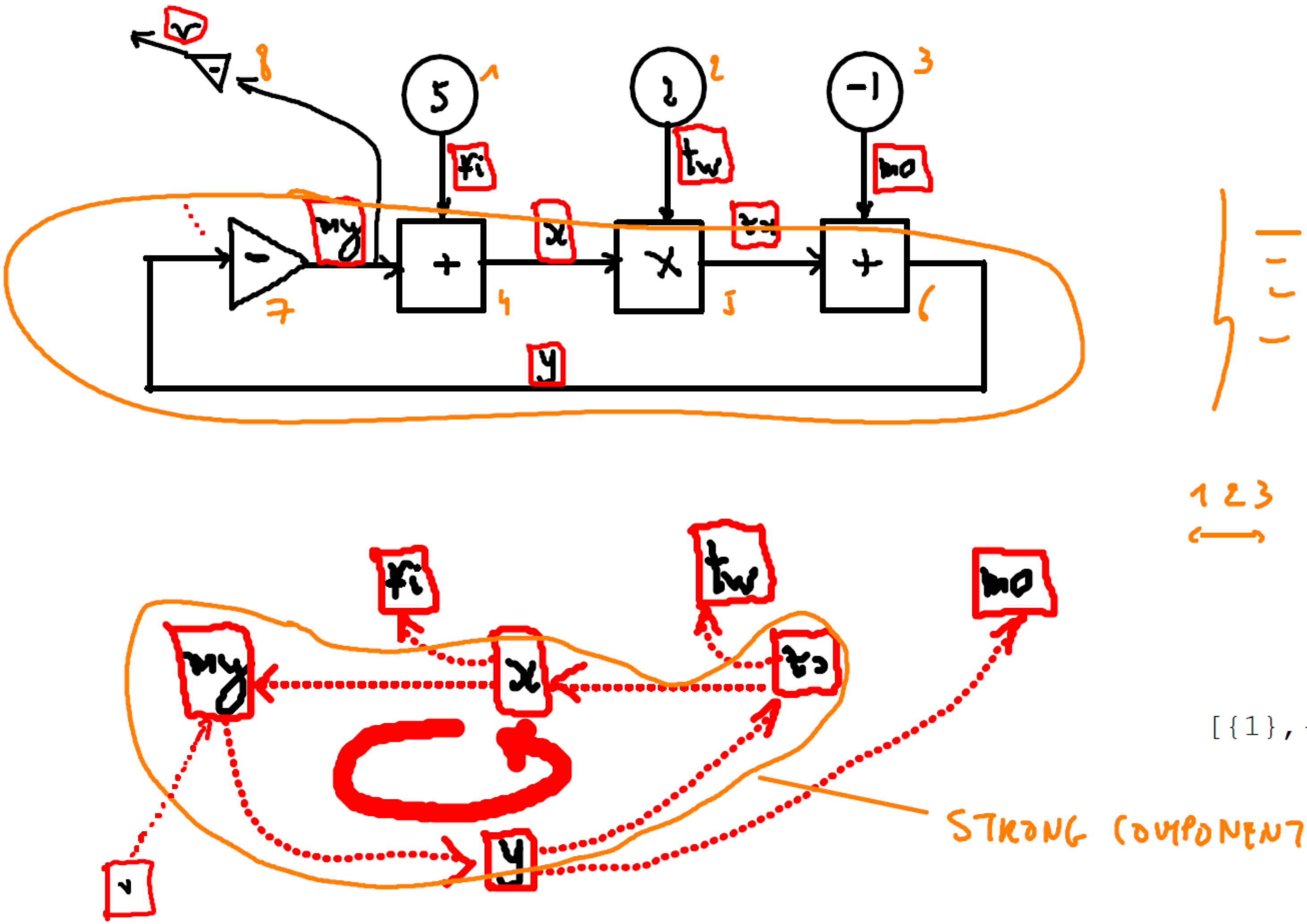
ALG-CBD

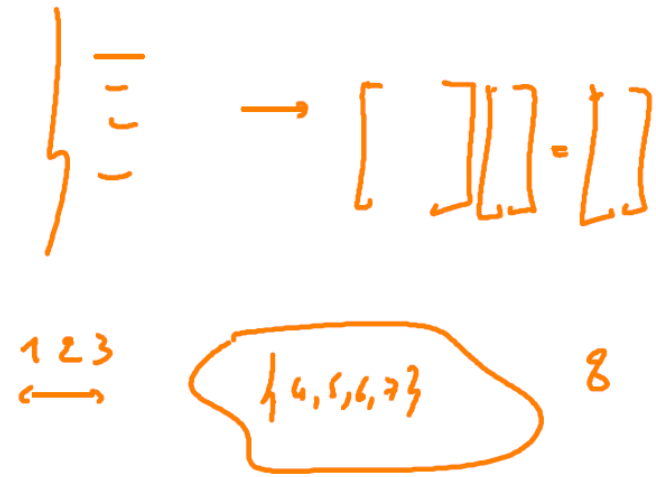
mvfi, mvx, mvtw, mvtx, mvmo, mvy, mvny $\in \mathbb{R}$

such that

$$
\begin{cases}
mvfi = 5 \\
mvx = mvfi + mvny \\
mvtw = 2 \\
mvtx = mvtw \times mvx \\
mvmo = -1 \\
mvy = mvmo + mvx \\
mvny = -mvy
\end{cases}
= (5, 2, 2, 4, -1, 3, -3) \in \mathbb{R}^7
$$

how to find an algebraic loop?





[{1},{2},{3}, {4,5,6,7}, {8}]

TARJAN 1974

O(N+E)

operational semantics

```
schedule = topologicalSortAndLoopDetect(depGraph(CBD))
```

$\Theta(N+E)$

```
for genBlock in schedule:
  genBlock.compute()
```

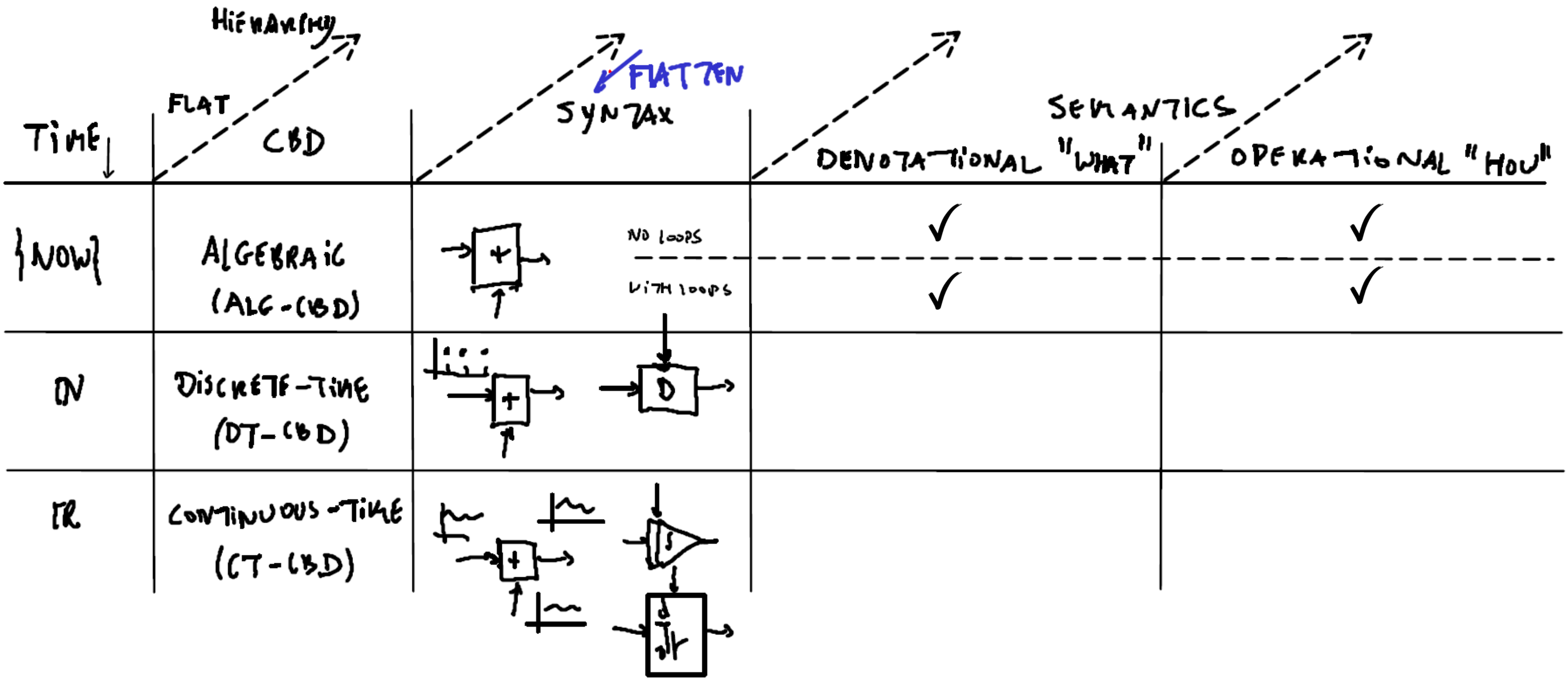
$\Theta(N)$?

some sets of coupled eqns (size m)

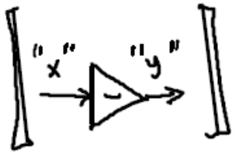
solver complexity $\leq \Theta(m^3)$

usually $m \ll N$

a family of Causal Block Diagram (CBD) formalisms



DISCRETE-TIME CBD (DT-CBD)



:= $[\![y]\!] = - [\![x]\!]$

$mvy(i) = -mvx(i), \ \forall i \in \mathbb{N}_0$

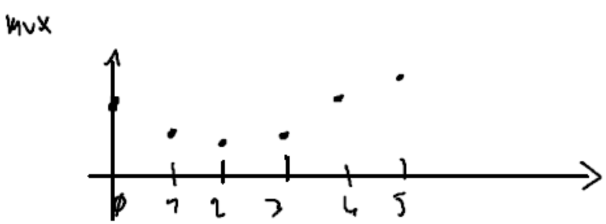
$mvx, mvy \in \mathbb{N}_0 \to \mathbb{R}$

$- : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$

x_i

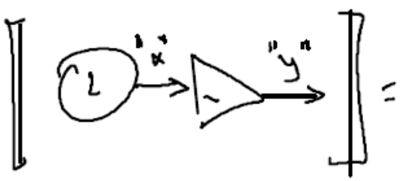
$x(i)$

$i \in \mathbb{N}_0$



$mvx = \{ 0 \to 10.3, \ 1 \to 5.5, \ 2 \to 3.33, \ \dots \}$

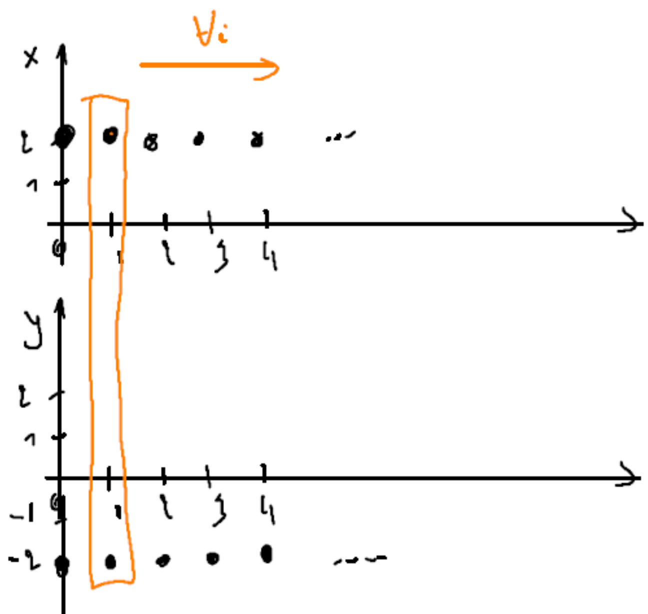
$[(0, 10.3), (1, 5.5), (2, 3.33), \dots]$

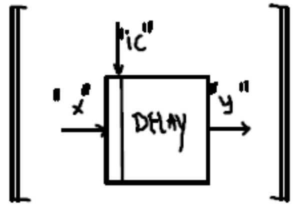


=

$x(i) = 2$

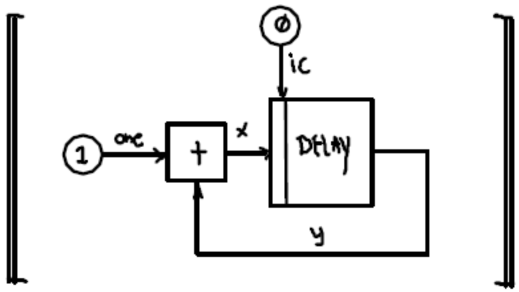
$y(i) = -x(i) \quad \forall i \in \mathbb{N}_0$





$$:= \begin{cases} v_y(i) = v_x(i-1), & \forall i \in \mathbb{N} \\ v_y(0) = v_{ic}(0) \end{cases}$$

$$v_{IC}, v_y, v_x, v_{one} \in \mathbb{N}_0 \to \mathbb{R}$$



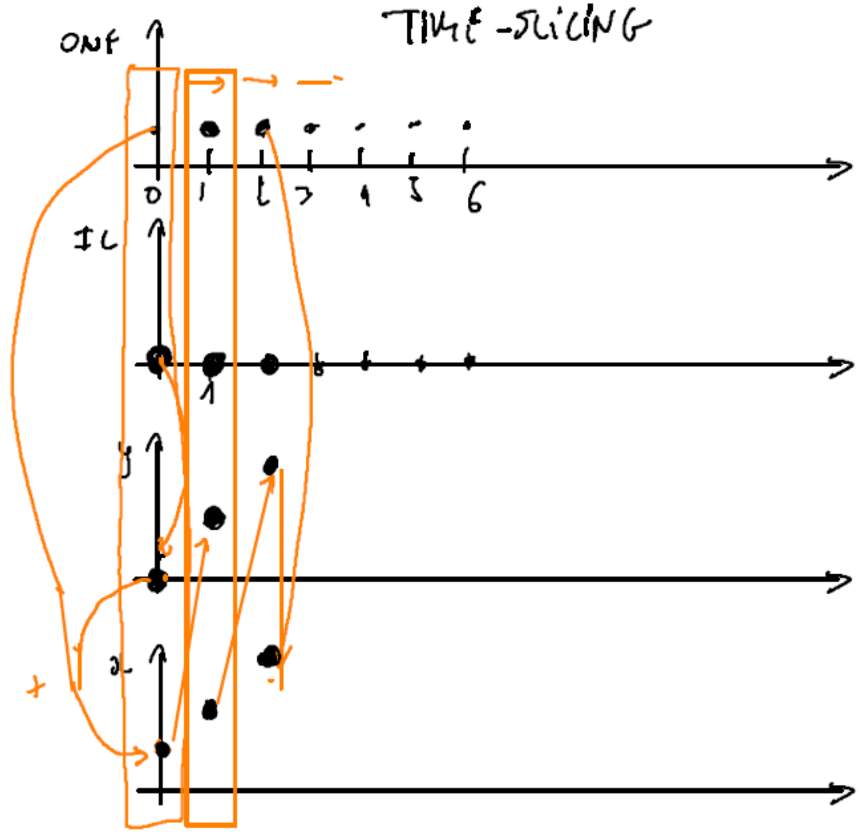
$$:= \begin{cases} v_{IC}(i) = 0, & \forall i \in \mathbb{N}_0 \\ v_y(i) = v_x(i-1), & \forall i \in \mathbb{N} \\ v_y(0) = ic(0) \leftarrow \\ v_x(i) = v_{one}(i) + v_y(i), & \forall i \in \mathbb{N}_0 \\ v_{one}(i) = 1, & \forall i \in \mathbb{N}_0 \end{cases}$$

$$= ([0, 0, 0, \ldots], [0, 1, 1, \ldots], [1, 2, 3, \ldots], [1, 1, 1, \ldots]) \in \mathbb{R}$$

↓ ALG

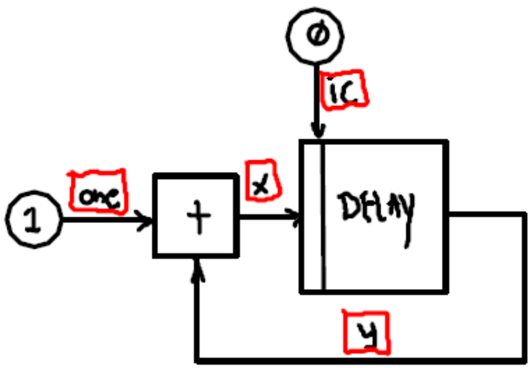
COUNTER

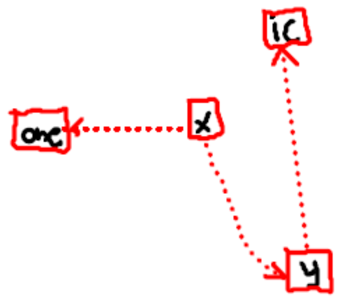
TIME-SLICING



operational semantics

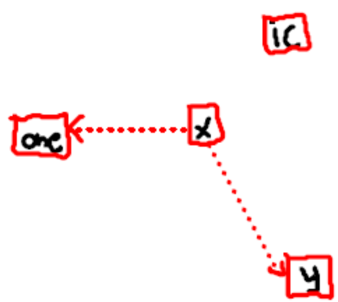
```
i = 0

while (not end_condition(i, ...)):

  depGraph = buildDepGraph(CBD)
  schedule = loopDetectAndTopSort(depGraph)

  for gblock in schedule:
    gblock.compute()

  i++
```





$i = \phi$

schedule = [one, IC, y, x]



$\forall i \in \mathbb{N}$

schedule = [y, one, x, IC]

operational semantics

```
i = 0

while (not end_condition(i, ...)):

  depGraph = buildDepGraph(CBD)
  schedule = loopDetectAndTopSort(depGraph)

  for gblock in schedule:
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  i++
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operational semantics

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  schedule = loopDetectAndTopSort(depGraph)

  for gblock in schedule:
    gblock.compute()

  i++
```
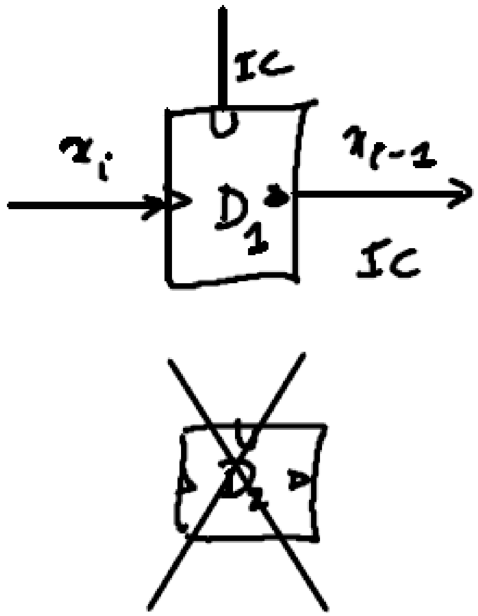
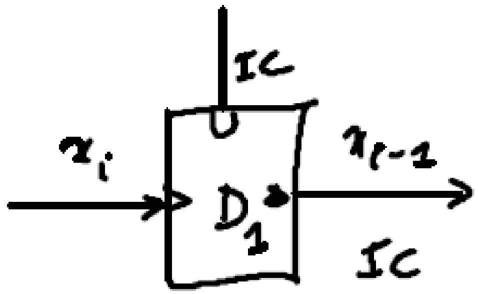
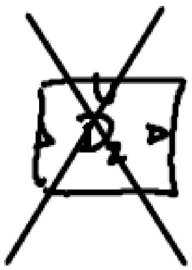

```
i = 0

if (not end_condition(i, ...)):
  depGraph = buildDepGraph(CBD)
  schedule = loopDetectAndTopSort(depGraph)

  for gblock in schedule:
    gblock.compute()
else:
  exit()

i = 1

while (not end_condition(i, ...)):

  depGraph = buildDepGraph(CBD)
  schedule = loopDetectAndTopSort(depGraph)

  for gblock in schedule:
    gblock.compute()

  i++
```

MEMORY OVER MORE THAN 1 TIME-SLICE



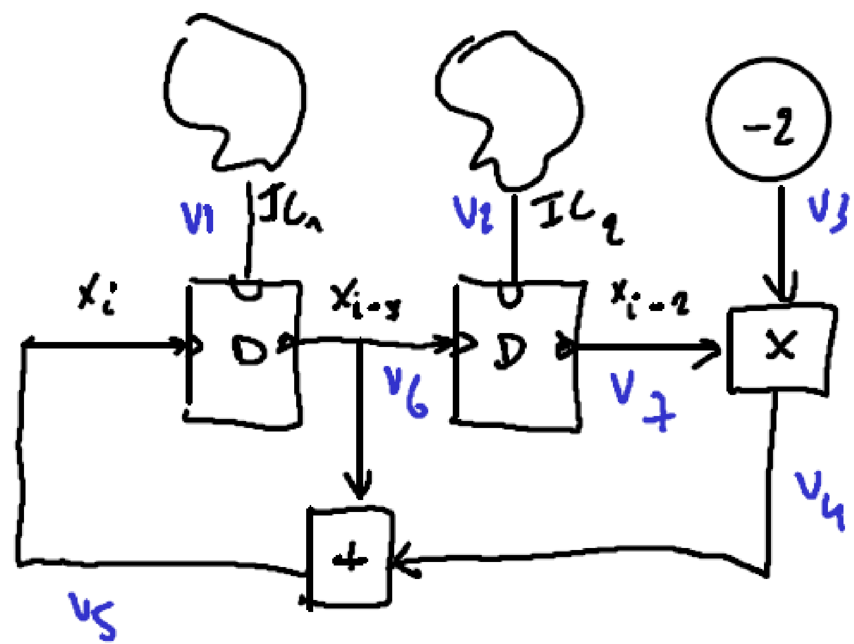
Memory over more than 1 time-slice



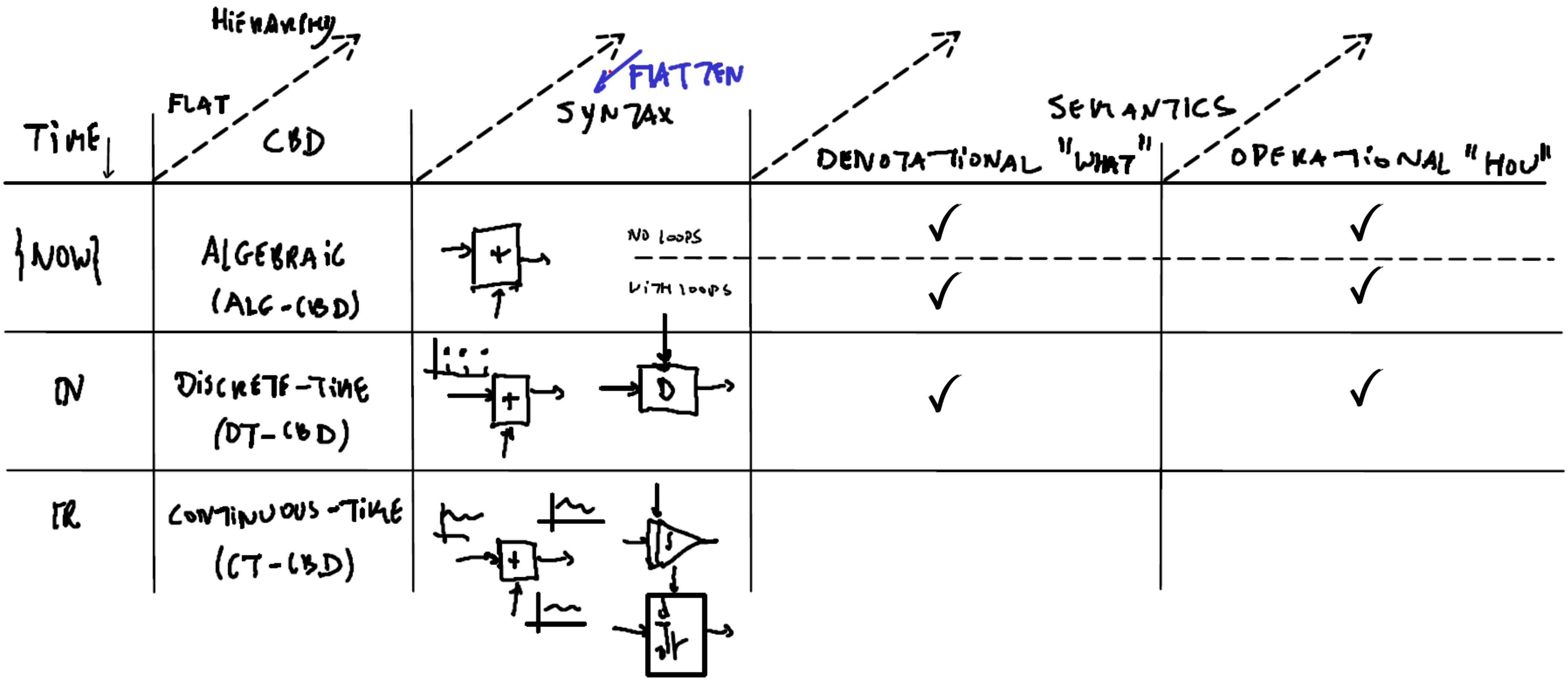


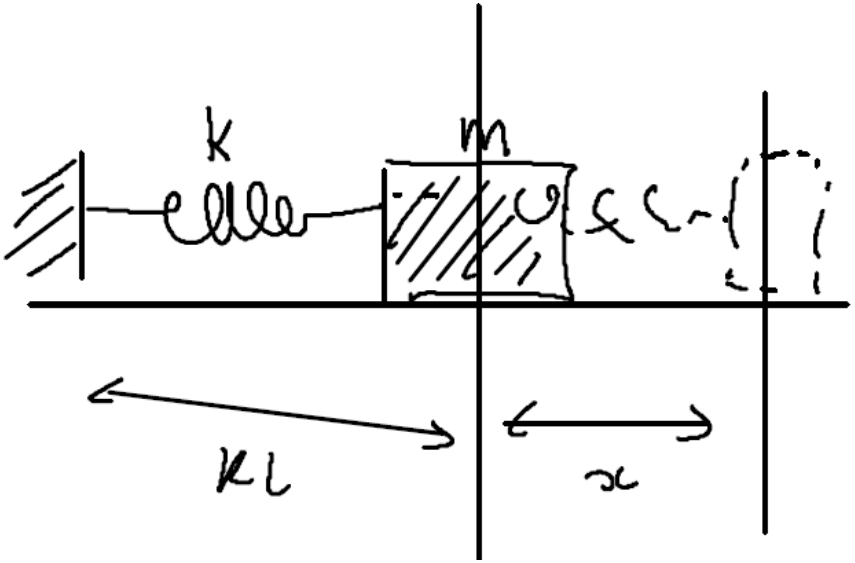
$$x_{i+1} = x_i - 2 \times x_{i-1}$$

$$x_i = x_{i-1} - 2 \times x_{i-2}$$



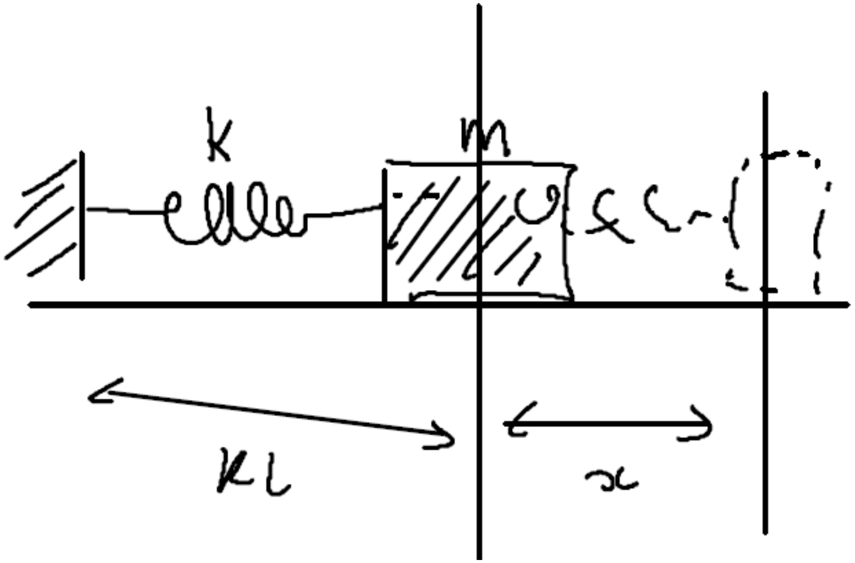
a family of Causal Block Diagram (CBD) formalisms





HOOKE'S LAW

$\underline{F} = -kx$ $x \ll$



HOOKE'S LAW

$$F = -kx \qquad x \ll$$

$$F = ma$$

$$F = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$$

$$v = \frac{dx}{dt}$$

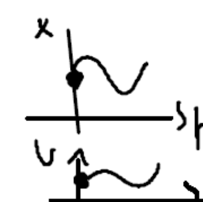
$$\frac{dv}{dt} = -\frac{k}{m}x, \; v(0)$$

$$\frac{dx}{dt} = v, \; x(0)$$

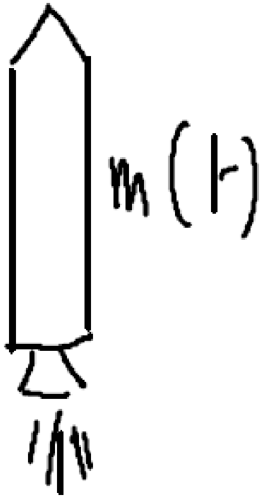
ODE
ORDINARY
DIFFERENTIAL
EQNS

PDE

$x(t)$, $v(t)$



F = m **a** is <u>not</u> “rocket science”

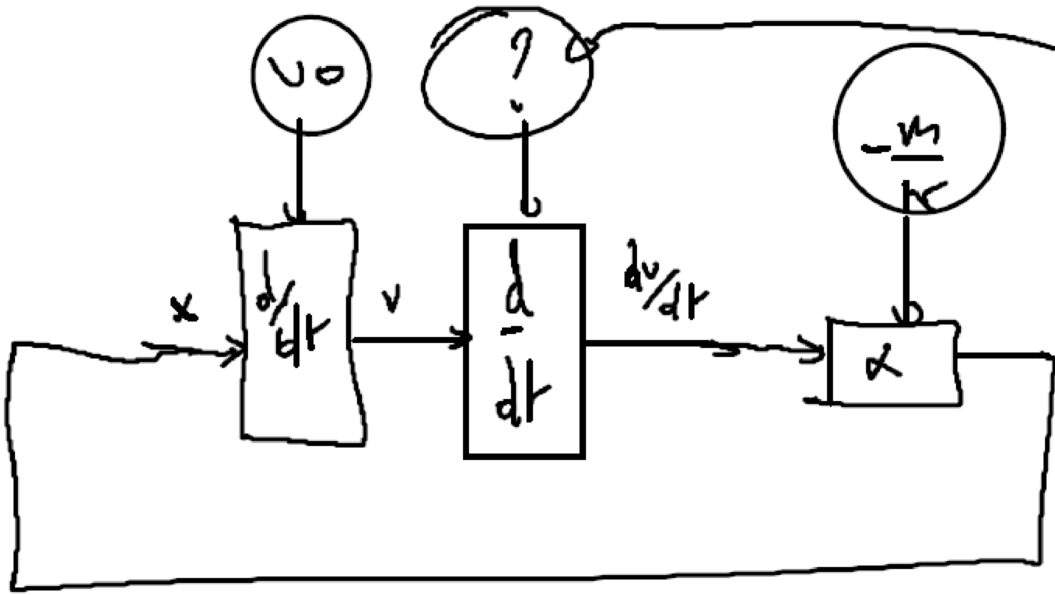


$$v = \frac{d}{dt} x$$

$$a = \frac{d}{dt} v = \frac{d^2}{dt^2} x$$

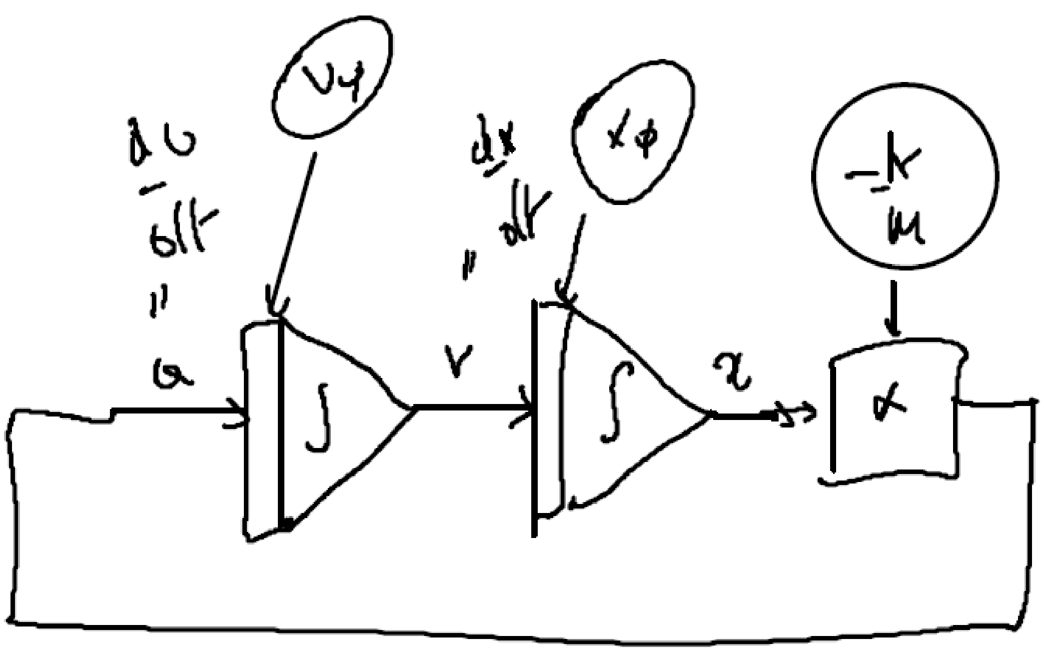
$$F = \frac{dp}{dt} = \frac{d}{dt}(mv)$$

$$= \underbrace{\frac{d}{dt} m \cdot v}_{} + m \overbrace{\frac{dv}{dt}}^{a}$$



$$x = -\frac{m}{k}\frac{dv}{dt}$$

$$\frac{dv}{dt}(0) = -\frac{k}{m}x(0)$$



$$\lim_{\Delta t \to 0}$$

Physical Systems Modelling

- Problem-Specific (technological)
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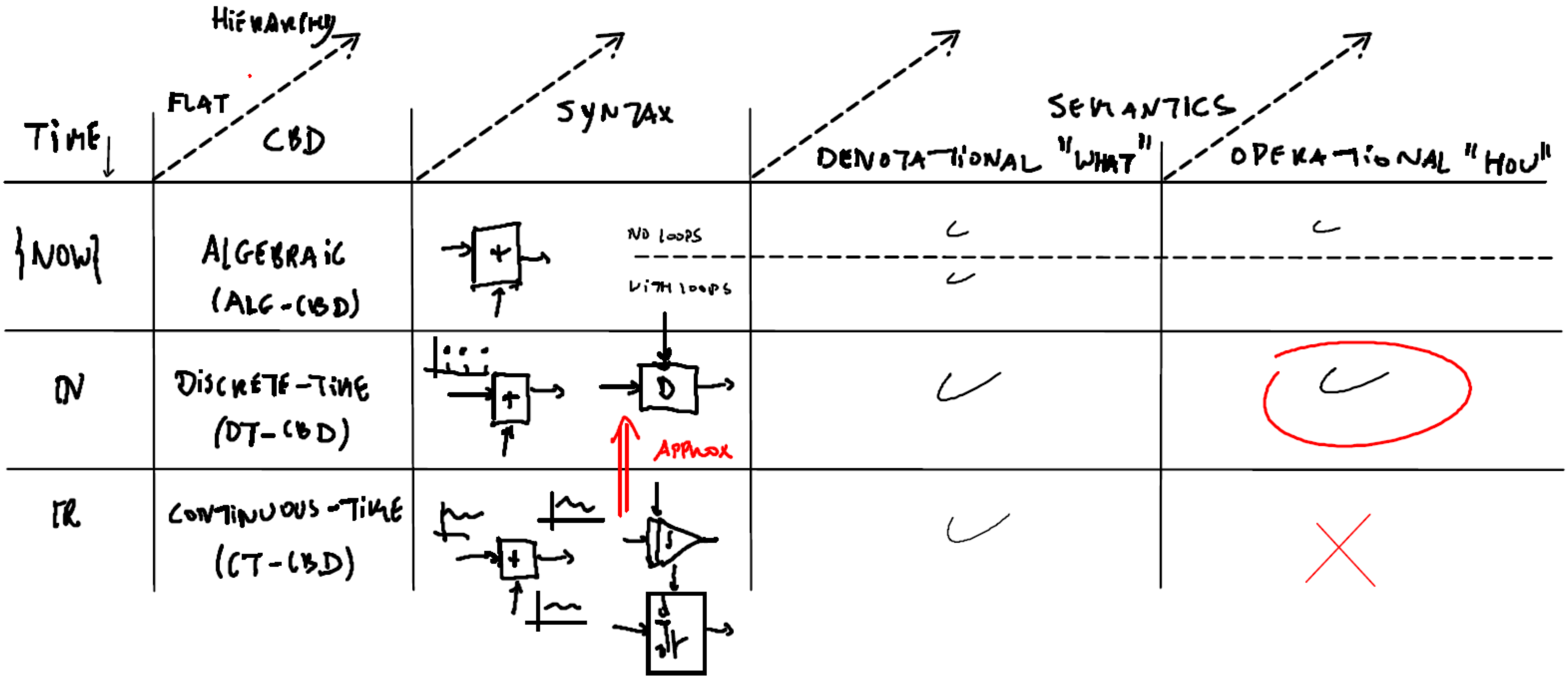
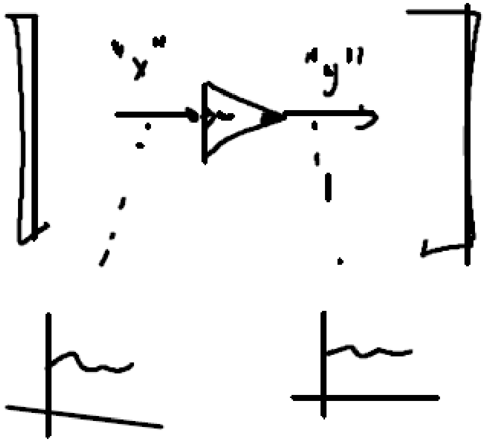
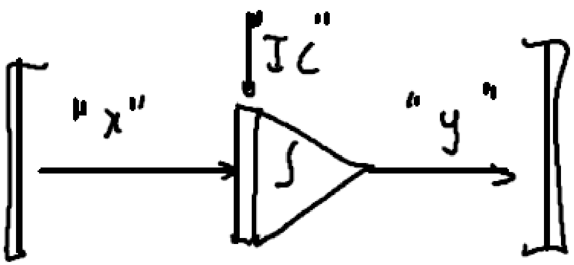
a family of Causal Block Diagram (CBD) formalisms



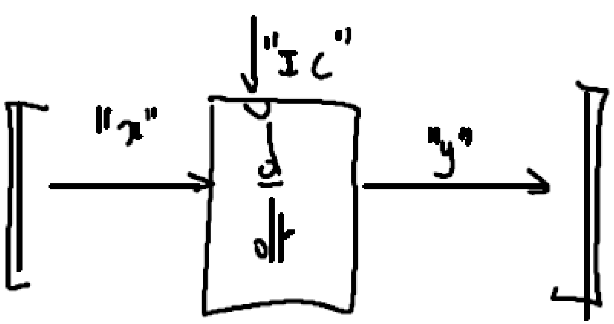


$$:= \quad mvy(t) = -mvx(t), \quad \forall t \in \mathbb{R}$$

$$- : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$

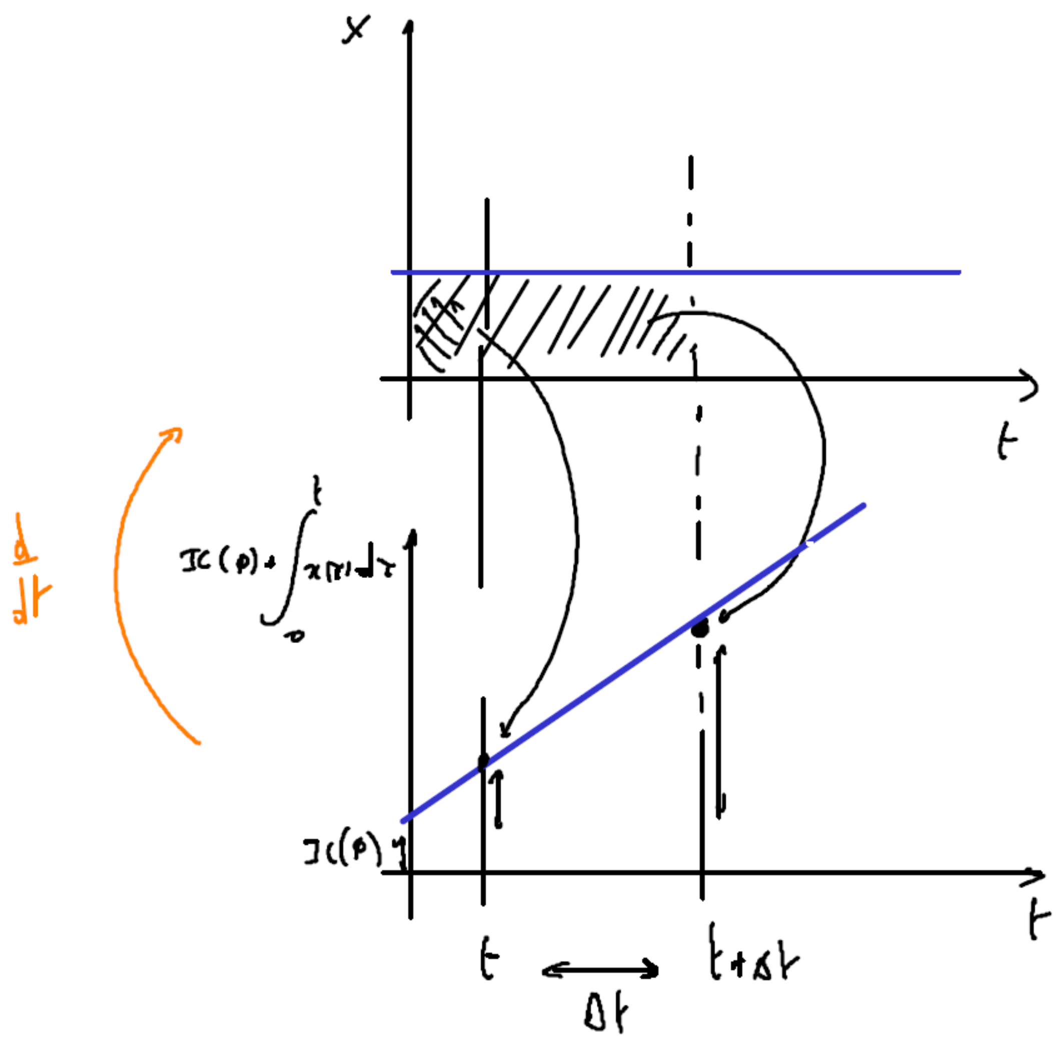


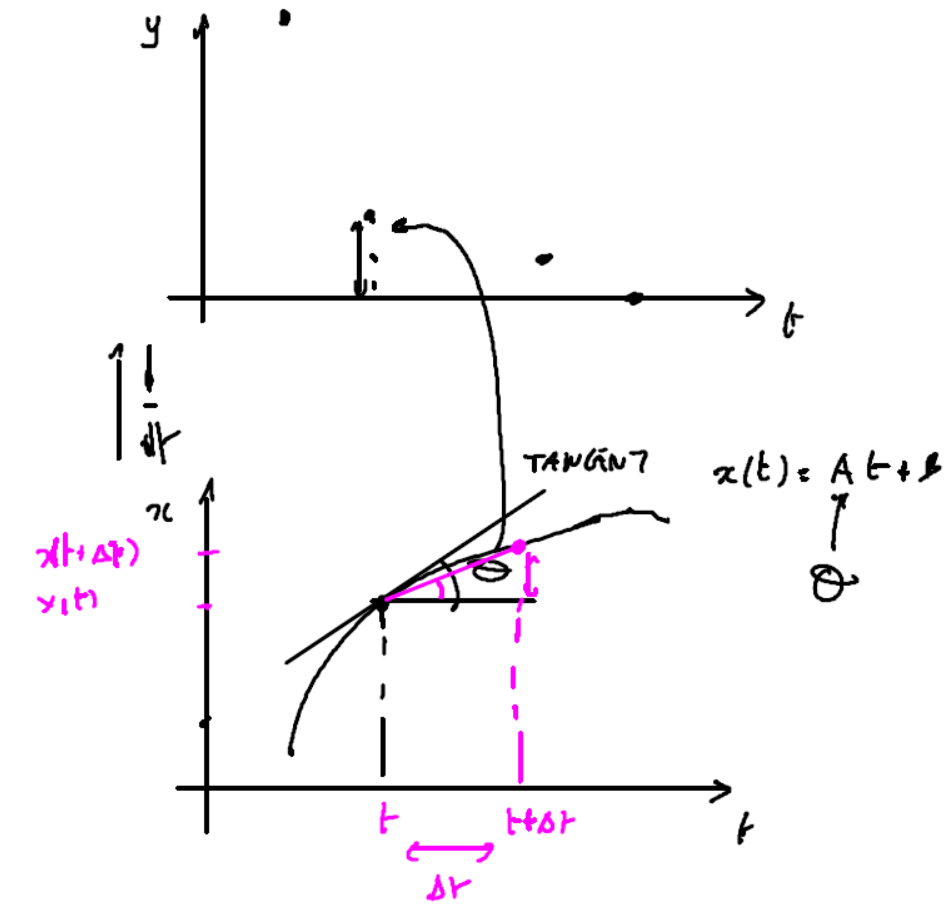
$$:= \quad mvy(t) = \underbrace{mvy(0)}_{IC(0)} + \int_0^t x(\tau)\,d\tau, \quad \forall t \in \mathbb{R}$$



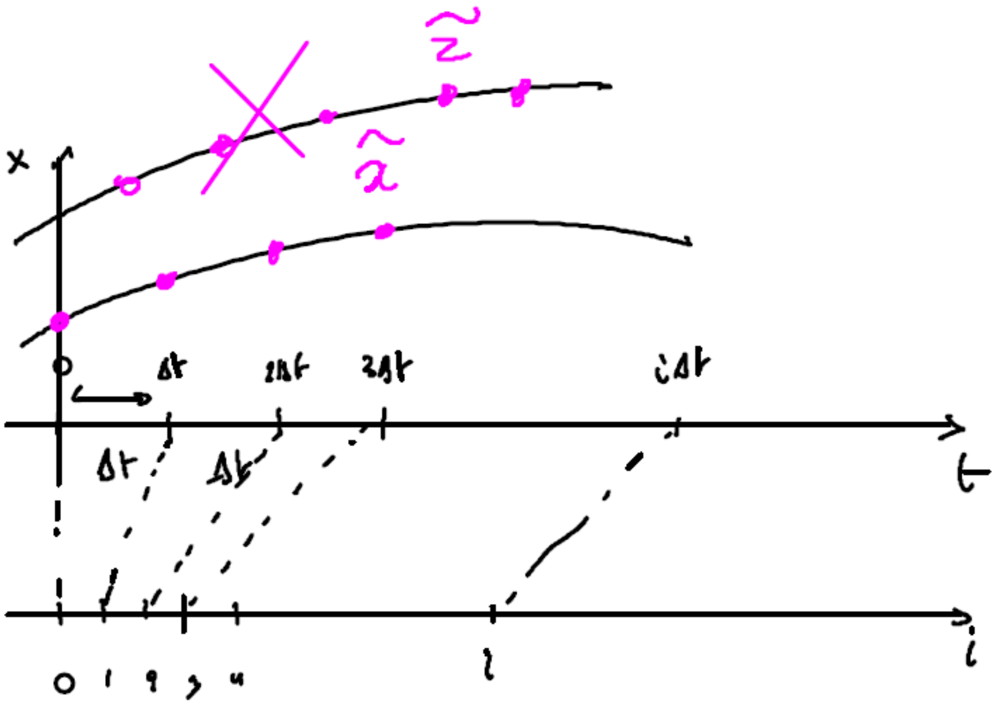
$$:= \quad mvy(t) = \frac{d}{dt} mvx(t), \quad \forall t \in \mathbb{R} \setminus \{0\}$$

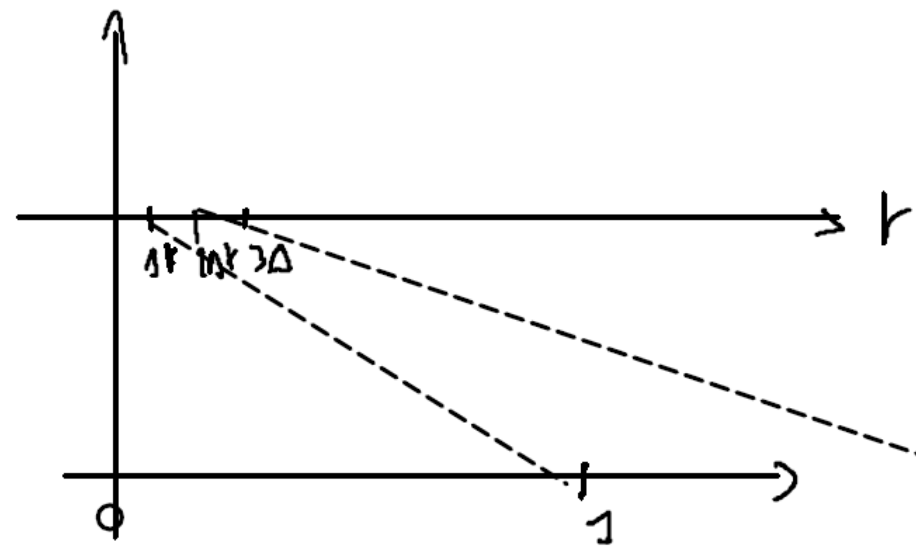
$$mvy(0) = IC(0)$$

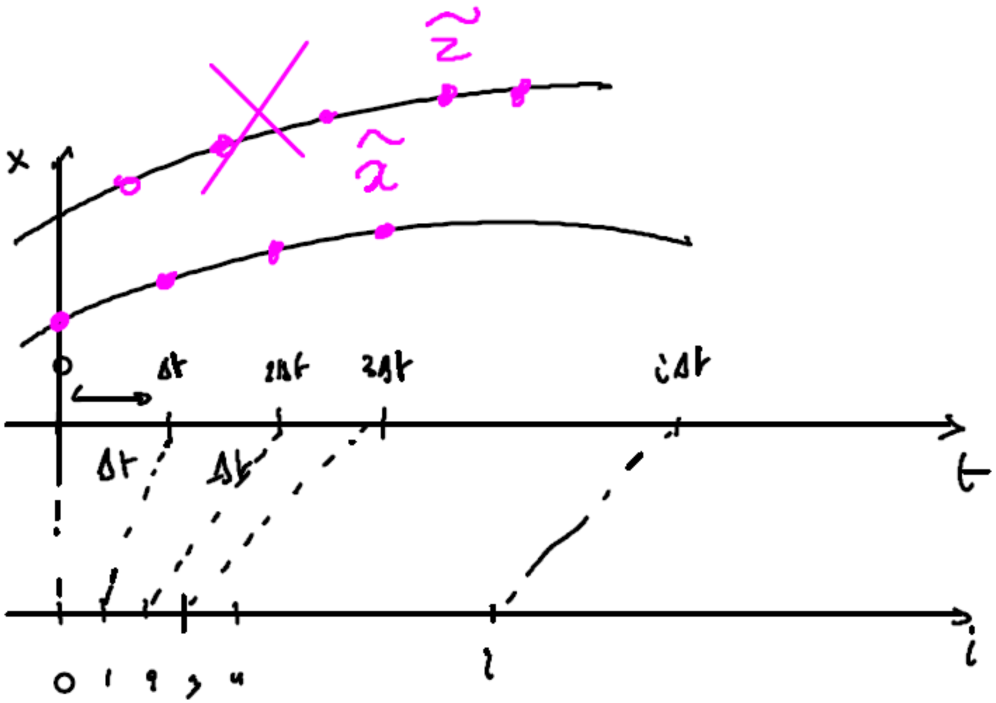


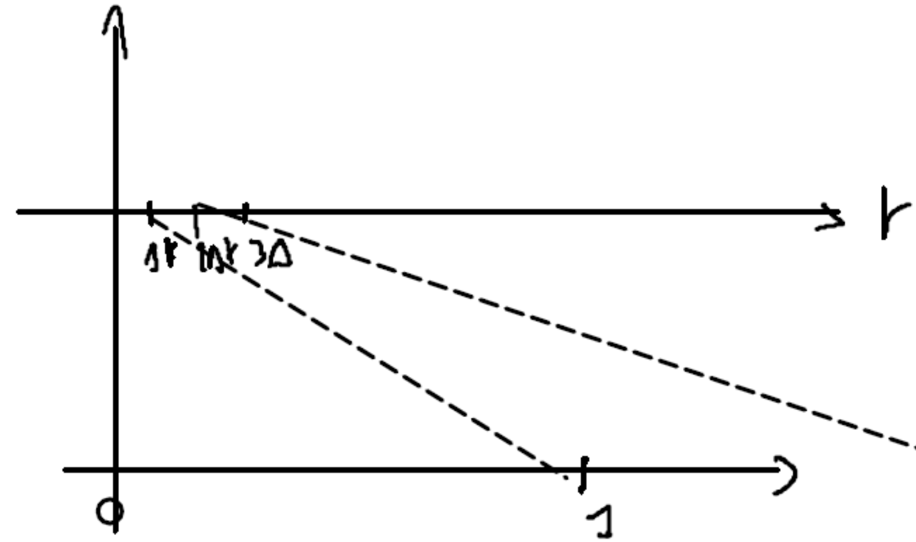


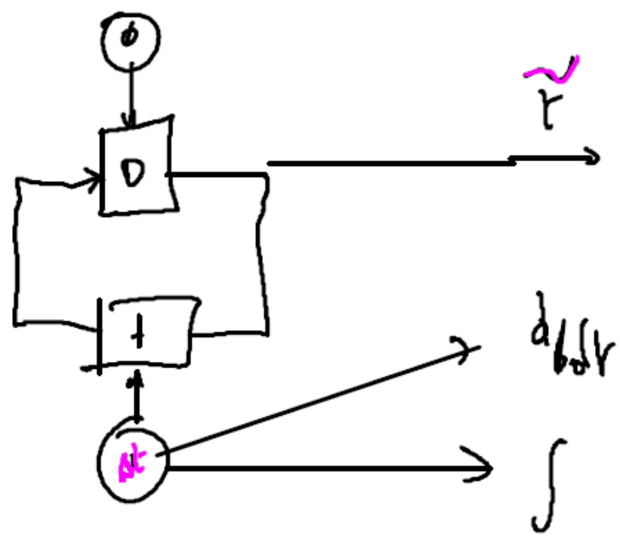
$$\lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$


x
0
Δt
2Δt
3Δt
iΔt
t
0
1
2
3
4
i


Δt
3Δ
0
1


x
t
0
Δt
2Δt
3Δt
iΔt

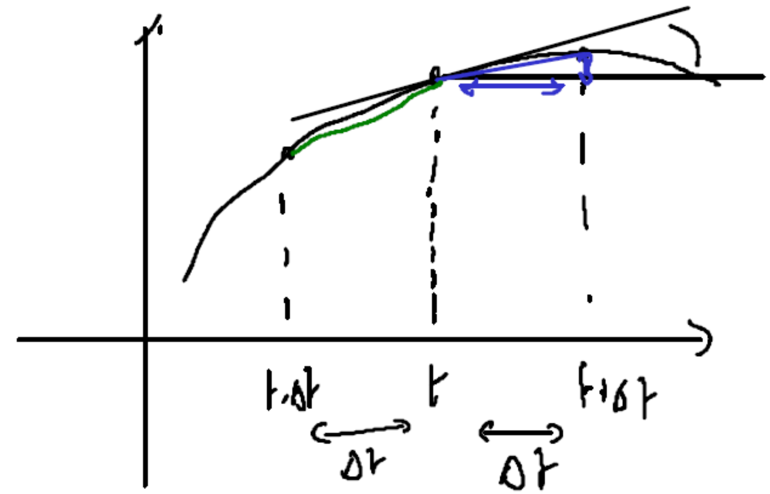

r
0
1


D
Δt

$$\tilde{y}(t) = \frac{\tilde{x}(t) - \tilde{x}(t-\Delta t)}{\Delta t}$$

$$\tilde{y}(t) = \frac{\tilde{x}(t+\Delta t) - \tilde{x}(t)}{\Delta t}$$

$\Delta t \ll$



$$\lim_{\Delta t \to 0}$$

$$\tilde{y}(t) = \frac{d\hat{x}}{dt}(t) = \frac{\hat{x}(t+\Delta t) - \hat{x}(t)}{\Delta t}$$

$$\hat{y}(t) = \frac{d\tilde{x}}{dt}(t) = \frac{\tilde{x}(t) - \tilde{x}(t-\Delta t)}{\Delta t}$$

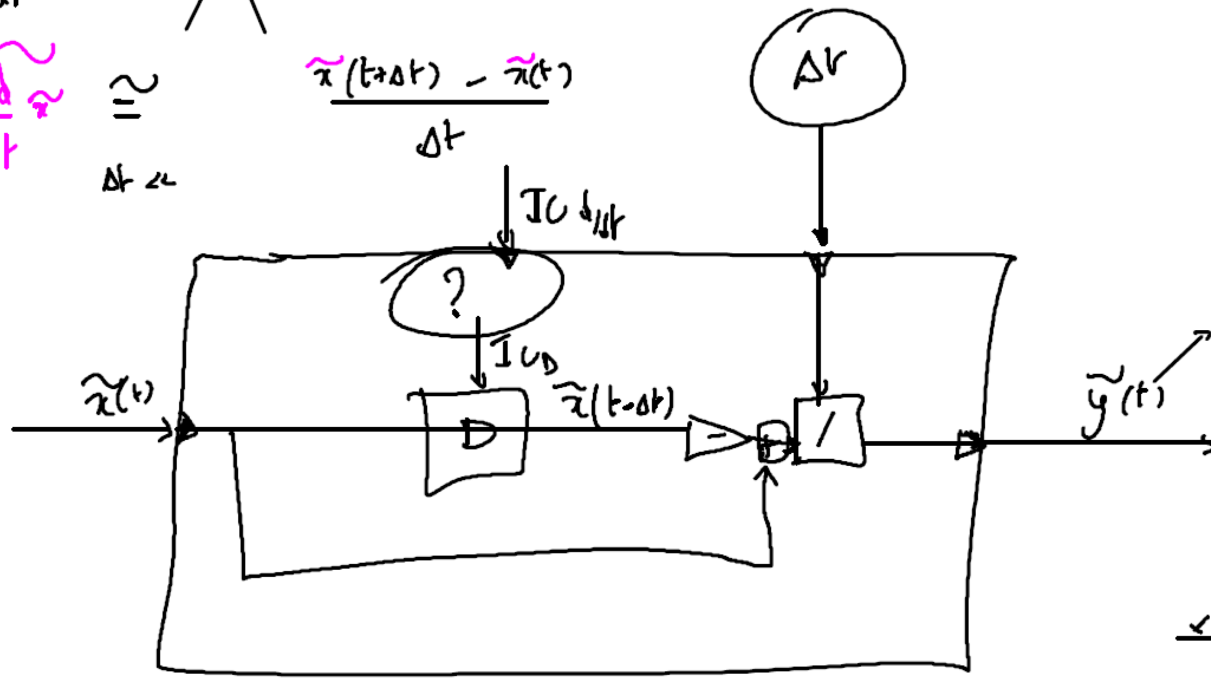
$$\frac{d}{dt}x(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$\tilde{y}(t) = \widetilde{\frac{d}{dt}\tilde{x}} \simeq \frac{\tilde{x}(t+\Delta t) - \tilde{x}(t)}{\Delta t}$$

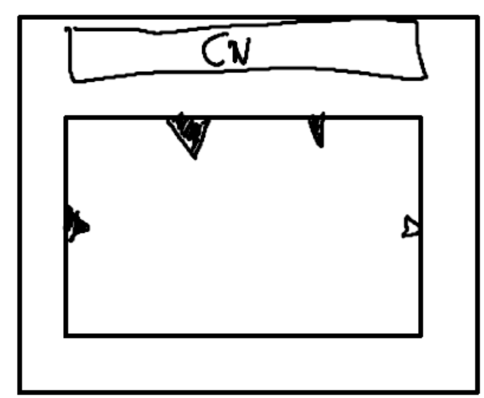
$\Delta t \ll$

$$\tilde{y}(t+\Delta t) = \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$\tilde{y}(t) = \frac{\tilde{x}(t) - \tilde{x}(t-\Delta t)}{\Delta t}$$

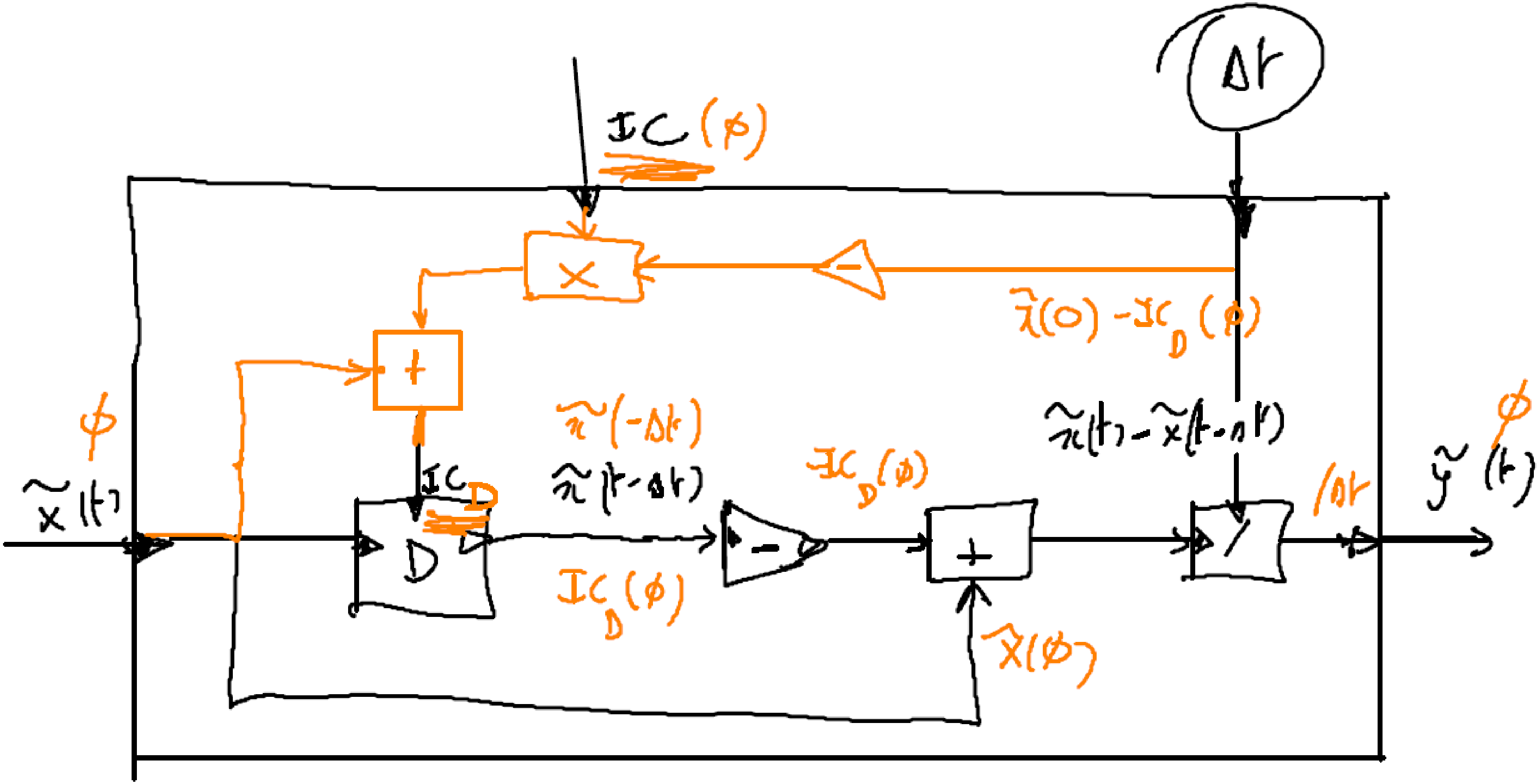


$$y = \frac{d}{dt}x \qquad y(0) = IC(0)$$



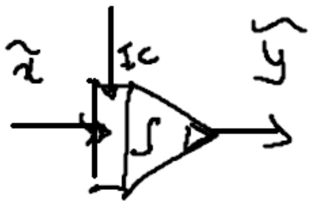
All blocks : $= \Delta t$

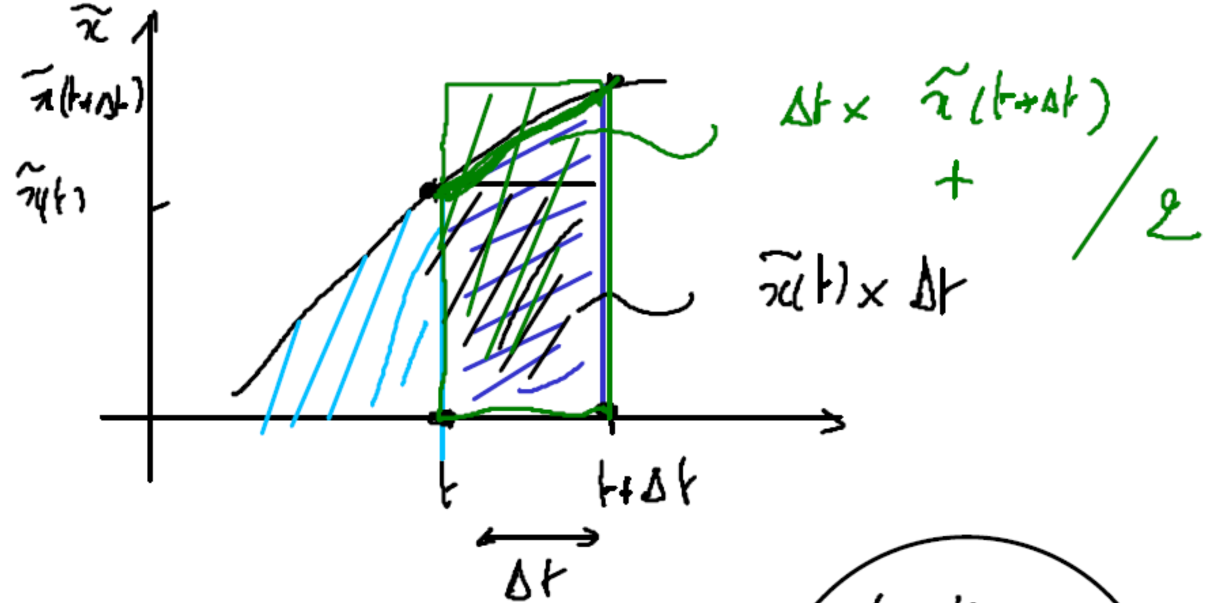
Synchronous block diagrams



$$\frac{\tilde{x}(0) - IC_D(0)}{\Delta t} = IC_{\frac{d}{dt}}(0) \qquad \tilde{y}(0) = IC_{\frac{d}{dt}}(0)$$

$$\tilde{x}(0) - IC_{\frac{d}{dt}}(0) \cdot \Delta t = IC_D(0)$$





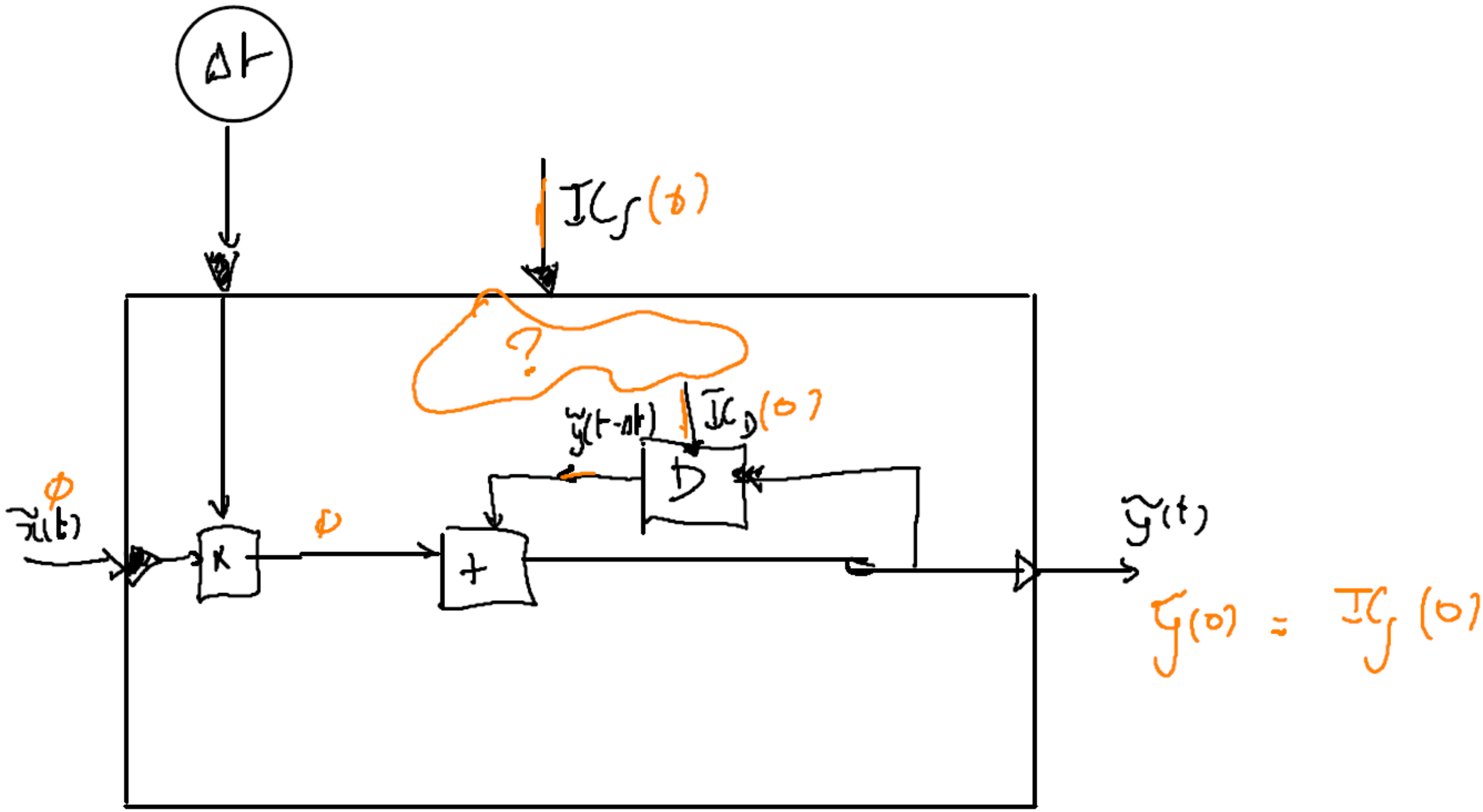
$$\tilde{y}(t+\Delta t) = \tilde{y}(t) + \int_{t}^{t+\Delta t} \tilde{x}(\tau)\,d\tau$$

$$\lim_{\Delta t \to 0} \quad \Delta t \; \tilde{x}(t)$$

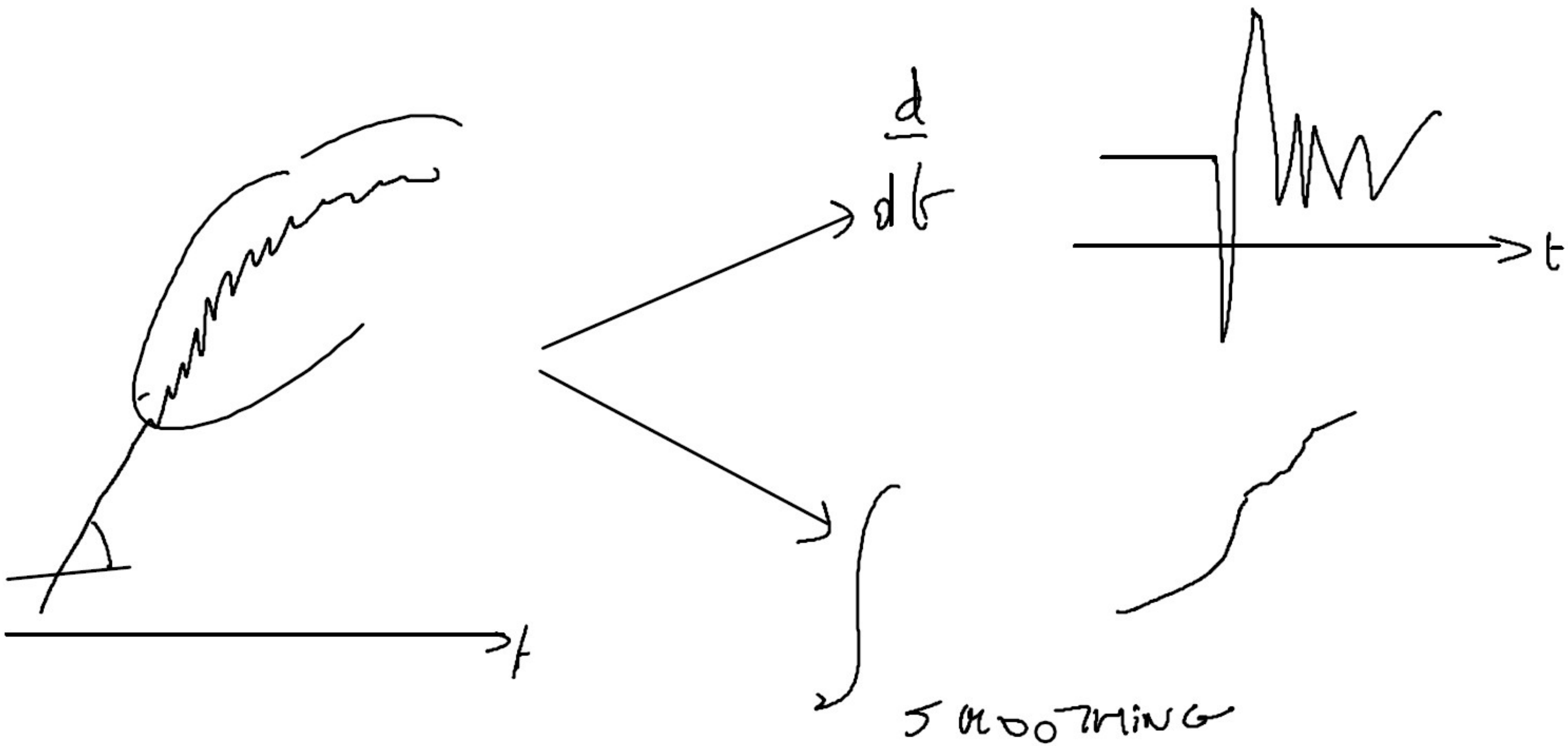
$$\Delta t \; \tilde{x}(t+\Delta t)$$

$$\tilde{y}(t) = y(t-\Delta t) + \Delta t \; \tilde{x}(t)$$

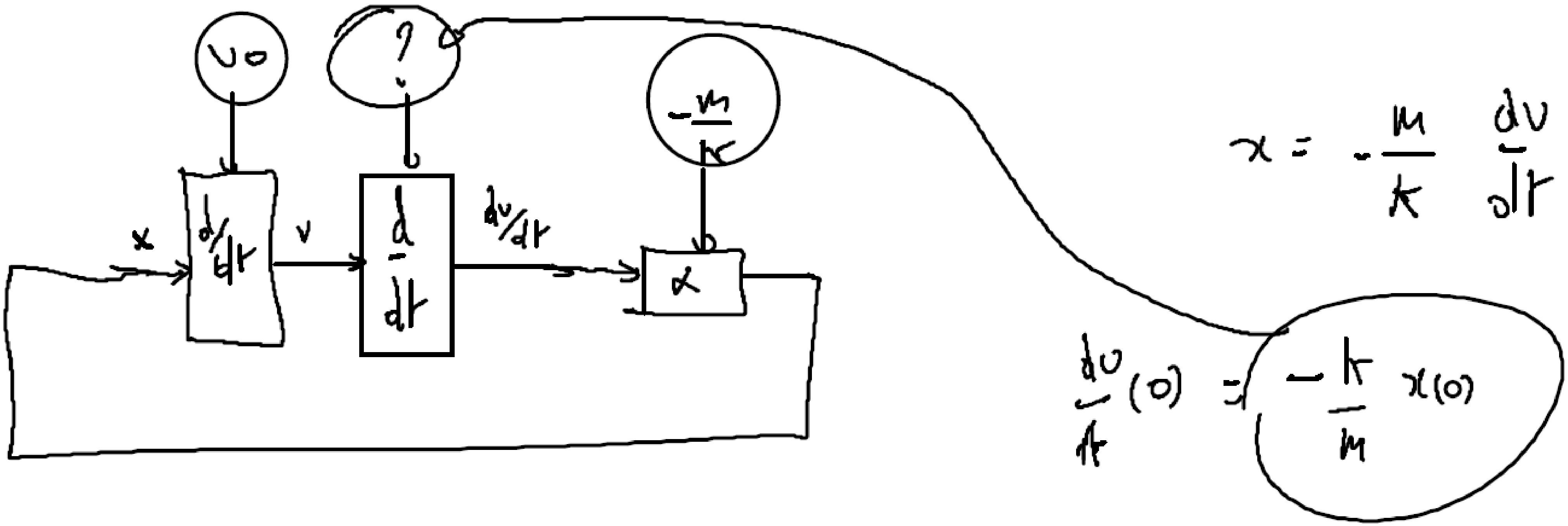
APPROX



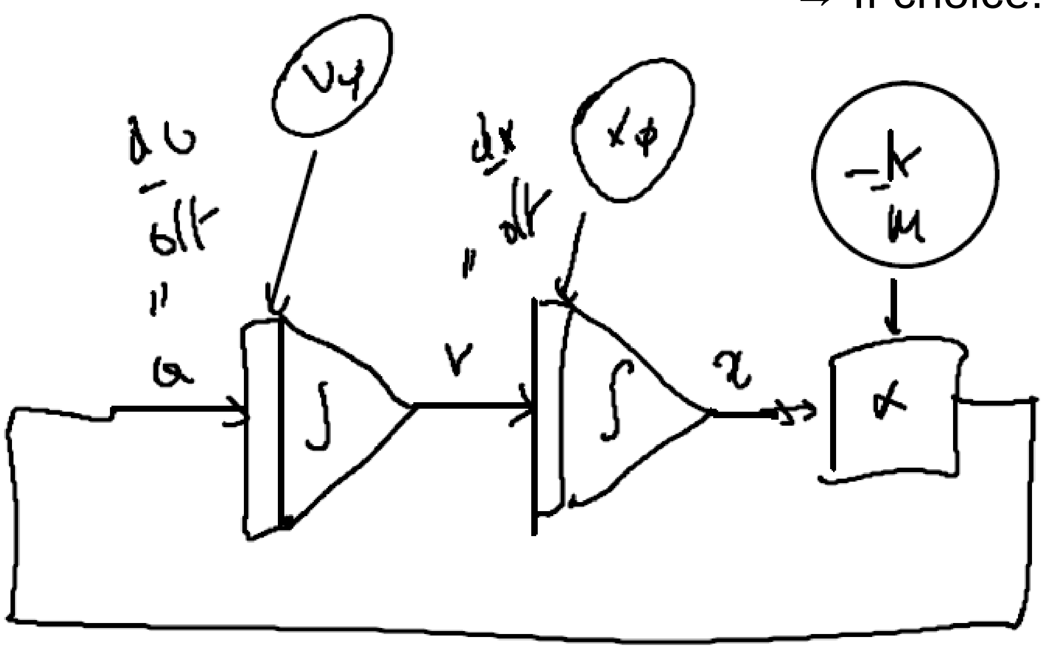
Δt
IC_J (0)
?
ỹ(t-Δt)
IC_D(0)
D
φ
x̃(t)
K
φ
+
ỹ(t)
ỹ(0) = IC_J (0)



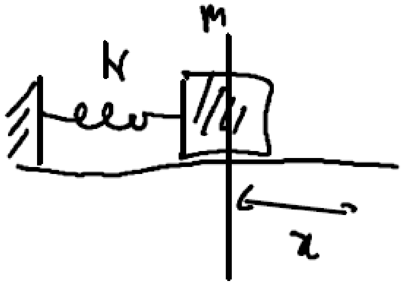
→ If choice: prefer intergration over differentiations



→ If choice: prefer integration over differentiation

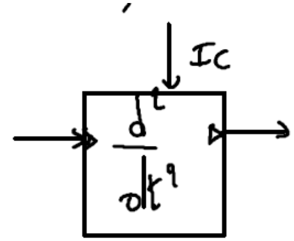


$$\lim_{\Delta t \to \emptyset}$$

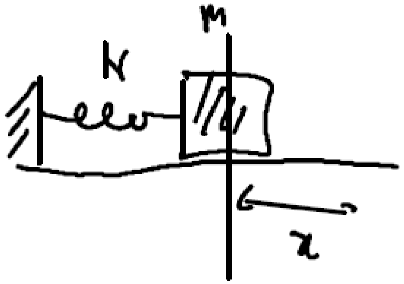


HARMONIC EQN.

$$\frac{d^2x}{dt^2} = -x \quad , \qquad x(0) = x_0 \quad , \qquad \frac{dx}{dt}(0) = v(0) = v_0 \qquad \text{ODE}$$



$$\begin{cases} \dfrac{dx}{dt} = v \quad , & x(0) = x_0 \\[2ex] \dfrac{dv}{dt} = -x \quad , & v(0) = v_0 \end{cases}$$



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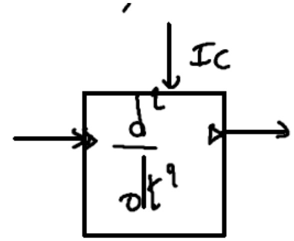
$$\frac{d^2x}{dt^2} = -x, \qquad x(0) = x_0, \qquad \frac{dx}{dt}(0) = v(0) = v_0 \qquad \text{ODE}$$



$$\begin{cases} \dfrac{dx}{dt} = v, & x(0) = x_0 \\[2ex] \dfrac{dv}{dt} = -x, & v(0) = v_0 \end{cases}$$

HIGHER-ORDER ODES

$$\frac{d^k x}{dt^k} = \overbrace{\frac{d}{dt}\left(\frac{d}{dt}\left(\cdots \frac{d}{dt}x\right)\right)}^{k}$$

$$\begin{cases} \dfrac{dx}{dt} = x\_der\_1 \\[2ex] \dfrac{d\,x\_der\_1}{dt} = x\_der\_2 \\[1ex] \vdots \\[1ex] \dfrac{d\,x\_der\_{k-1}}{dt} = x\_der\_k \end{cases}$$

$$\frac{d^2x}{dt^2} = -x \quad , \quad x(0) = \phi \quad , \quad \frac{dx}{dt}(0) = 1$$

$$\frac{dx}{dt} = x$$

$$x(t) = e^x + C$$

$$\begin{cases} x(t) = A\sin(t) + B\cos(t)\left(+\ C\right) \\ x(0) = \phi \quad , \quad \frac{dx}{dt}(0) = 1 \end{cases}$$

$$\frac{dx}{dt} = A\cos(t) - B\sin(t)$$

$$\frac{d^2x}{dt^2} = -A\sin(t) - B\cos(t) = -\left(A\sin(t) + B\cos(t)\right) = -x(t) \qquad \text{q.e.d.}$$

$$x(0) = \quad B = \phi$$

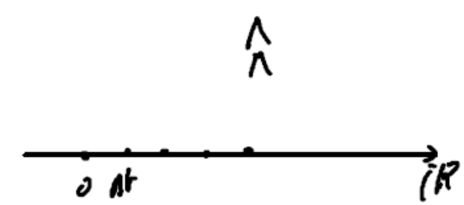
$$\frac{dx}{dt}(0) = \quad A = 1$$

$$\begin{cases} x(t) = \sin(t) \\ \frac{dx}{dt}(t) = \cos(t) \end{cases}$$

$$\tilde{x}(\hat{t})$$

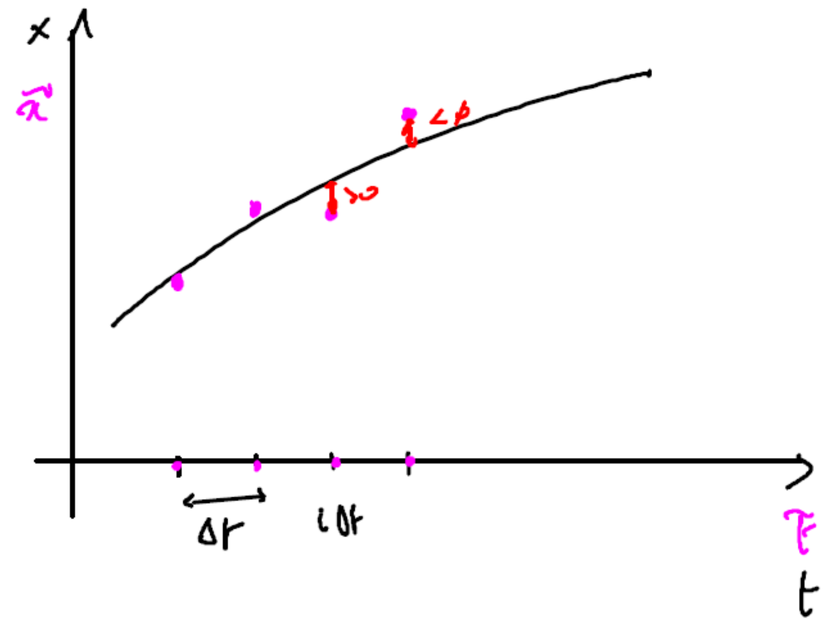
$$\widetilde{\frac{dx}{dt}} = \tilde{v}(\hat{t})$$

$$\hat{t} = i \times \Delta t$$



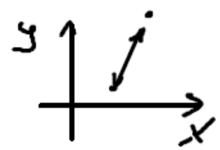
ERROR ~ $e(i\Delta t) = x(i\Delta t) - \tilde{x}(i\Delta t)$

TOTAL ERROR ~ $\sum_{i=0}^{i\_END} e(i\Delta t)$



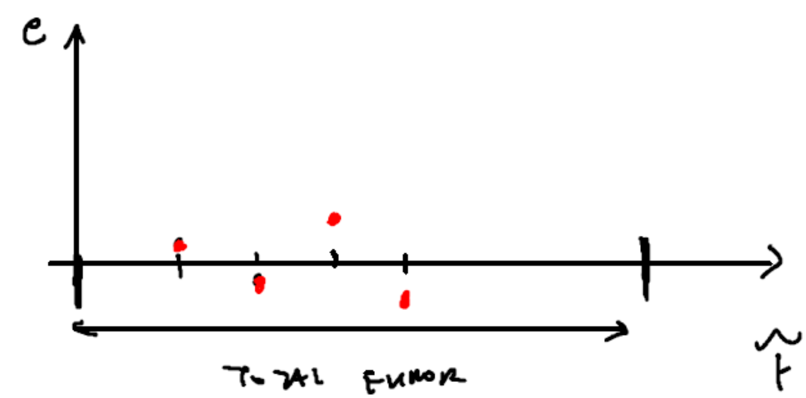
$e(i\Delta t) = |x(i\Delta t) - \tilde{x}(i\Delta t)|$

$e(i\Delta t) = (x(i\Delta t) - \tilde{x}(i\Delta t))^2$



"SUM OF SQUARES"

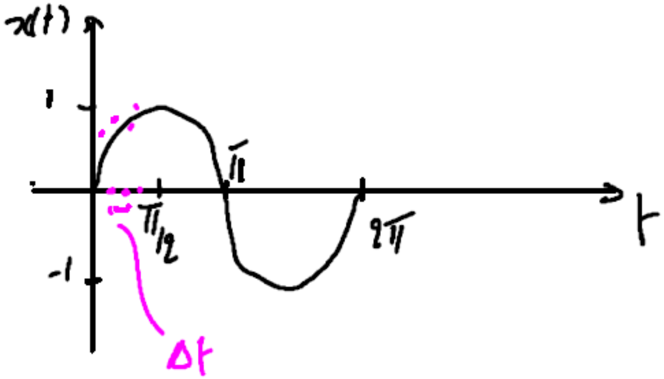
TOTAL ERROR $= \sum_{i=\emptyset}^{i\_END} (x(i\Delta t) - \tilde{x}(i\Delta t))^2$

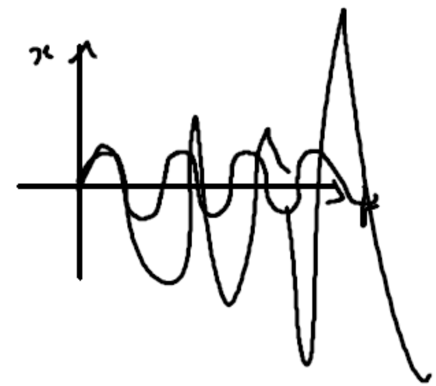


$= \int (x(\tau) - \tilde{x}(\tau))^2 \, d\tau$

sin(T)
CBD

CBD



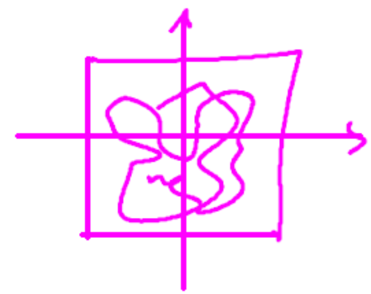


UNSTABLE

SYSTEM ?

NUMERICAL ?

BOUNDED REGION

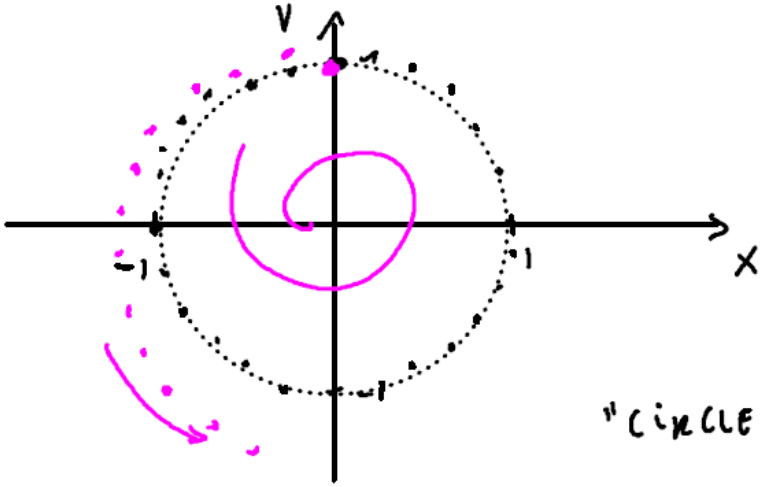


PHASE PLOT

~ STABILITY

- SYSTEM (SHO)
- NUMERICAL

~ Δt, SCHEME

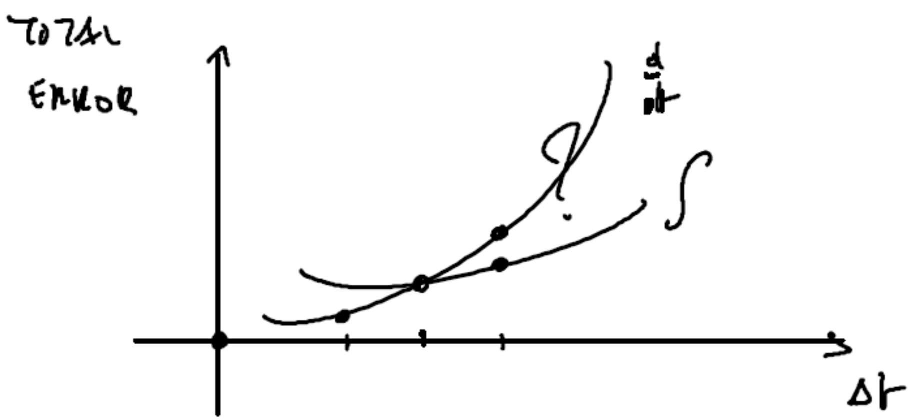


$(x(t), v(t))$

$x(0) = 0 \quad , \quad v(0) = 1$

$x\left(\frac{\pi}{2}\right) = \quad , \quad v\left(\frac{\pi}{2}\right) =$

"CIRCLE TEST"



DISCRETIZATION
SCHEME
FIXED

$$\text{TOTAL ERROR}(\Delta t) \overset{?}{=} f(\Delta t)$$

$$x(t+\Delta t) = x(t) + \frac{x'(t)\,\Delta t}{1!} + \frac{x''(t)}{2!}\Delta t^2 + \frac{x^{(3)}(t)}{3!}\Delta t^3 + \dots$$

TAYLOR EXPANSION

$$\longrightarrow O(\Delta t^2)$$

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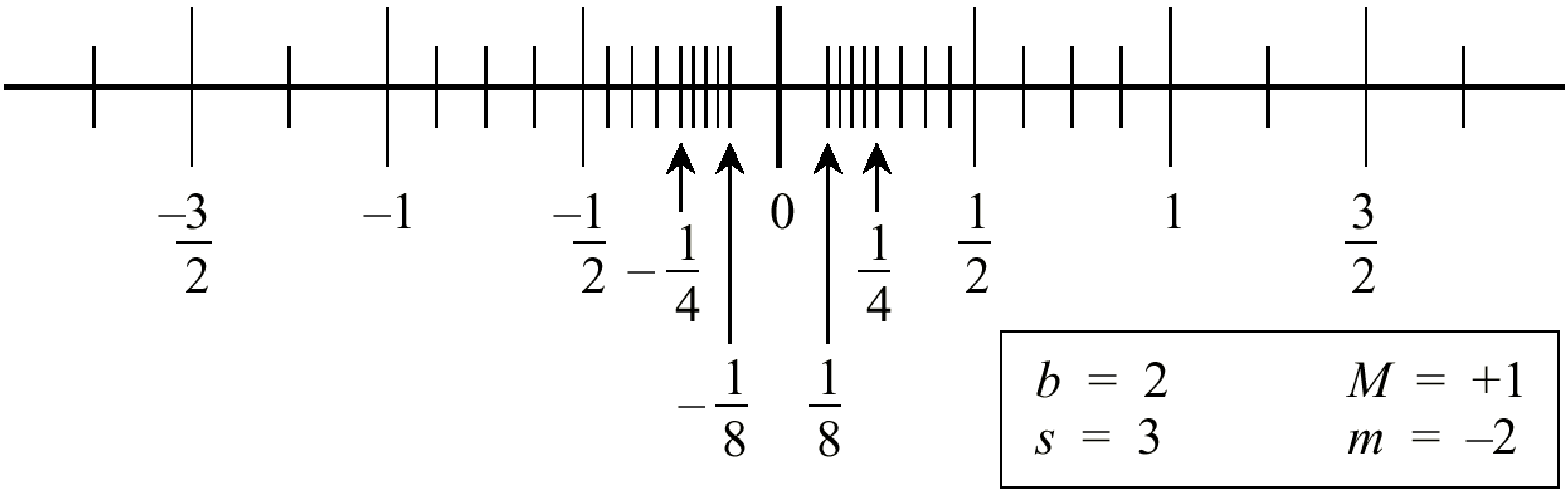
Example Floating Point Format



- Smallest non-zero positive number = b^m x b^{-1} = 1/8
- Largest non-zero positive number = b^M x $(1 – b^{-s})$ = 7/4
- Smallest gap = b^m x b^{-s} = 1/32
- Largest gap = b^M x b^{-s} = 1/4
- Number of representable numbers = **2x((M-m)+1)x(b-1)xb^{s-1}+1** = 33

  ... fits into available bits? Optimal number of bits?
- Note: fill the gap around 0: **de-normalized**

Physical Systems Modelling

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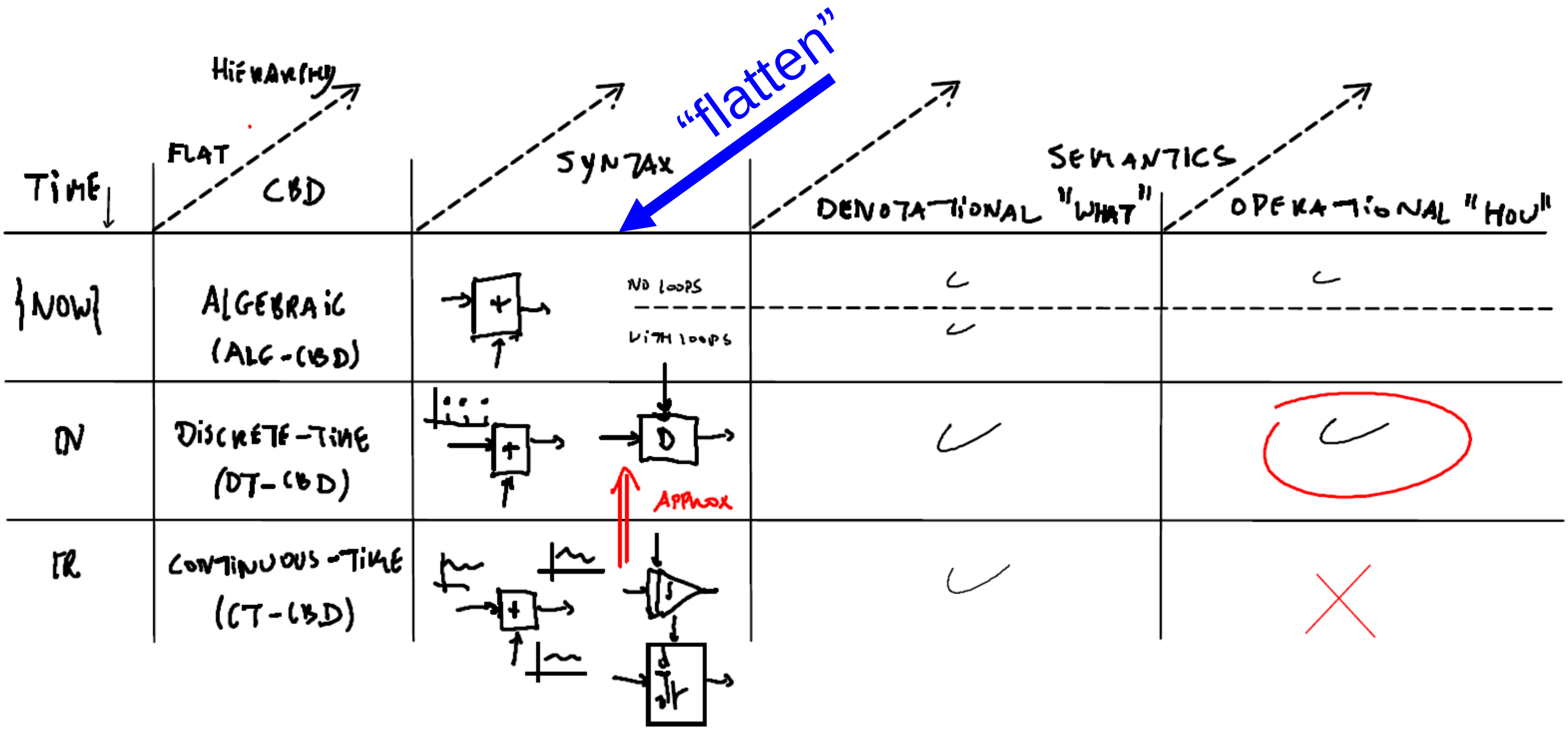
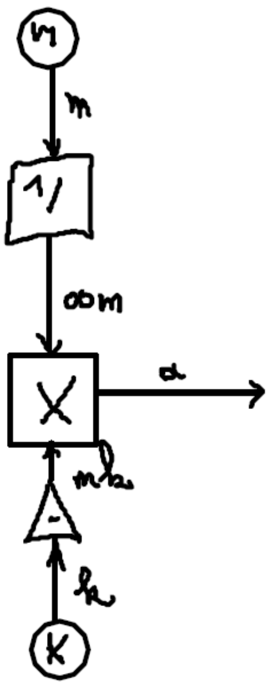
++ hierarchy

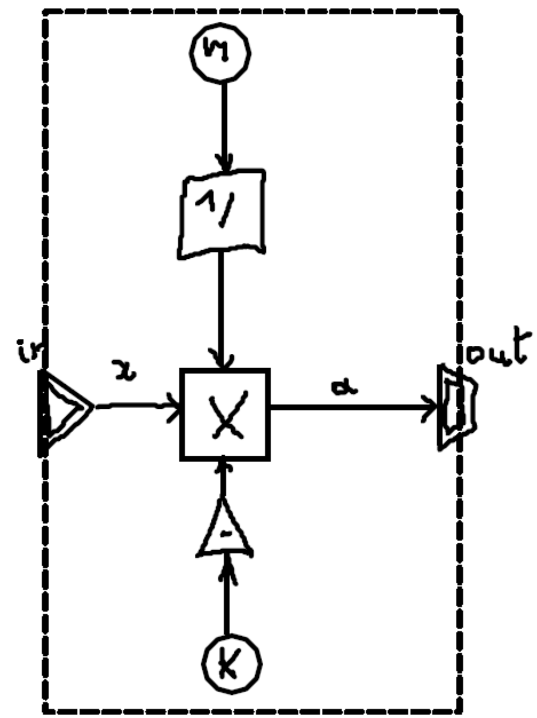
adding hierarchy to CBD formalisms



adding hierarchy to CBD formalisms

modularizing: introducing ports/interfaces

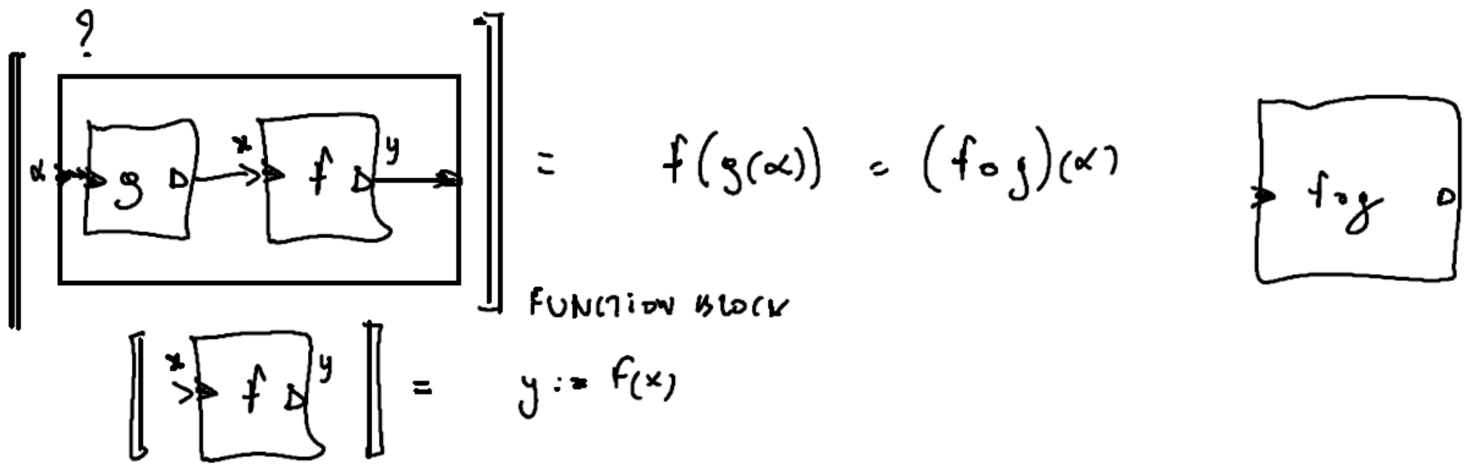




adding hierarchy to CBD formalisms

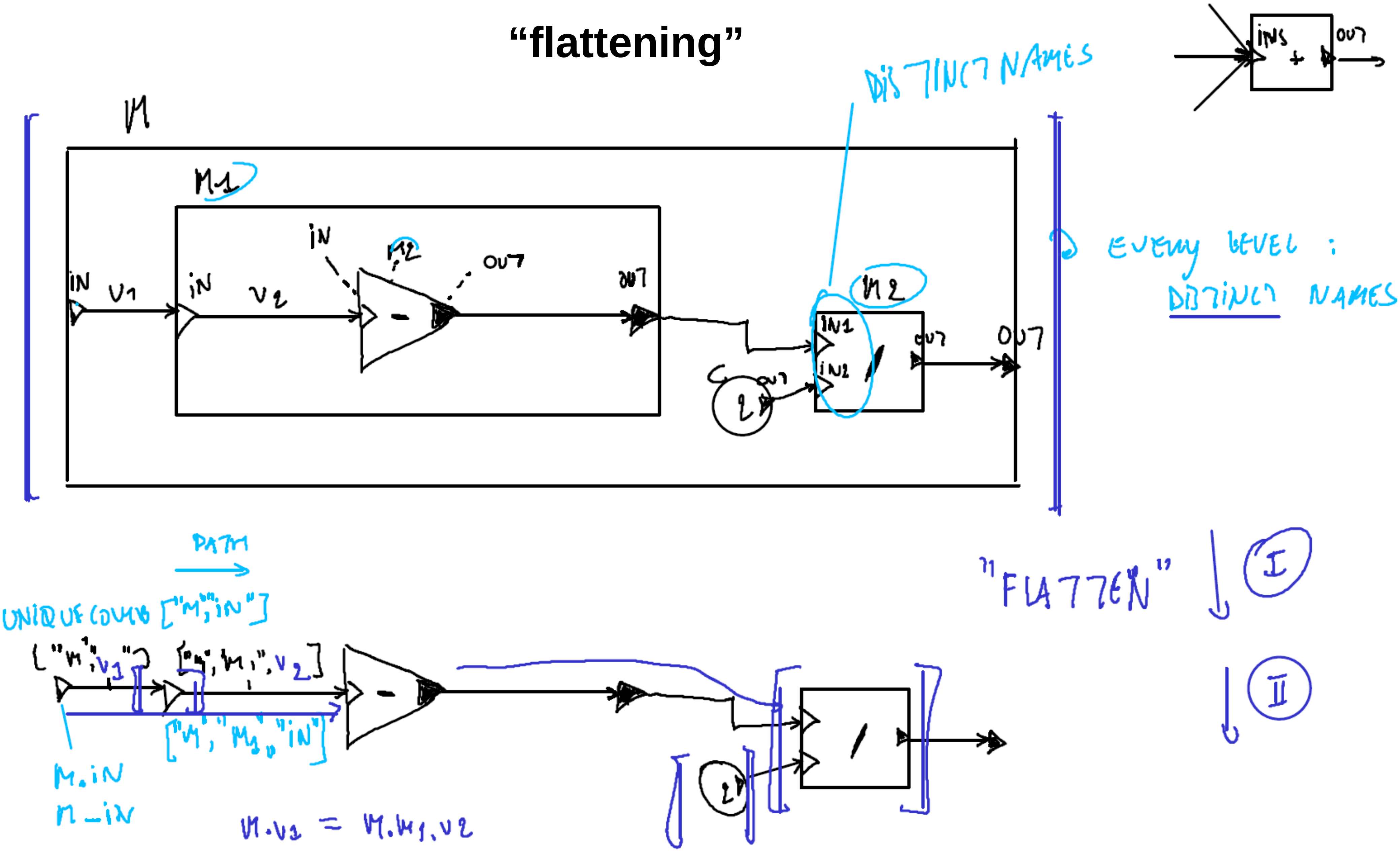
semantics?

HIERARCHY



adding hierarchy to CBD formalisms

“flattening”



Physical Systems Modelling

- Problem-Specific (technological)
- Domain-Specific (e.g., translational mechanical)
- (general) Laws of Physics
- Power Flow/Bond Graphs (physical: energy/power)
- Computationally a-causal
  (Mathematical and Object-Oriented) ← **Modelica**
- Causal Block Diagrams (data flow)
- Numerical (Discrete) Approximations
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  (Floating Point vs. Fixed Point)
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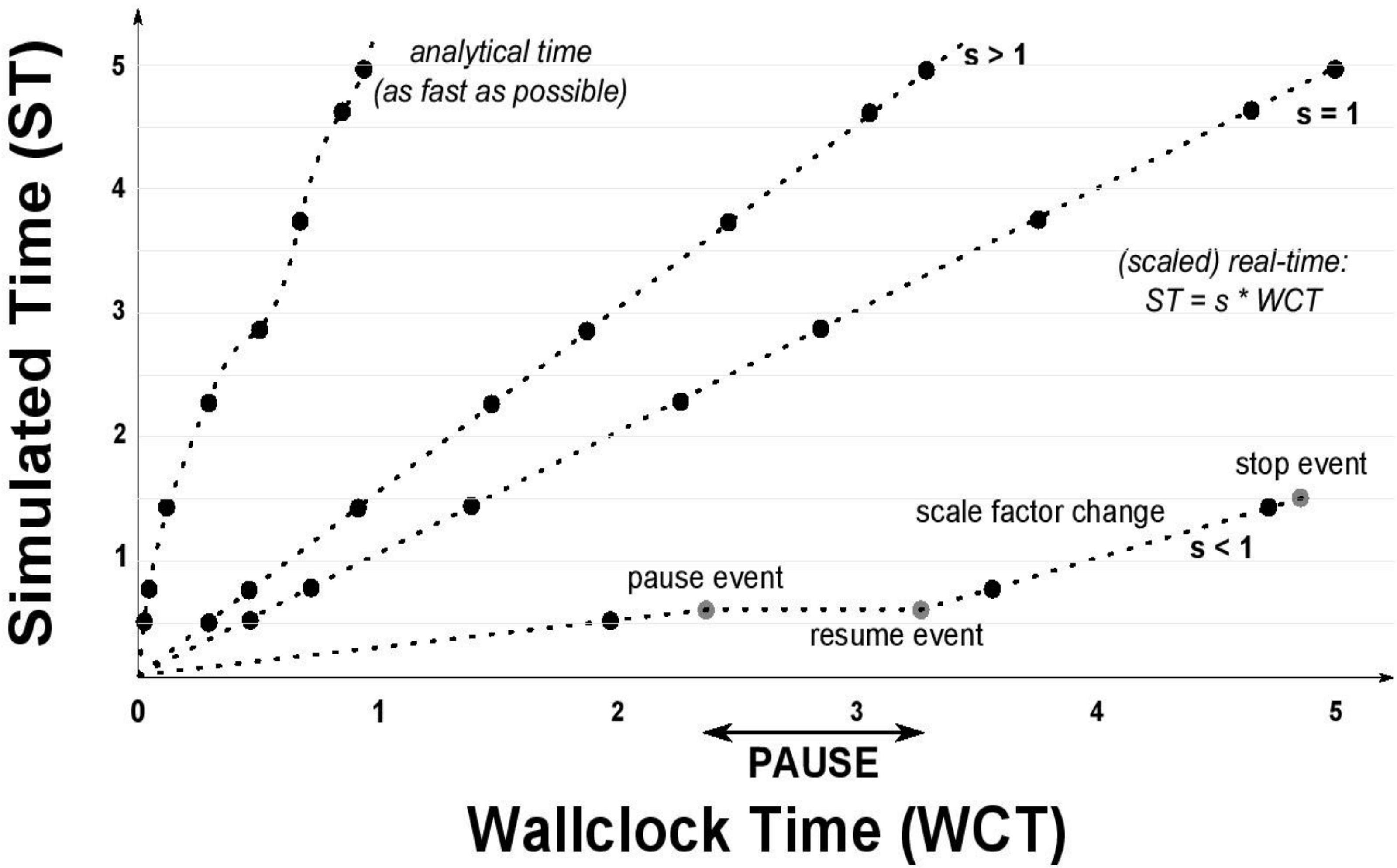
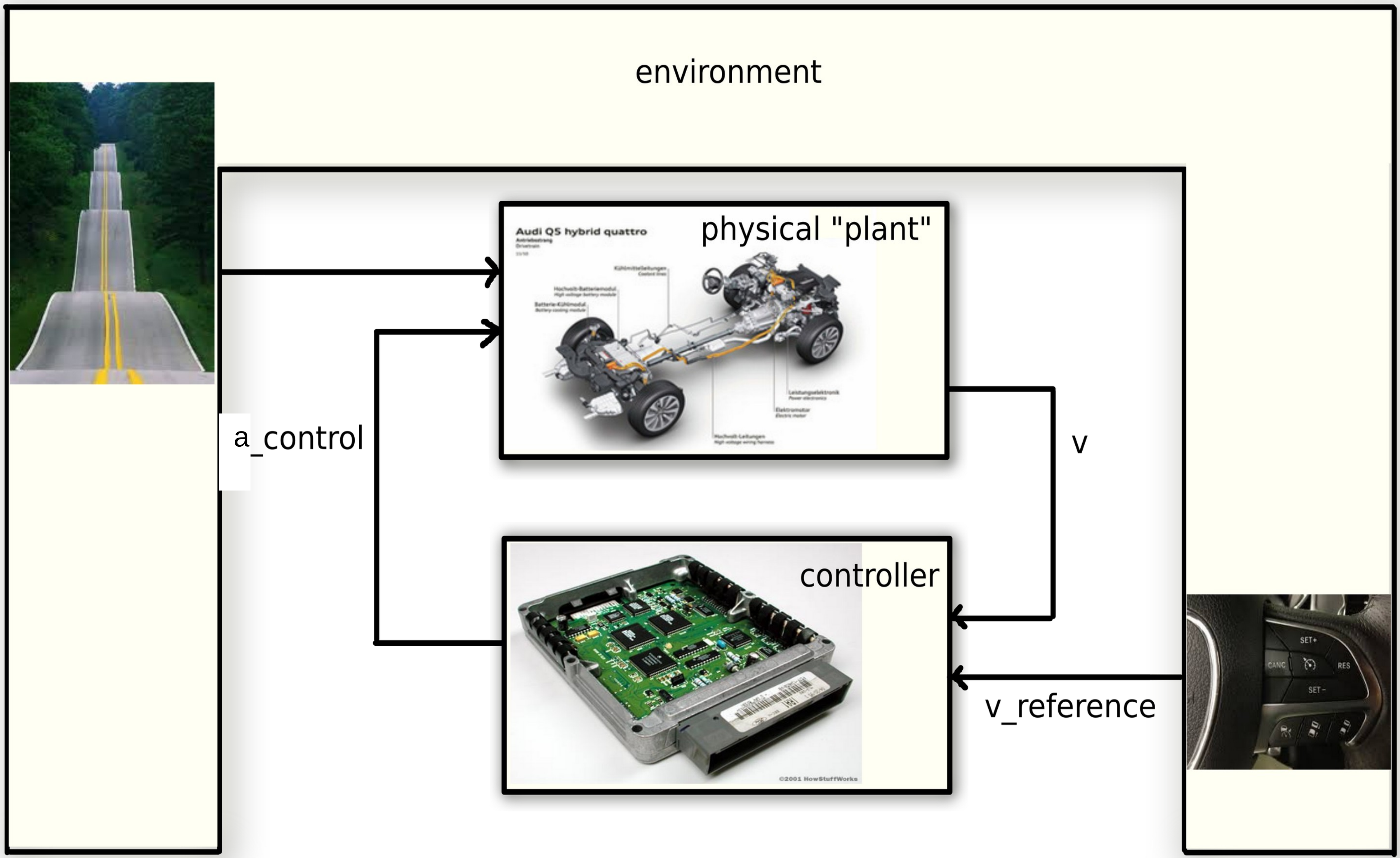
- Dynamic Structure

As-Fast-As-Possible vs. Real-time



Model-Based Systems Engineering (MBSE)



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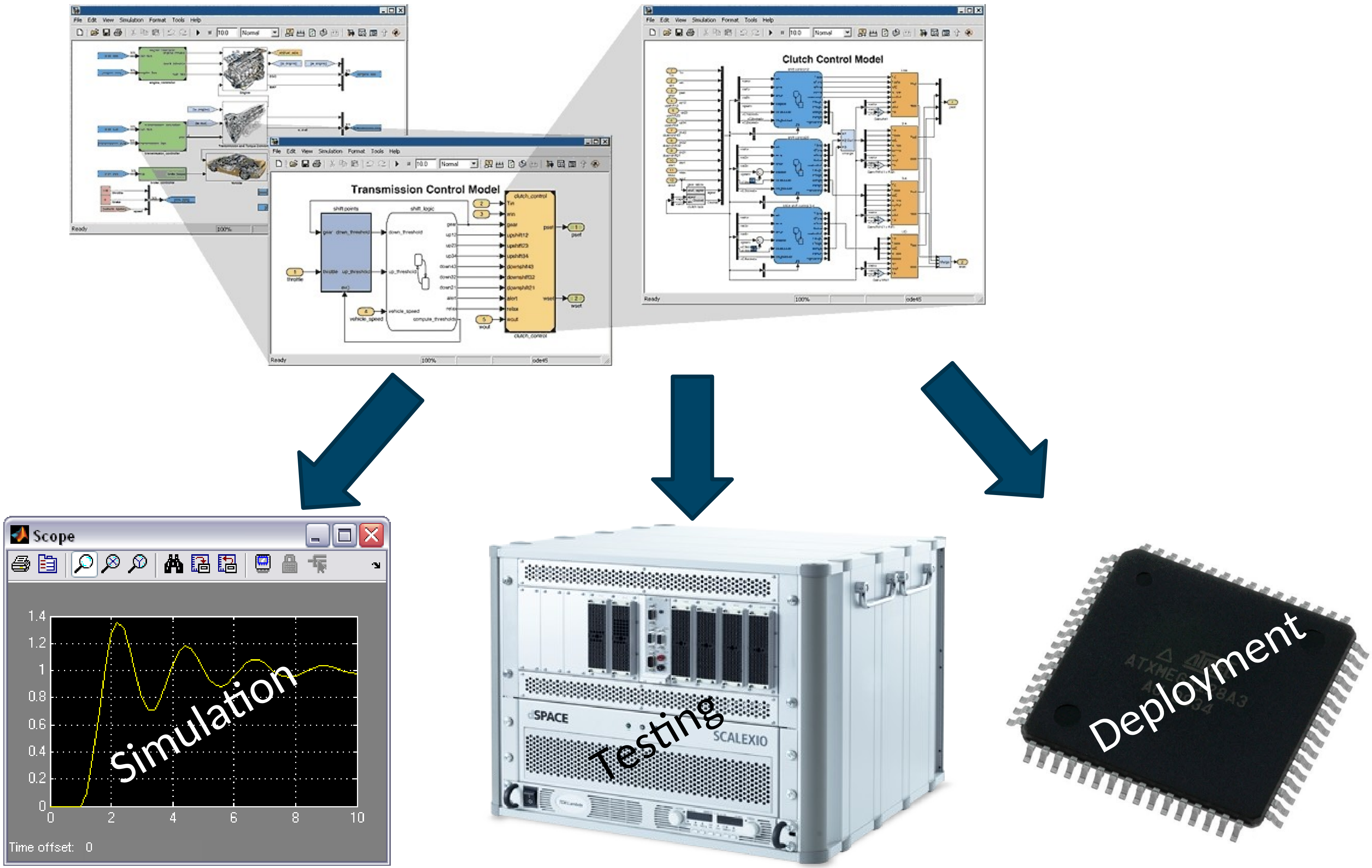
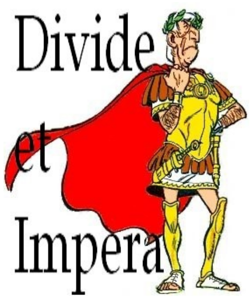
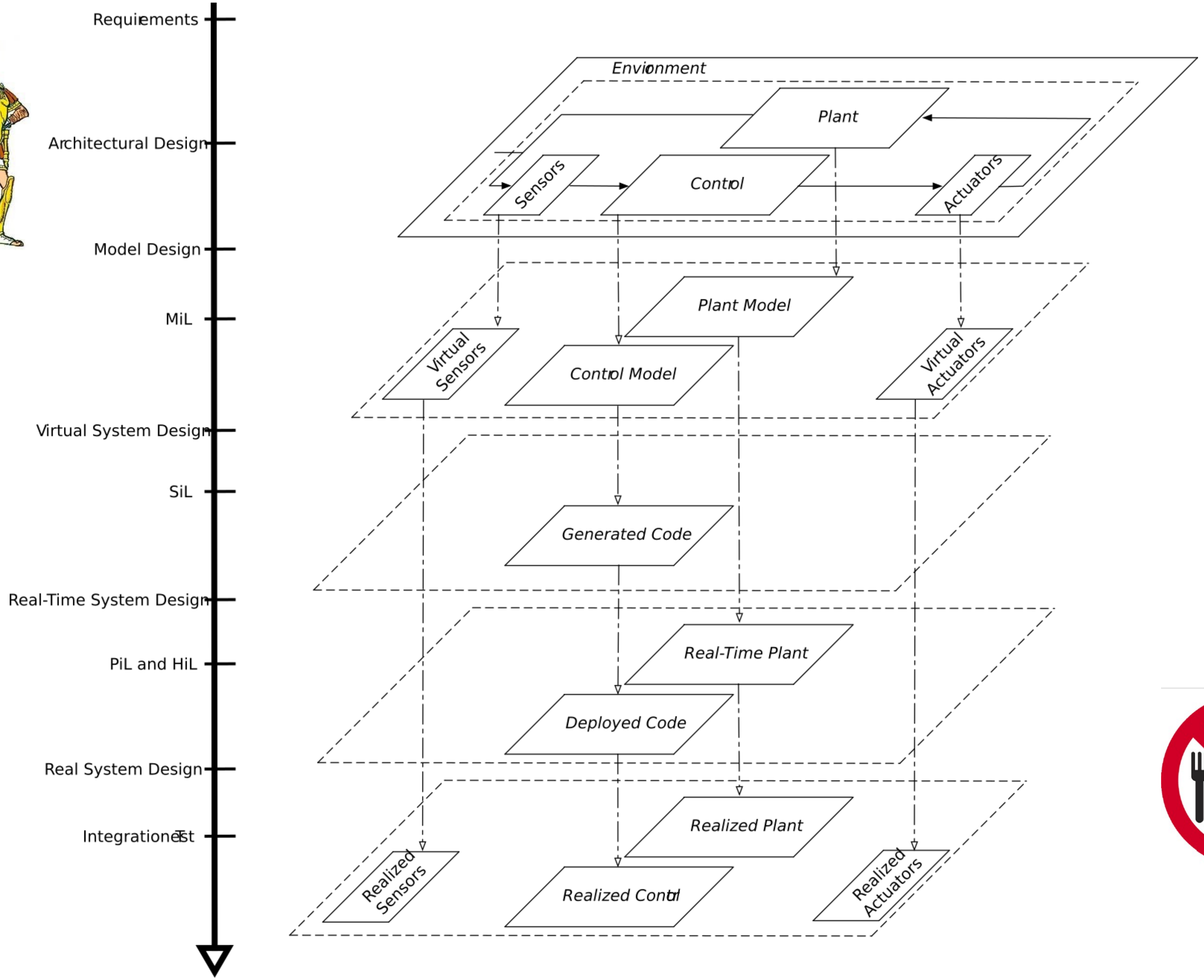


MiL, HiL, SiL, …

XiL: X = Model, Software, Processor, Hardware





vertical consistency!

Ken Vanherpen. A contract-based approach for multi-viewpoint consistency in the concurrent design of cyber-physical systems. PhD thesis University of Antwerp. 2018.

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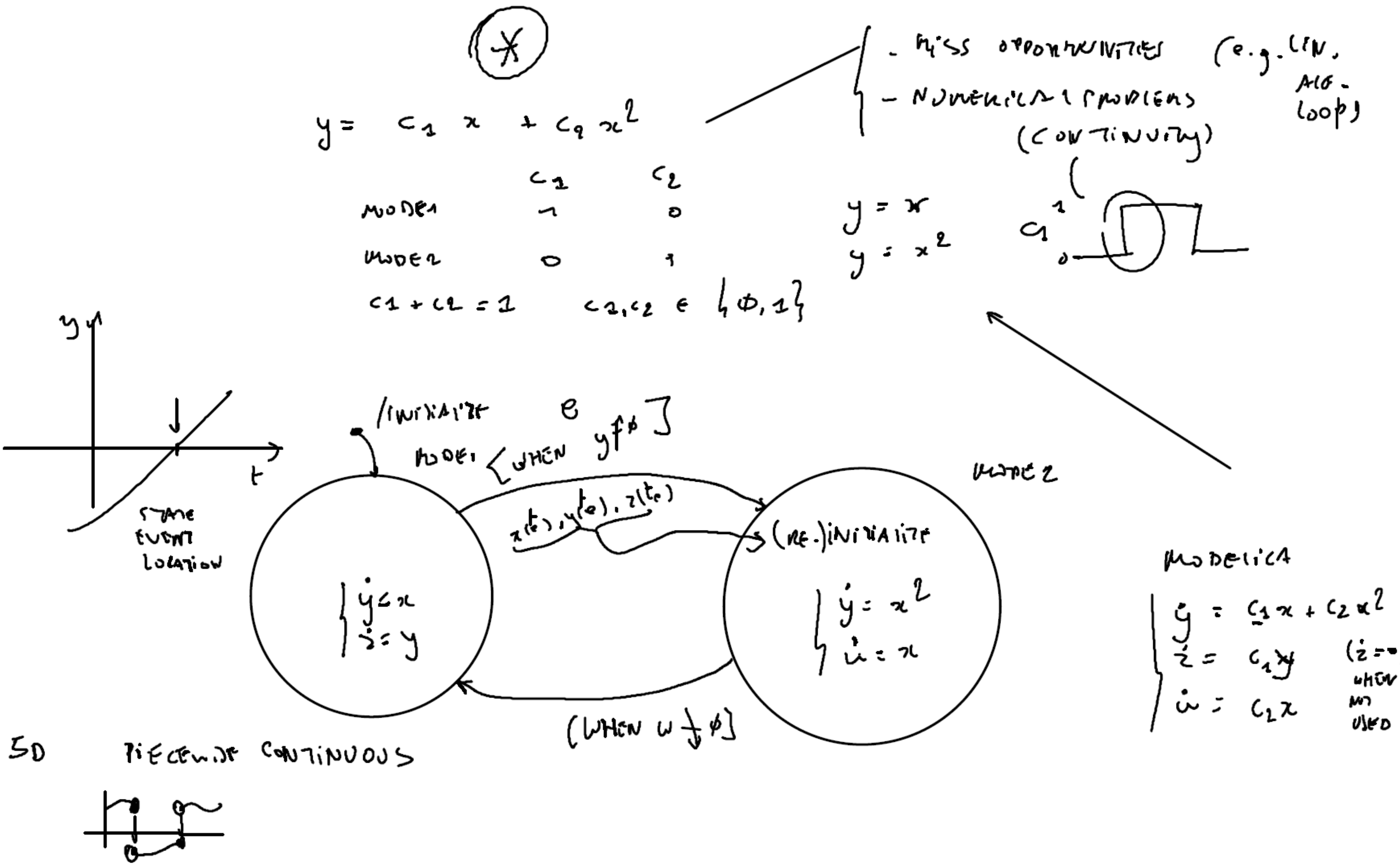
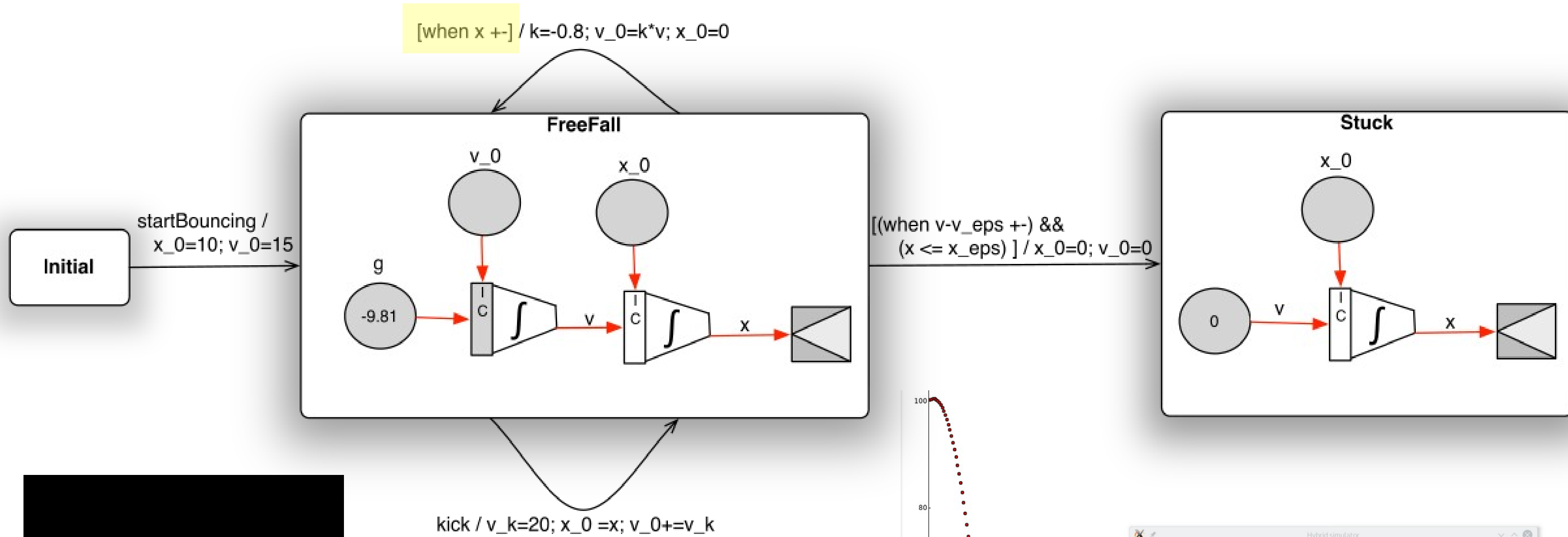
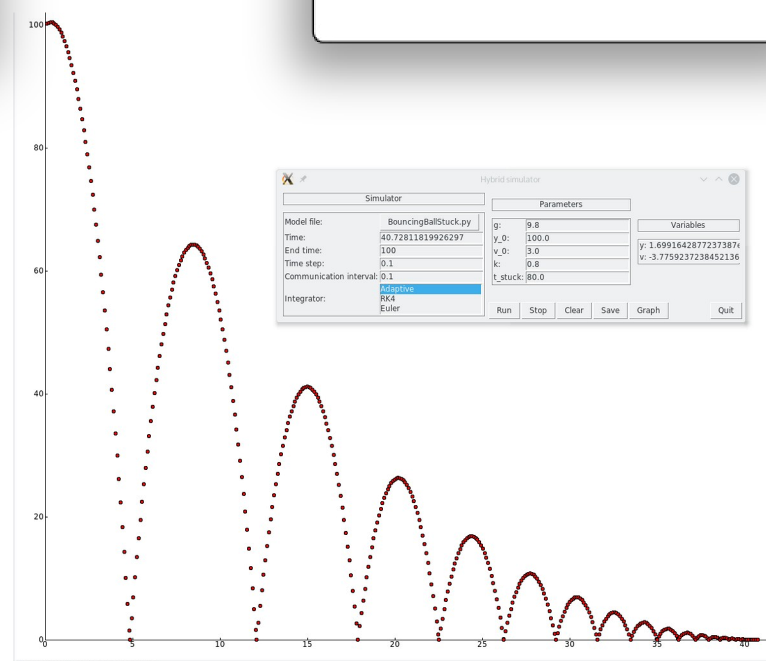
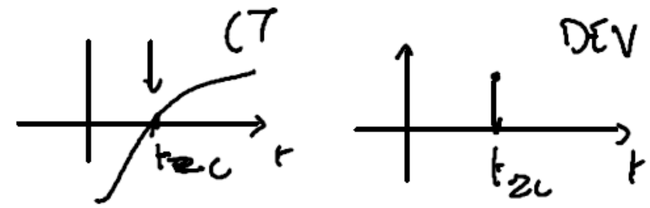
Hybrid (discrete-continuous) modelling/simulation



“bouncing ball” hybrid abstraction

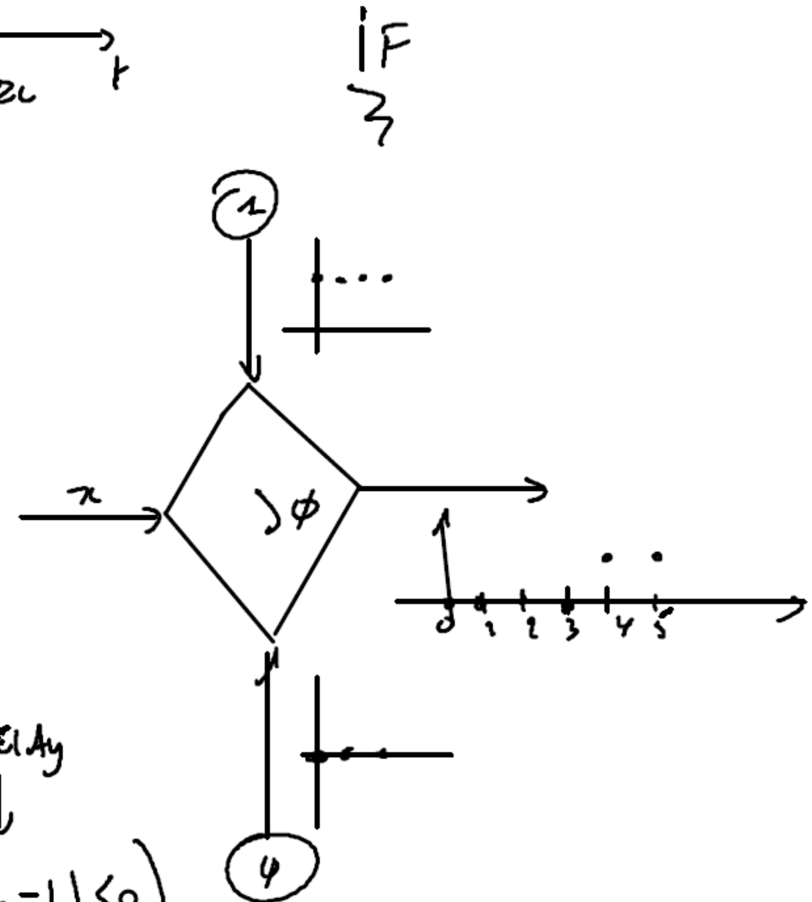


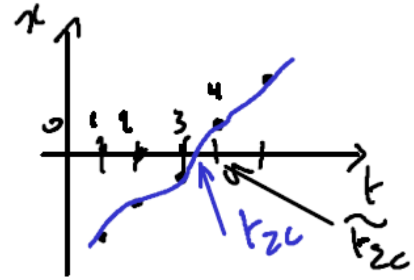




WHEN: (l?? → ??) → DEV SIGNAL

WHEN ($x(t) \neq 0$)

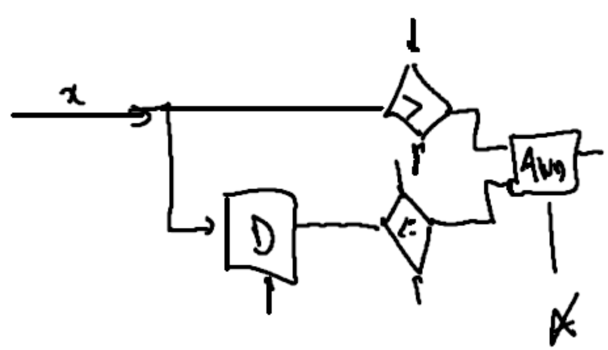


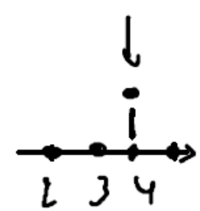


DISCR

DELAY

$$\tilde{t}_{zc}? : (\tilde{x}(\tilde{t}_{zc}) > 0) \,\&\, (\tilde{x}(\tilde{t}_{zc}-1) \leq 0)$$





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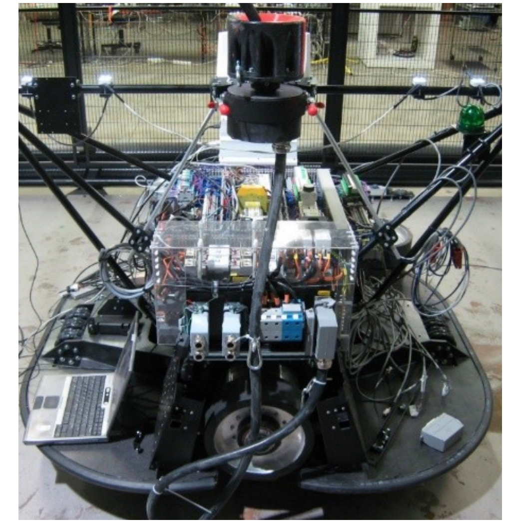
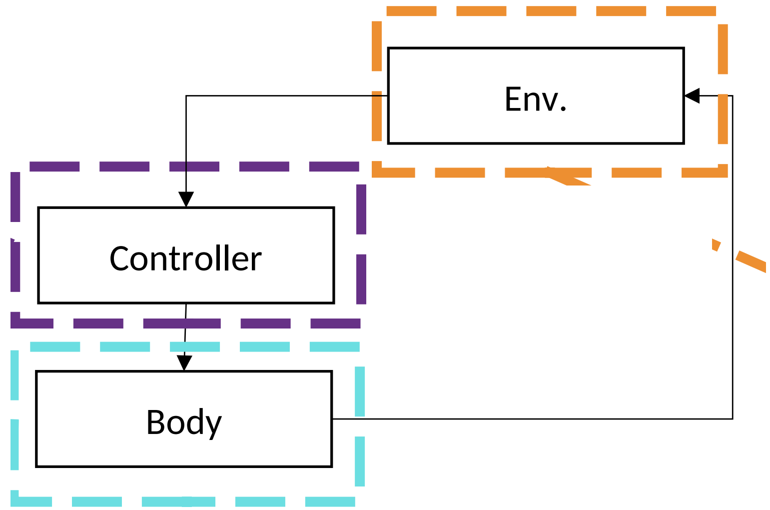
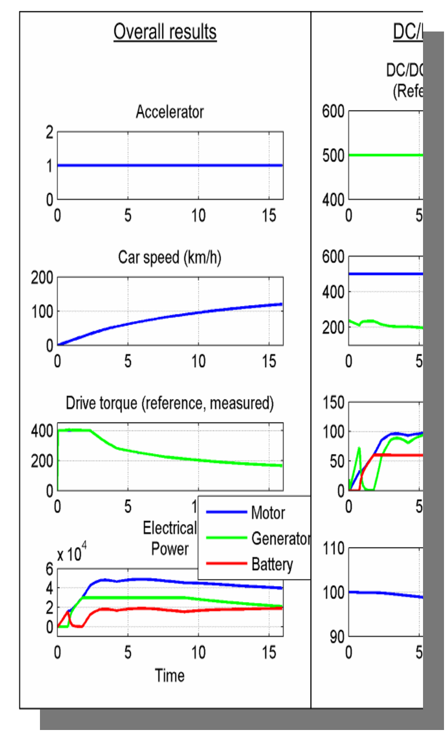
problem: **full-system analysis**

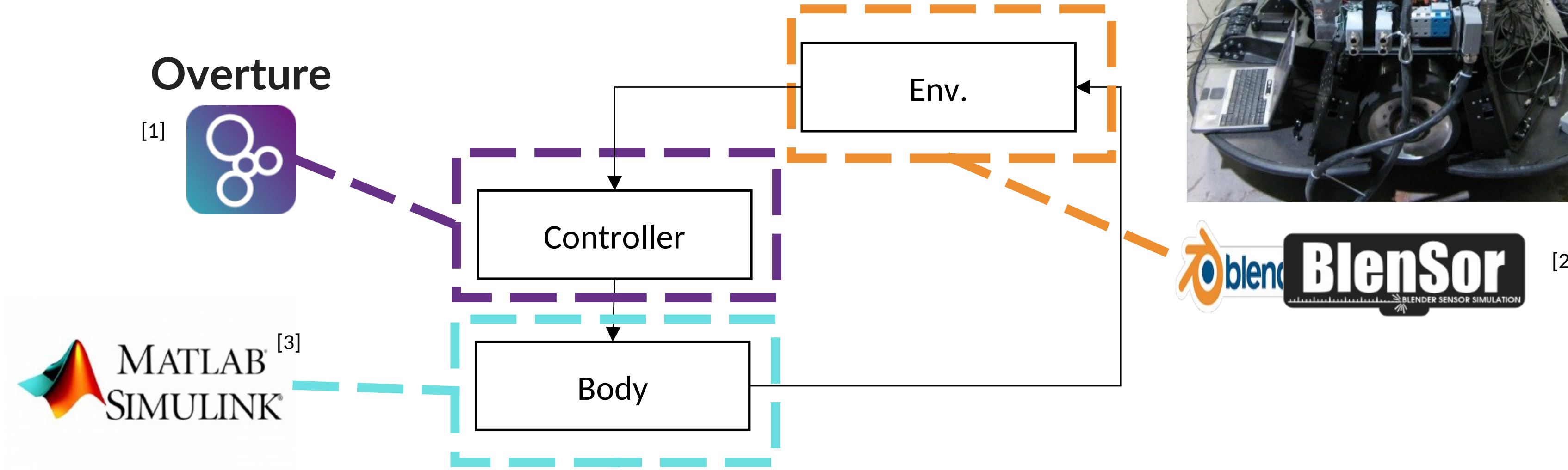


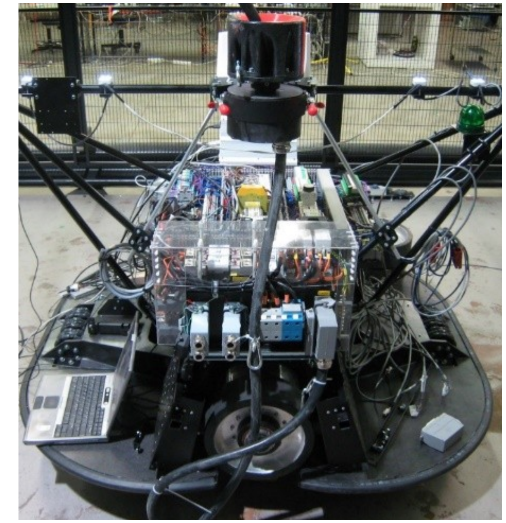


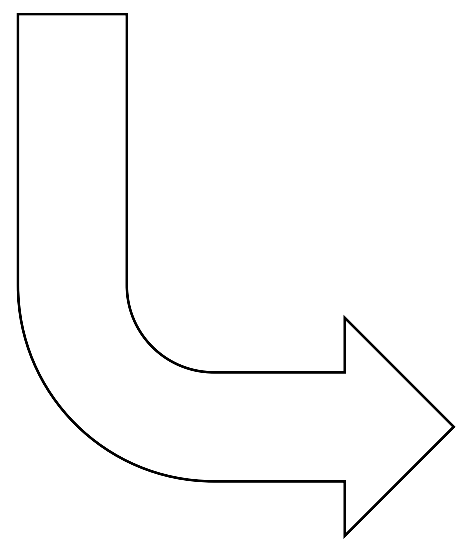


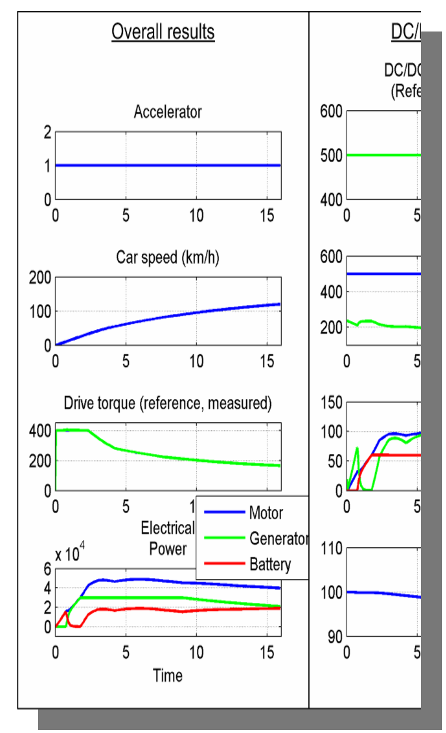
problem: **full-system analysis**

(also when IP protected)



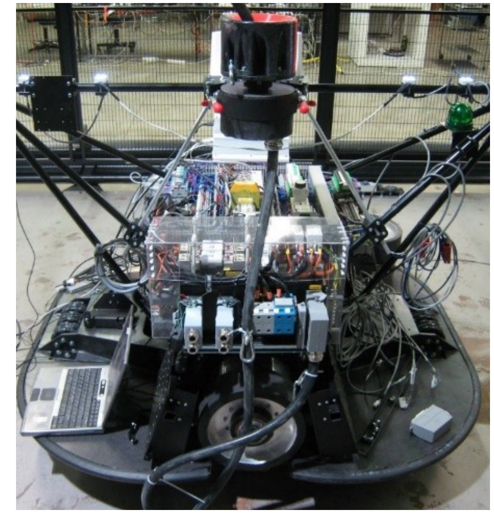




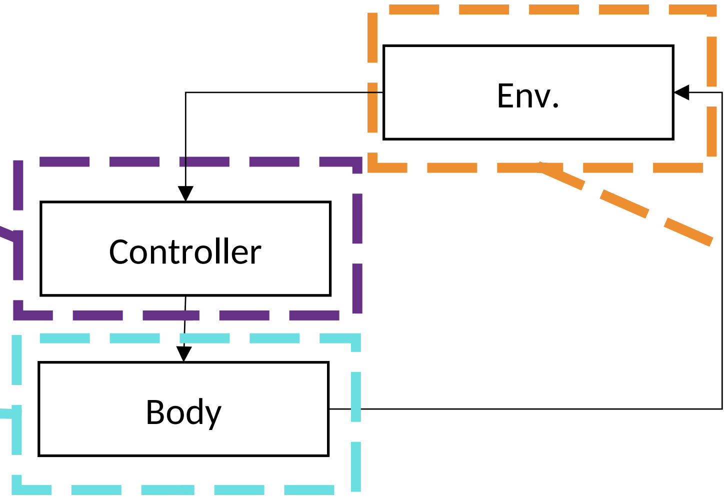


problem: **full-system analysis**

(also when IP protected)

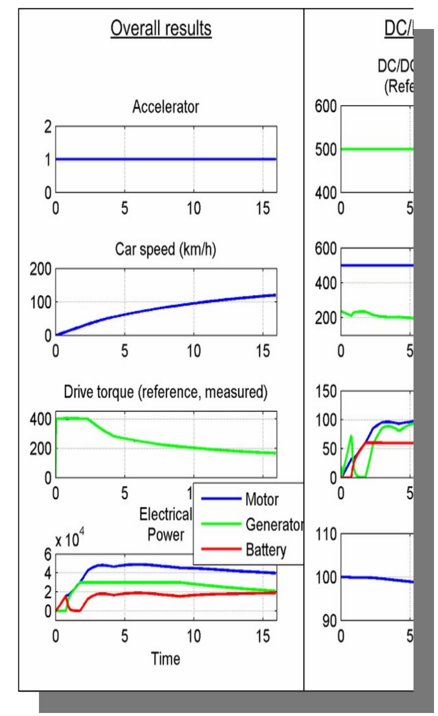


Overture [1]



MATLAB SIMULINK [3]

BlenSor [2]



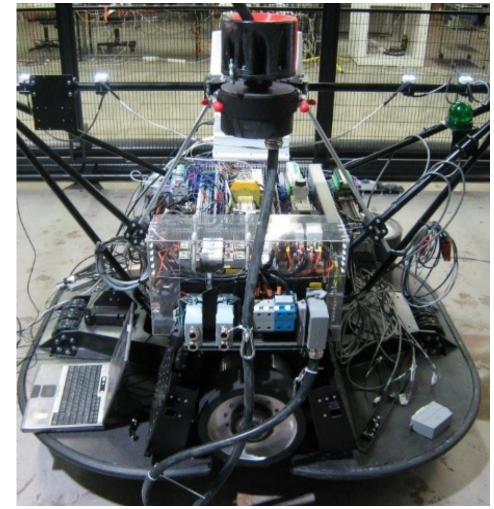
solution: combine sub-system **simulators**

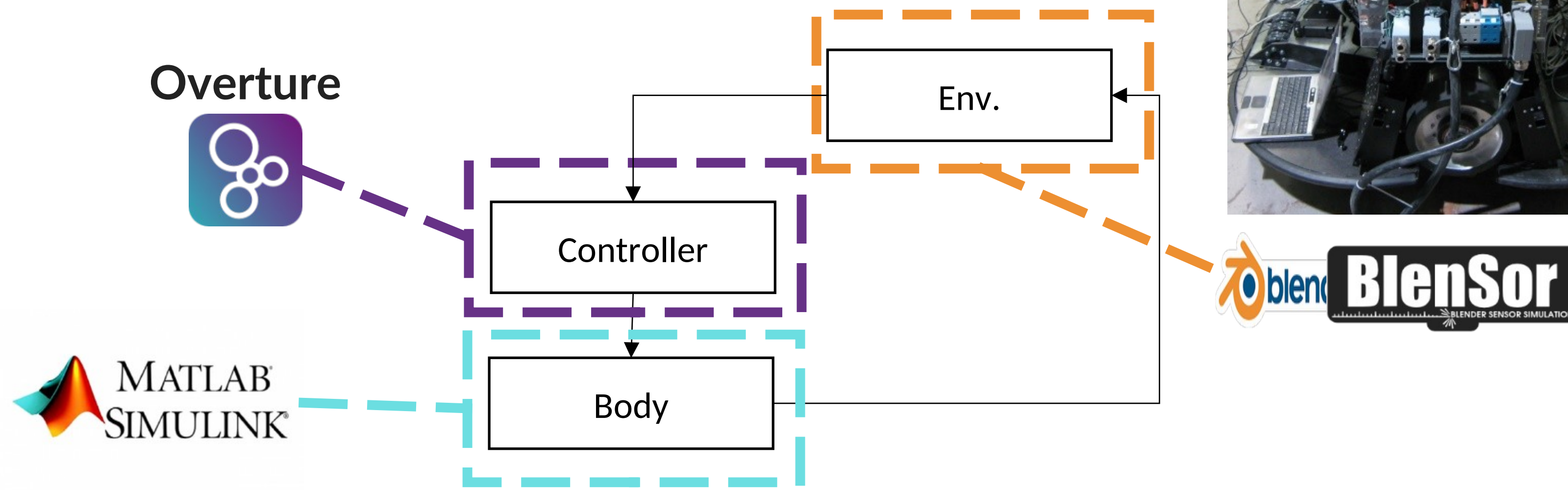
aka **co-simulation**

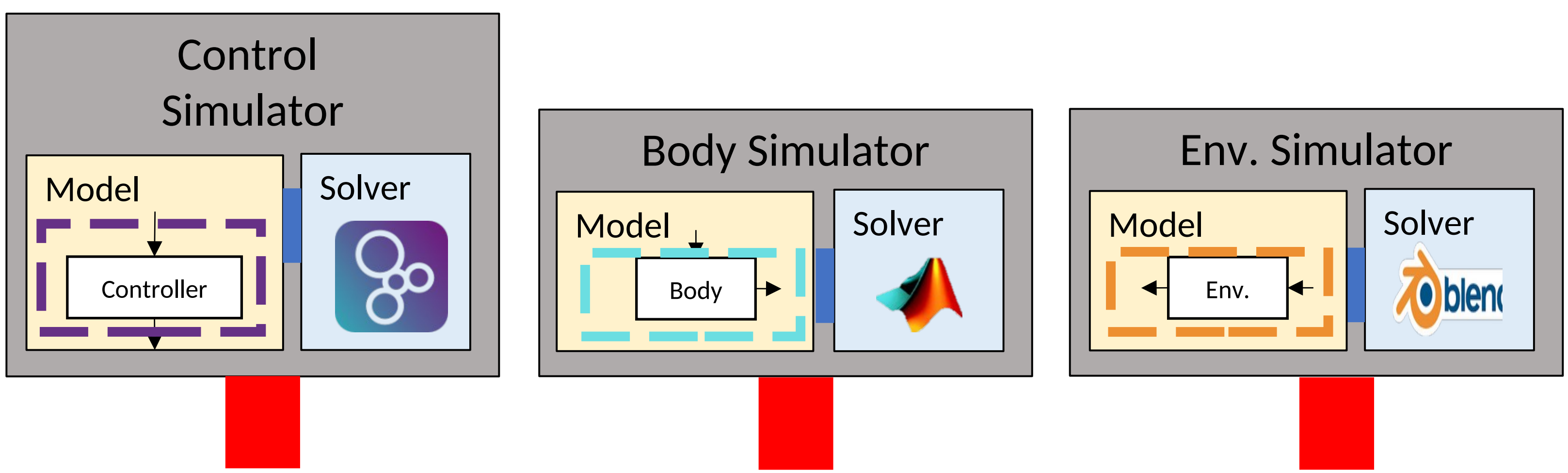
Cláudio Gomes, Casper Thule, David Broman, Peter Gorm Larsen, Hans Vangheluwe
Co-Simulation: A Survey. ACM Comput. Surv.51(3): 49:1-49:33 (2018)

co-simulation: how?

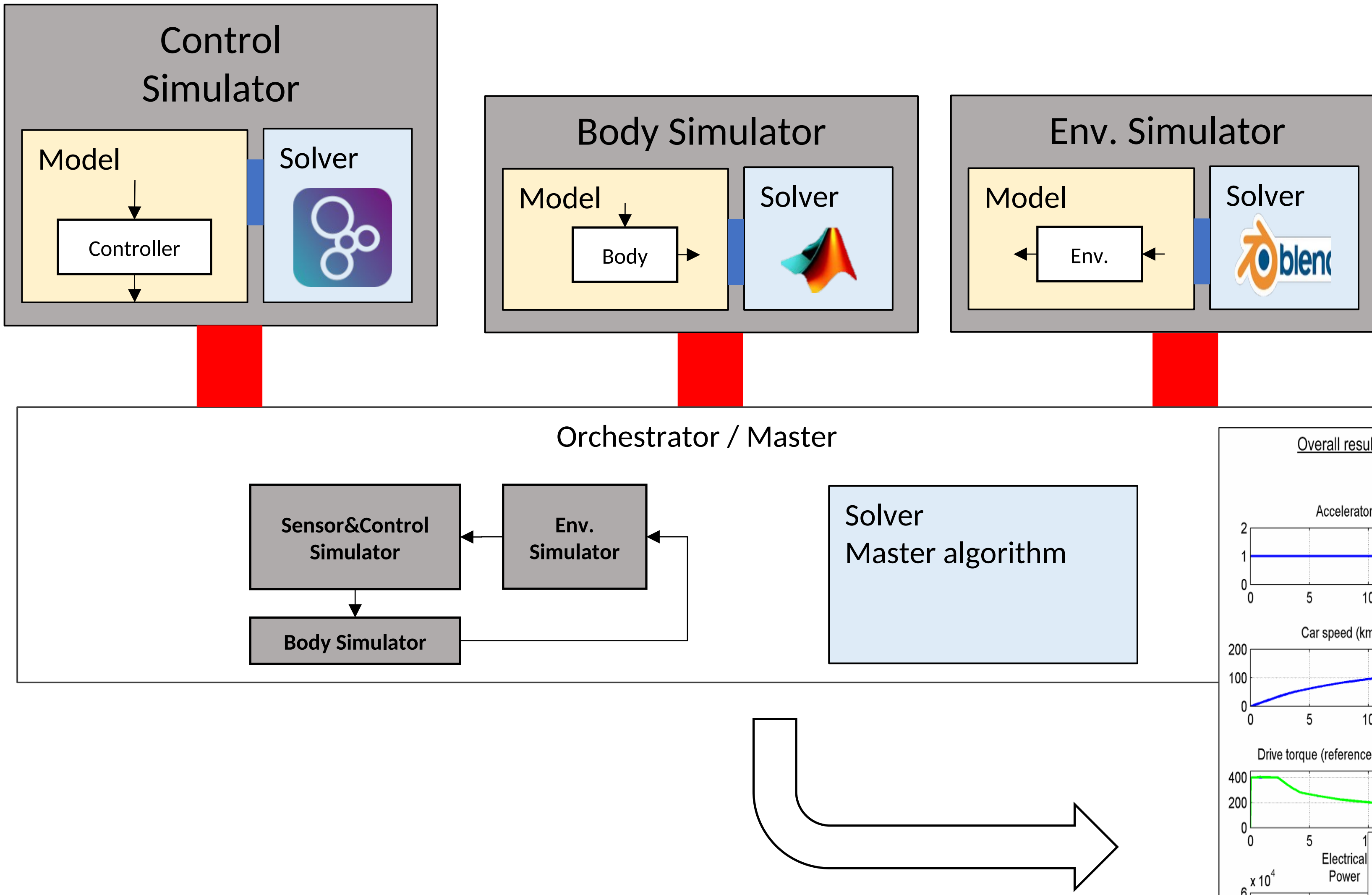
(when IP protected)

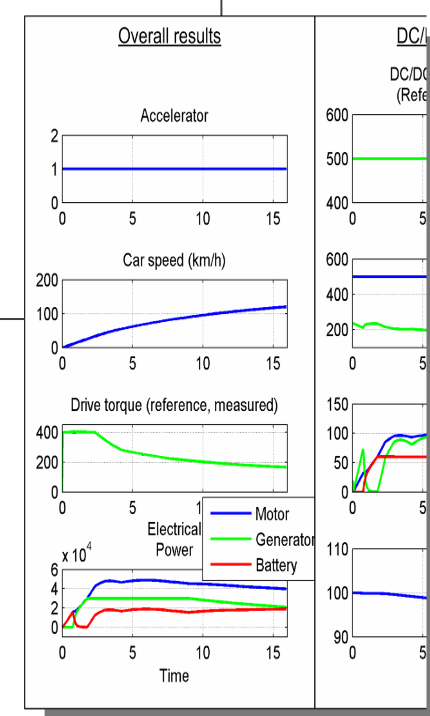






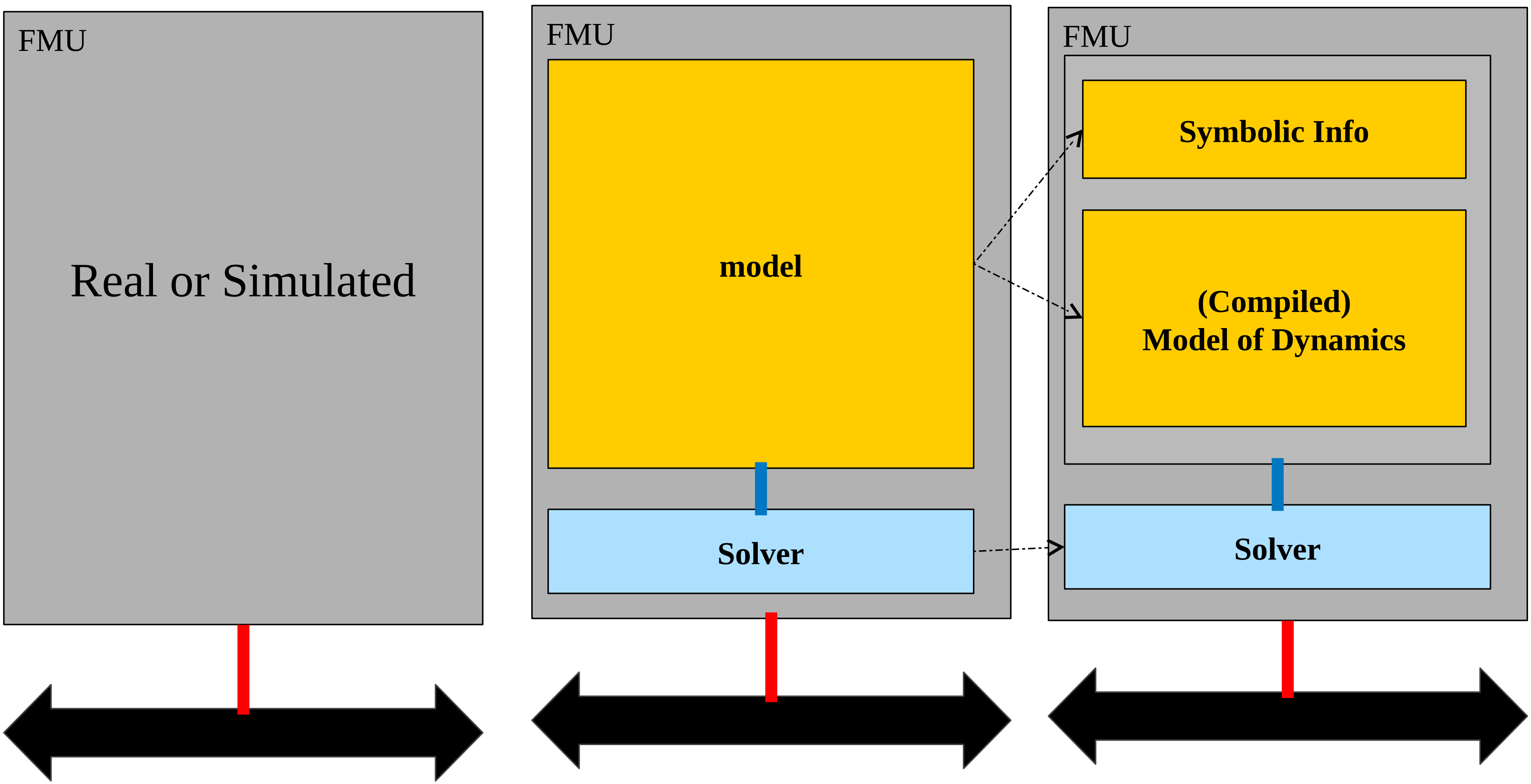
co-simulation: how?
(when IP protected)



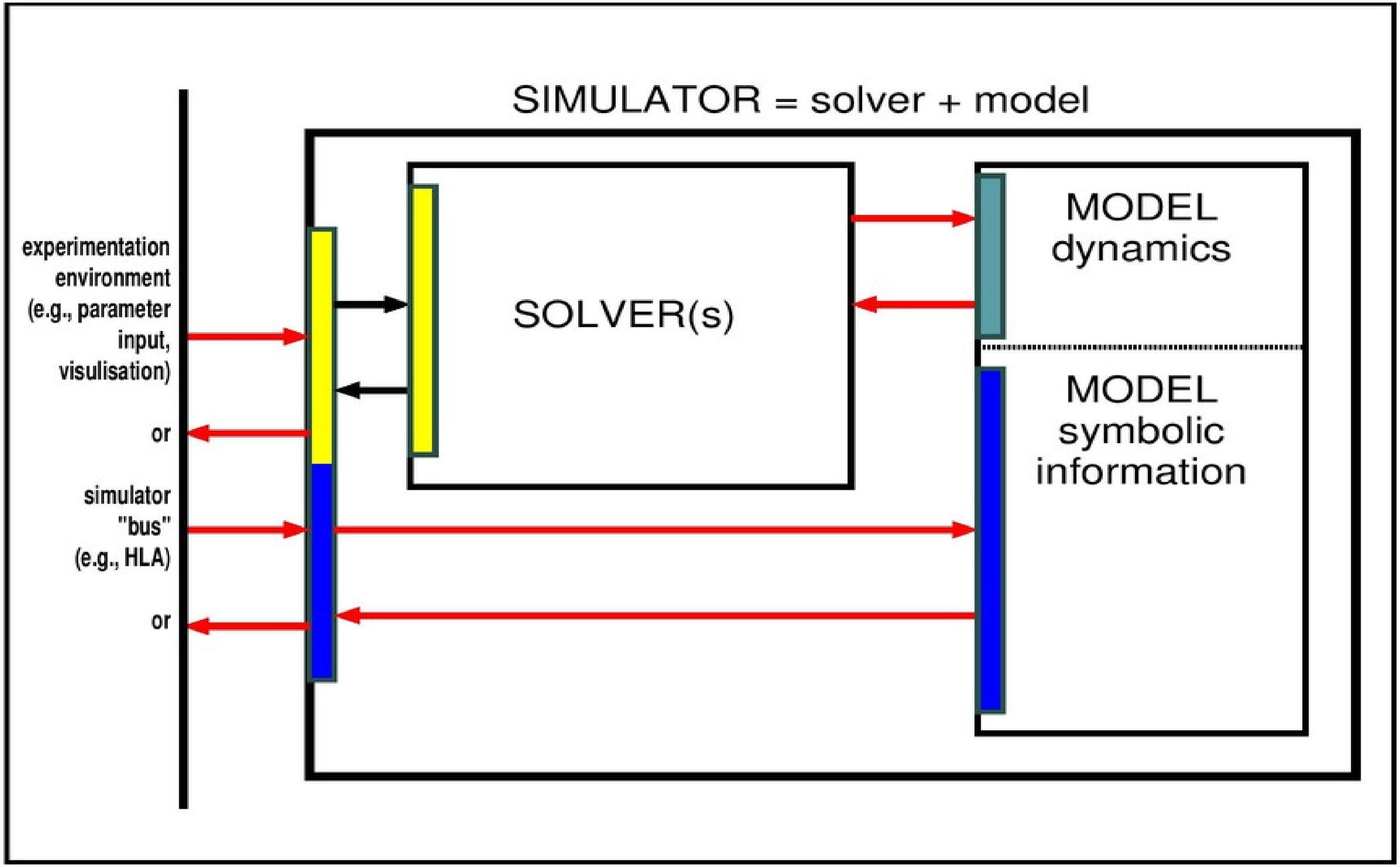


Minimally, Constrained Stable Switched Systems and Application to Co-Simulation
C Gomes, RM Jungers, B Legat, H Vangheluwe 2018
IEEE Conference on Decision and Control (CDC), 5676-5681

Deconstructing an FMU



Model-Solver Interface
Simulator-Environment Interface



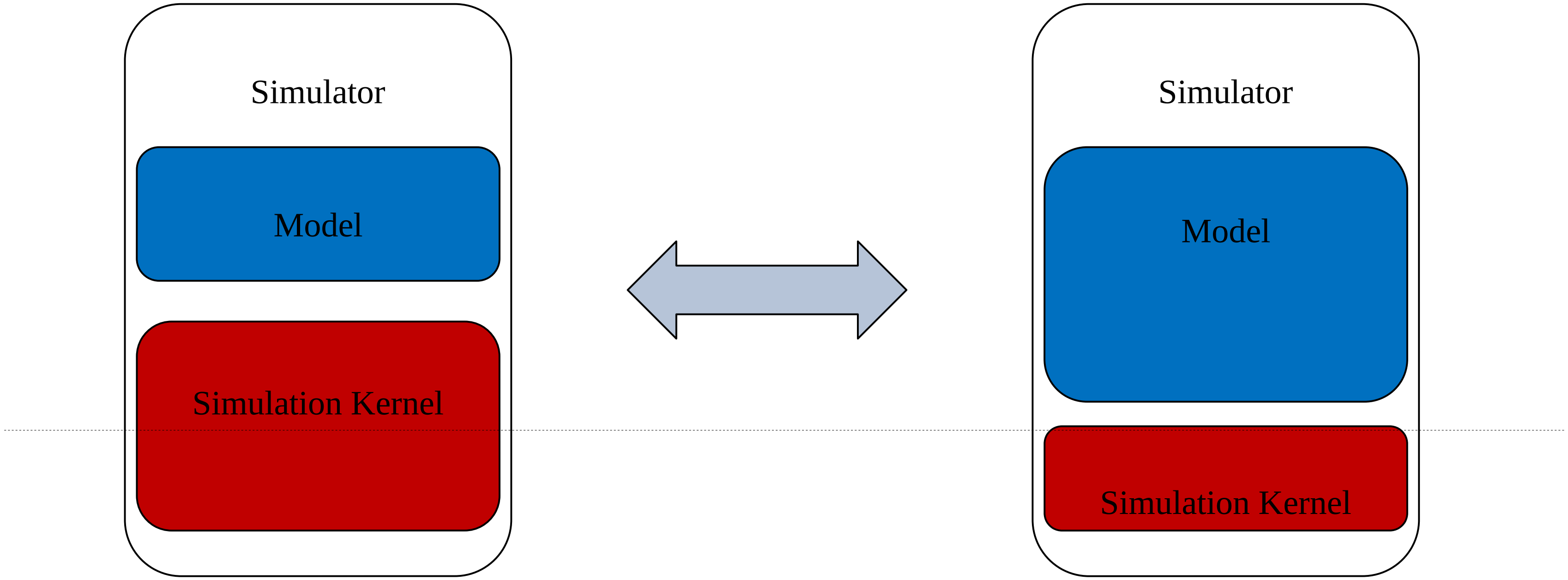
DSblock

Martin Otter and Hilding Elmquist.
The DSblock interface for exchanging model components. Eurosim '95 Simulation Congress. pp. 505- 510. 1995.

MSL-EXEC

Henk Vanhooren, Jurgen Meirlaen, Youri Amerlinck, Filip Claeys, Hans Vangheluwe, and Peter A. Vanrolleghem.
WEST: Modelling biological wastewater treatment. Journal of Hydroinformatics , 5(1):27--50, 2003.

meaningful operational semantics
(Models of Computation)



Traditional
Simulator

Explicit
Computational Semantics
“inline integration” (Cellier)

modelisar

fmi FUNCTIONAL MOCK-UP INTERFACE

fmi Functional Mock-up Interface

The leading standard to exchange dynamic simulation models

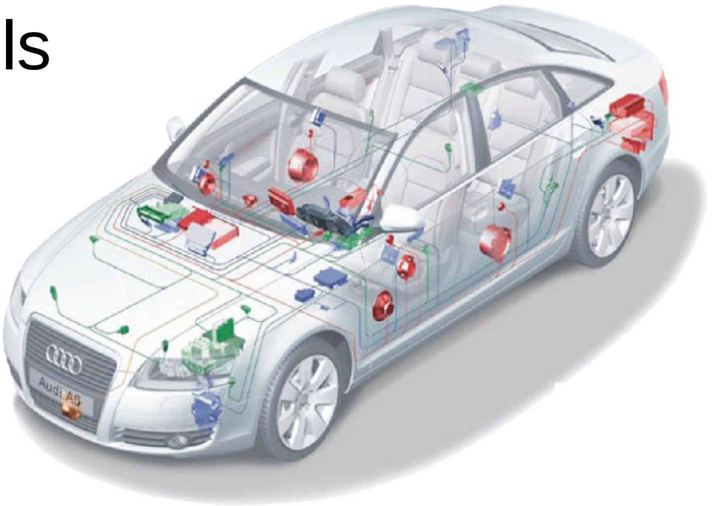
The Functional Mock-up Interface is a free standard that defines a container and an interface to exchange dynamic simulation models using a combination of XML files, binaries and C code, distributed as a ZIP file. It is supported by 180+ tools and maintained as a Modelica Association Project.

Why FMI | Complete Package 3.0.1 | Specification 3.0.1 | Implementers' Guide

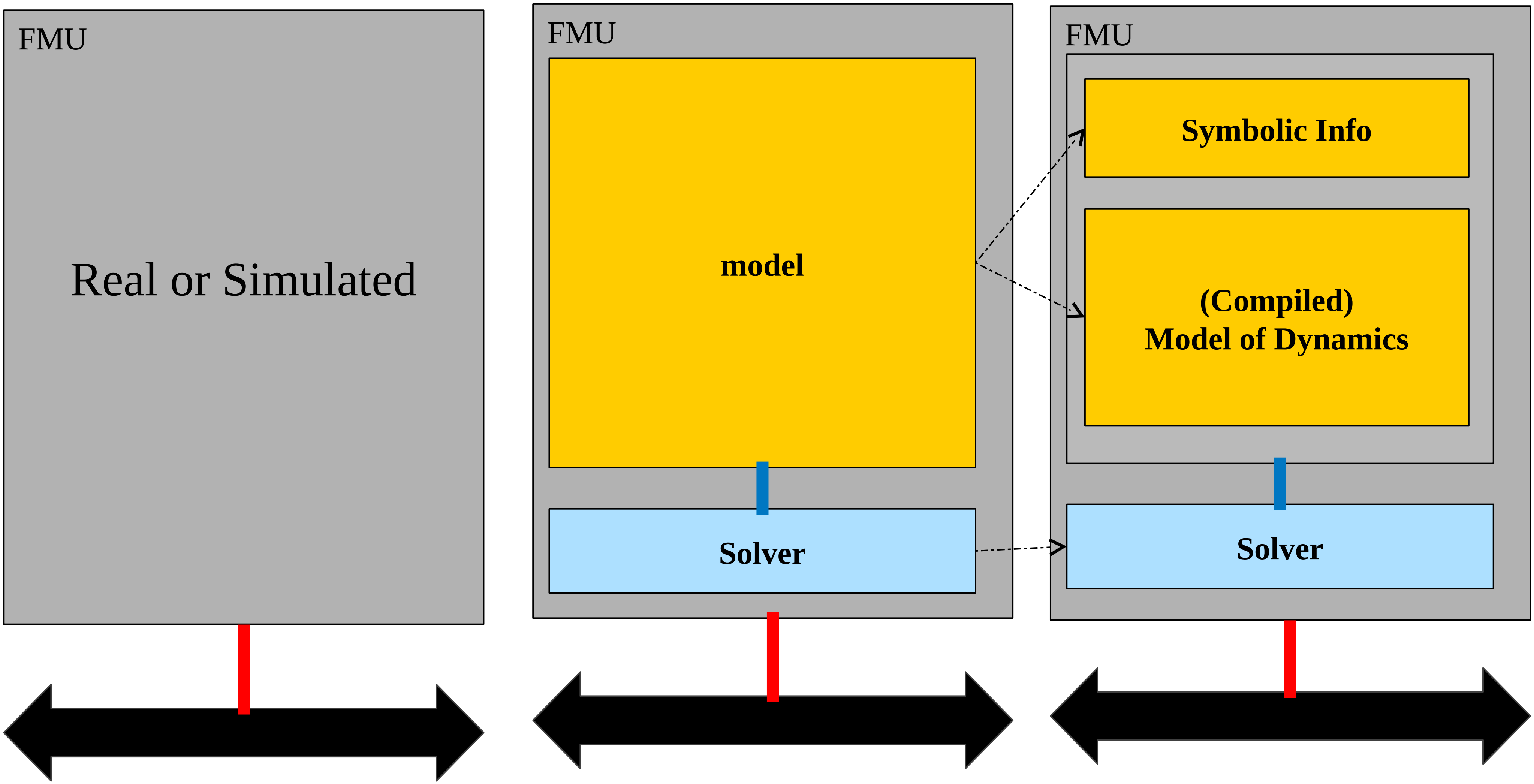
https://fmi-standard.org/

Functional Mock-up Interface (FMI)

- **XML** + Binary Representation for Models
  - Standard
  - Modelling Tool Independent
  - +/- Black box …



Deconstructing an FMU

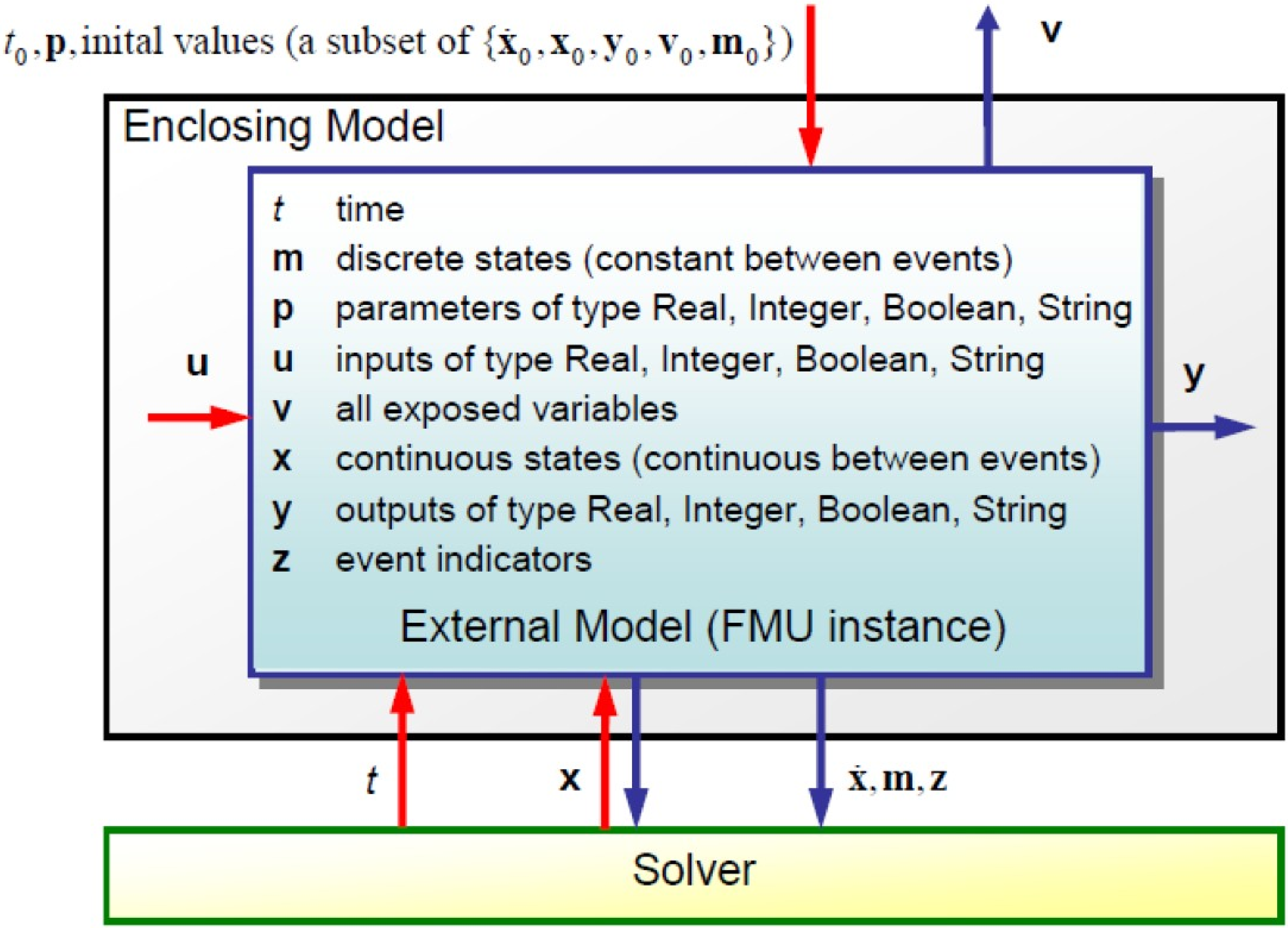


Symbolic information: FMU - XML

```
<?xml version="1.0" encoding="iso-8859-1"?>
<fmiModelDescription fmiVersion="1.0"
 modelName="BackEulerExmpl"
 modelIdentifier="BackEulerExmpl"
 guid="{7c4e810f-3da3-4a00-8276-176fa3c9f000}"
 numberOfContinuousStates="1"
 numberOfEventIndicators="0">
 <ModelVariables>
  <ScalarVariable
         name="Delay" valueReference="0">
    <Real start="0.0" fixed="true" />
  </ScalarVariable>
  <ScalarVariable
         name="stepSize" valueReference="1">
    <Real start="0.1" fixed="true" />
  </ScalarVariable>
  <ScalarVariable
         name="Product" valueReference="2">
    <Real start="0.0" fixed="true" />
  </ScalarVariable>
  <ScalarVariable
         name="Negator" valueReference="3">
    <Real start="0.0" fixed="true" />
  </ScalarVariable>
  <ScalarVariable
         name="Adder" valueReference="4">
    <Real start="0.0" fixed="true" />
  </ScalarVariable>
 </ModelVariables>
</fmiModelDescription>
```

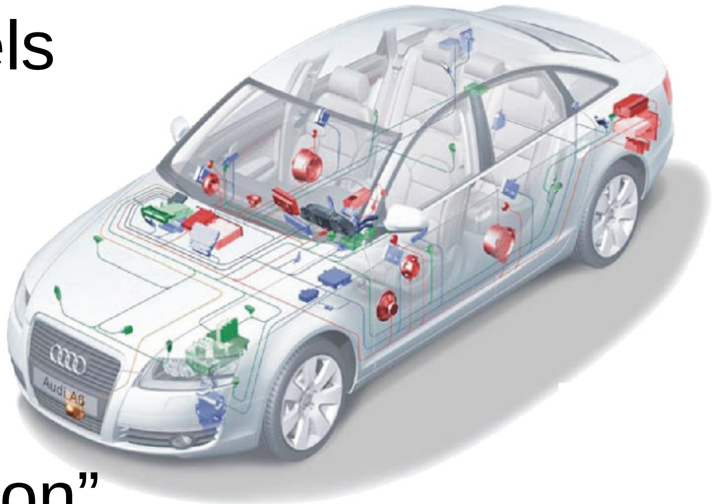
Behaviour information: FMU - C

```
fmi2Status fmi2DoStep(fmi2Component fc , fmi2Real currentCommPoint, fmi2Real commStepSize, fmi2Boolean
    noPrevFMUState)
{
    FMUInstance* fi = (FMUInstance *)fc;
    fmi2Status simStatus = fmi2OK;
    printf("%s in fmiDoStep()\n",fi->instanceName);
    fi->currentTime = currentCommPoint + commStepSize;
    printf("Motor_in: %f\n", fi->r[_motor_in]);
    printf("slave CBD_PART2 now at time: %f\n", fi->currentTime);

    fi->r[_position] =  fi->r[_position] + fi->r[_velocity] * commStepSize;
    fi->r[_velocity] = fi->r[_velocity] + fi->r[_acceleration_after_friction] * commStepSize;
    fi->r[_friction] = fi->r[_velocity] * 5.81;
    fi->r[_motor_acceleration] = fi->r[_motor_in] * 40;
    fi->r[_acceleration_after_friction] = fi->r[_motor_acceleration] - fi->r[_friction];

    return simStatus;
}

fmi2Status fmi2GetReal(fmi2Component fc, const fmi2ValueReference vr[], size_t nvr, fmi2Real value[])
{
    FMUInstance* comp = (FMUInstance *)fc;
    int i;
    for (i = 0; i < nvr; i++)
    {
        value[i] = comp->r[(vr[i])];
    }
    return fmi2OK;
}
```



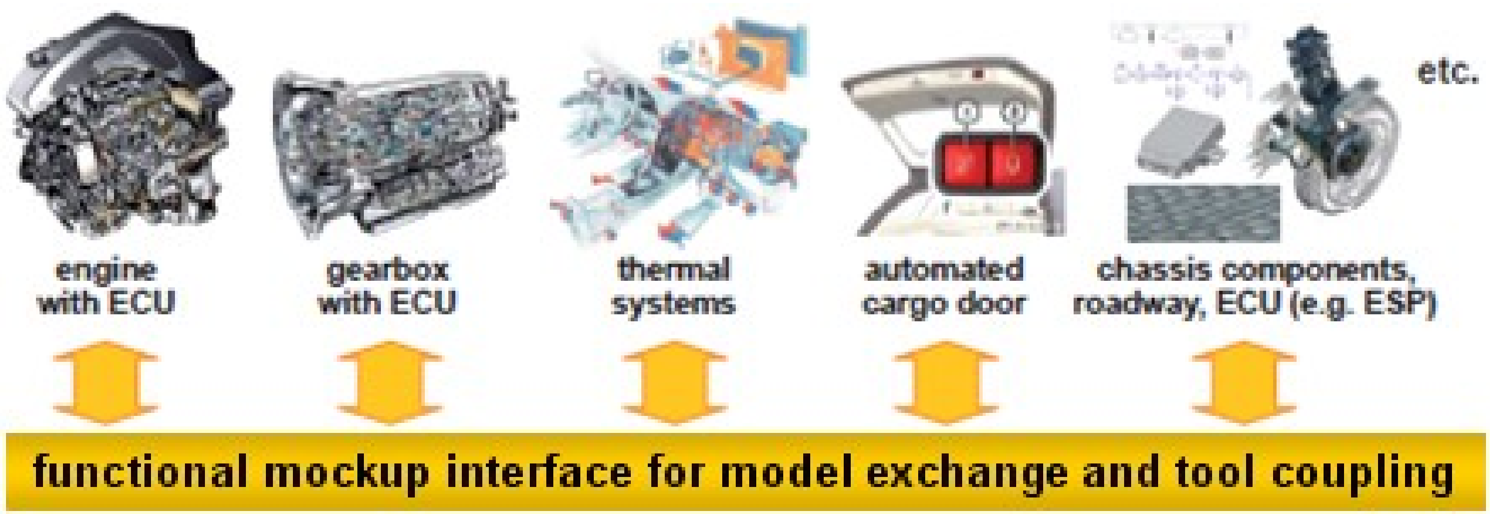
t_0, **p**, inital values (a subset of $\{\dot{\mathbf{x}}_0, \mathbf{x}_0, \mathbf{y}_0, \mathbf{v}_0, \mathbf{m}_0\}$)
v
Enclosing Model
t time
m discrete states (constant between events)
p parameters of type Real, Integer, Boolean, String
u inputs of type Real, Integer, Boolean, String
v all exposed variables
x continuous states (continuous between events)
y outputs of type Real, Integer, Boolean, String
z event indicators
External Model (FMU instance)
u
y
t
x
$\dot{\mathbf{x}}, \mathbf{m}, \mathbf{z}$
Solver

Functional Mock-up Interface (FMI)



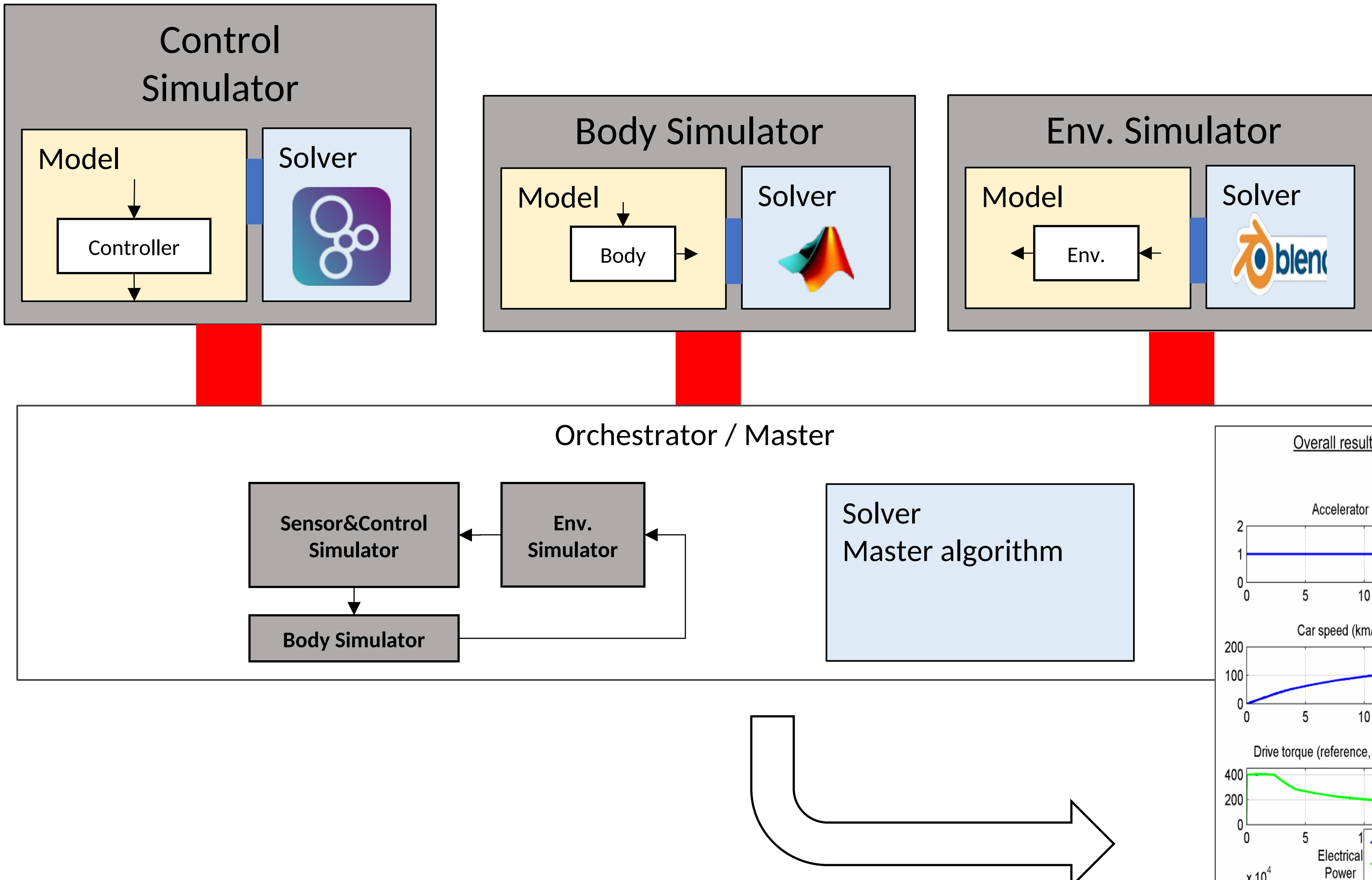
- **XML** + Binary Representation for Models
  - Standard
  - Modelling Tool Independent
  - +/- Black box …

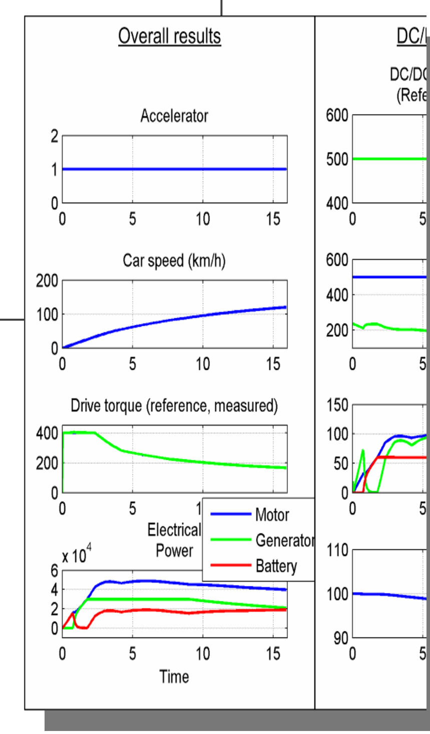
- Composed FMUs still need “orchestration”



www.fmi-standard.org

co-simulation: how?
(when IP protected)





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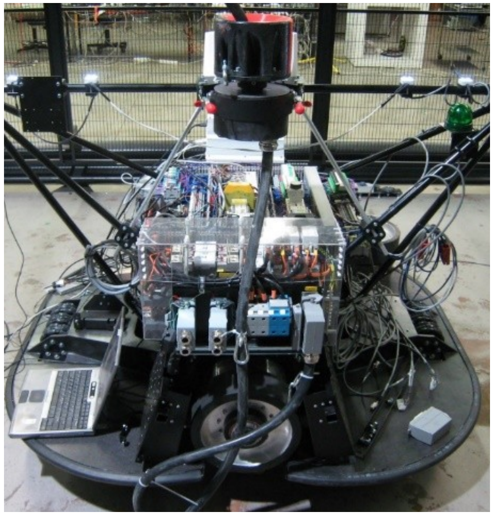
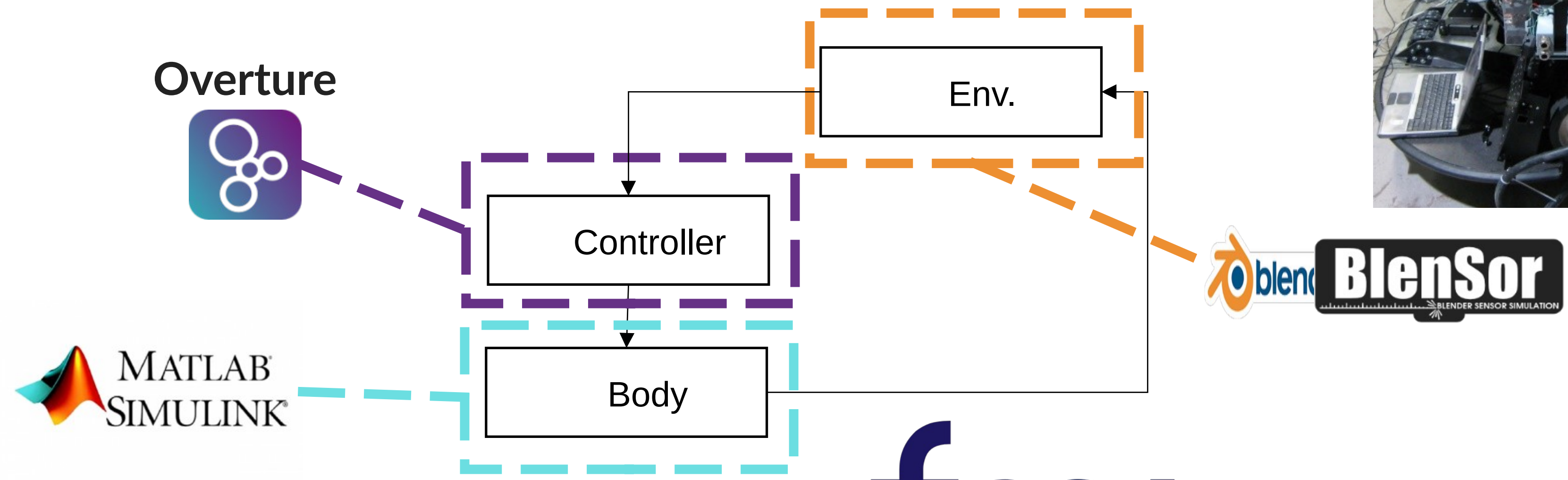




fmi FUNCTIONAL MOCK-UP INTERFACE [1]

Control Simulator

FMU.zip
|-- .dll or .so
|-- .xml

Body Simulator

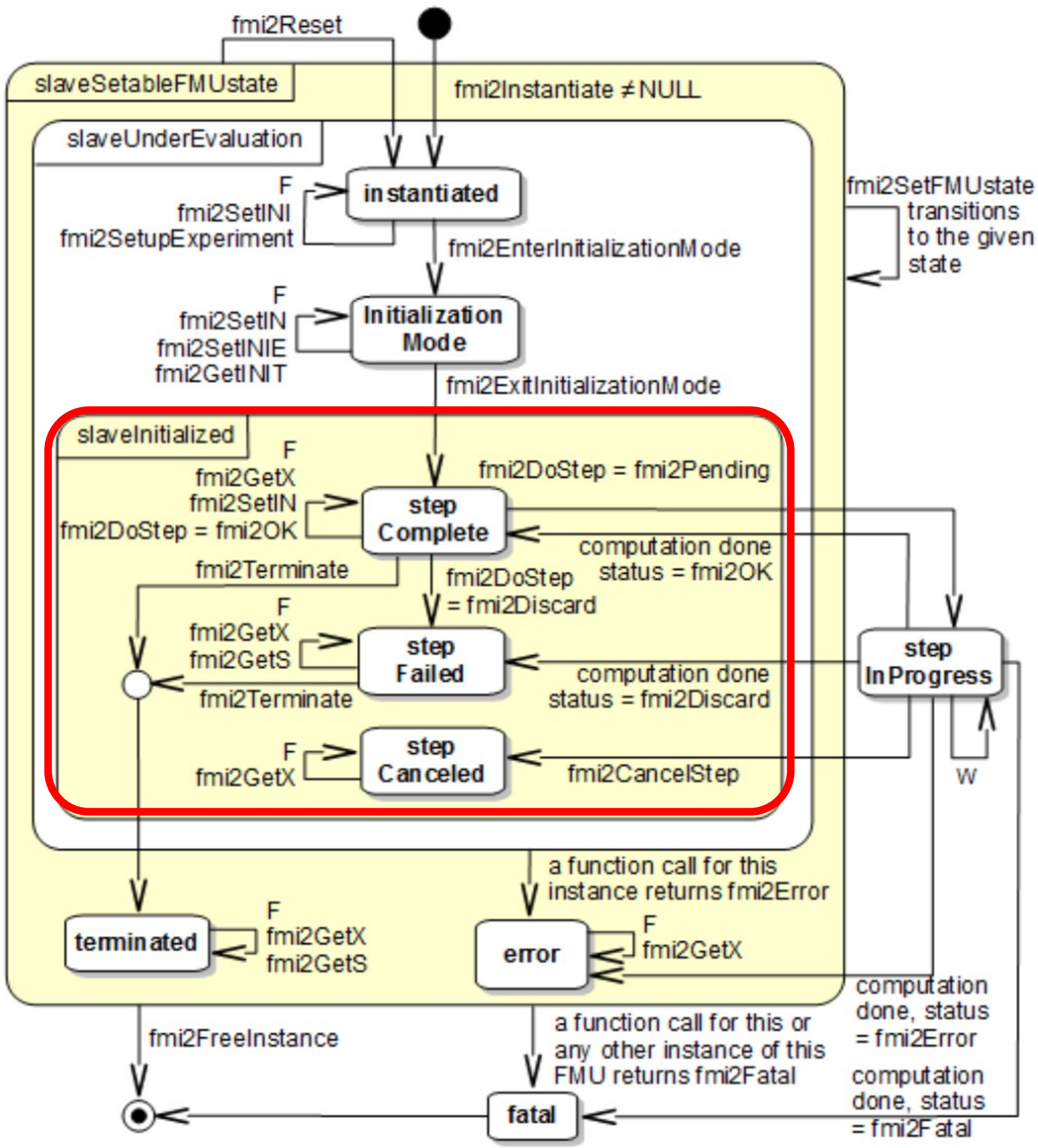
Env. Simulator

fmi2DoStep(…), fmi2SetReal(…), …

[1] www.fmi-standard.org

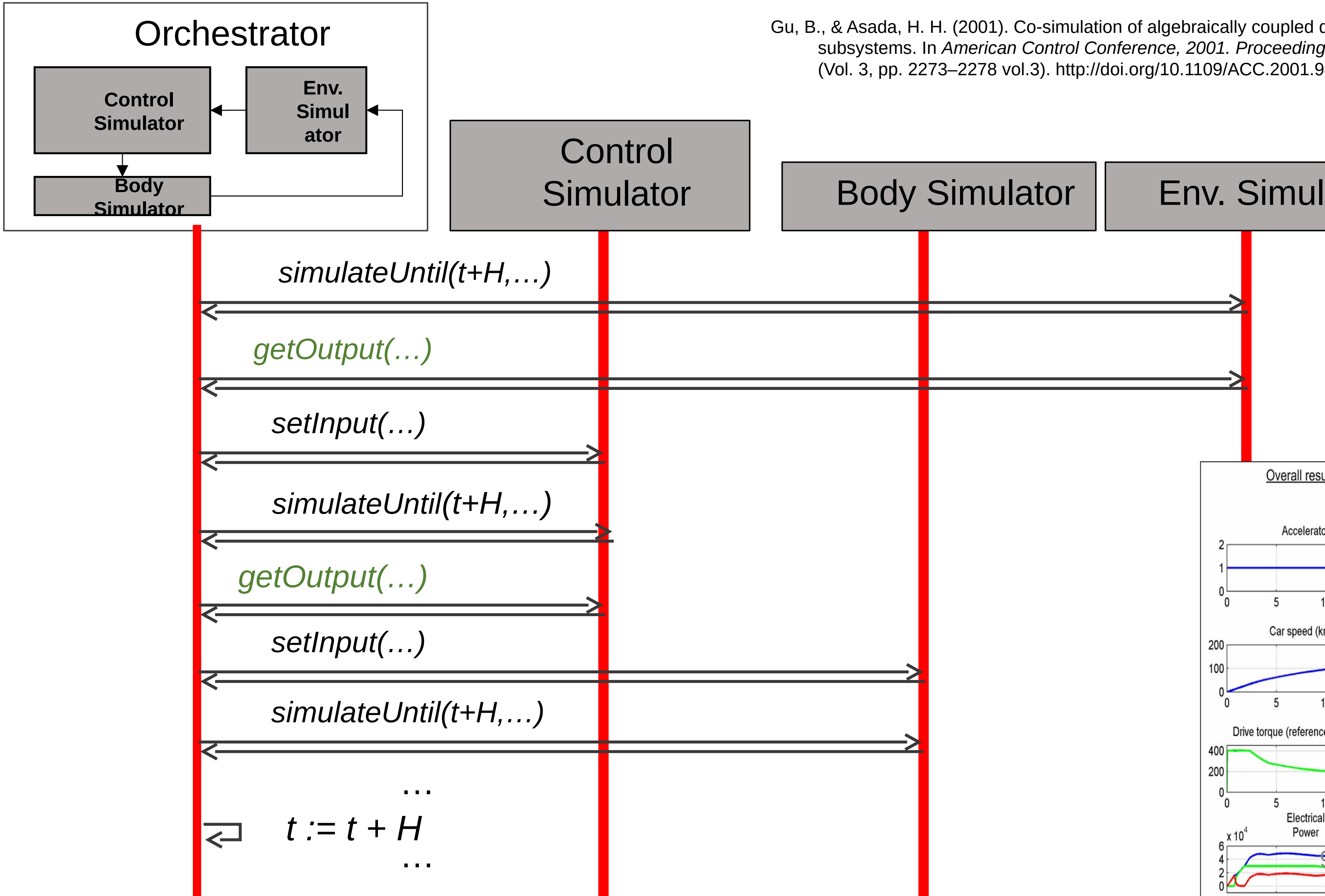
Co-simulation master

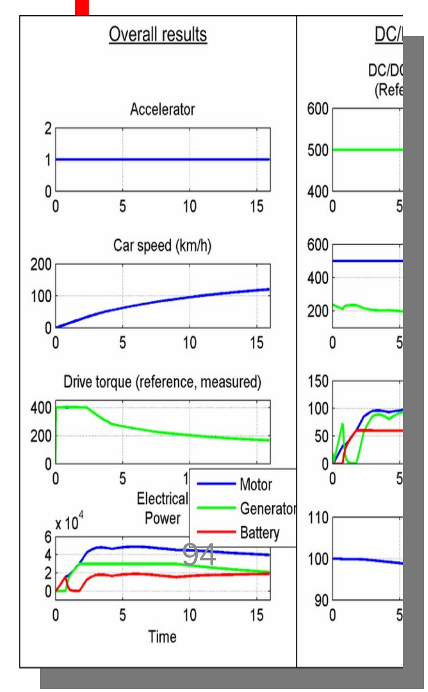
State machine calling sequence Master-Slave



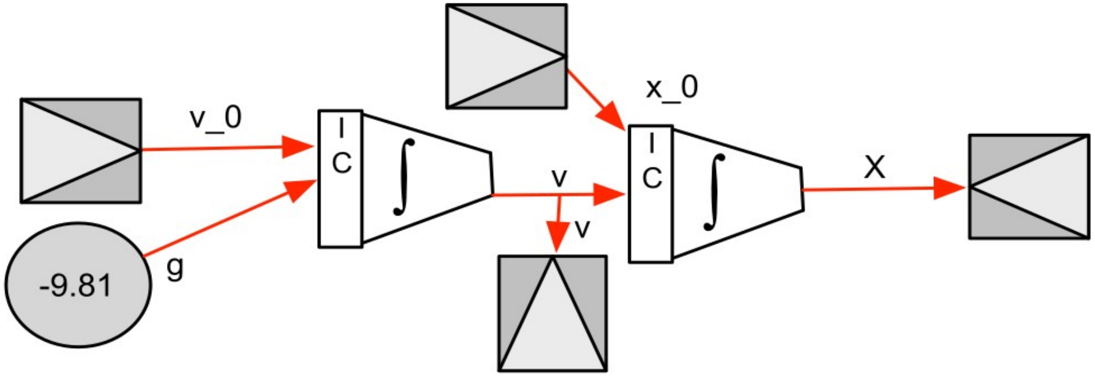
Co-simulation: how?

Gu, B., & Asada, H. H. (2001). Co-simulation of algebraically coupled dynamic subsystems. In *American Control Conference, 2001. Proceedings of the 2001* (Vol. 3, pp. 2273–2278 vol.3). http://doi.org/10.1109/ACC.2001.946089





Looking inside a simulator/solver for Causal Block Diagrams (CBDs)



$logicalTime \leftarrow 0$

while not end_condition **do**

  $schedule \leftarrow LOOPDETECT(DEPGRAPH(cbd))$

  for gblock **in** schedule **do**

    $COMPUTE(gblock)$

  end for

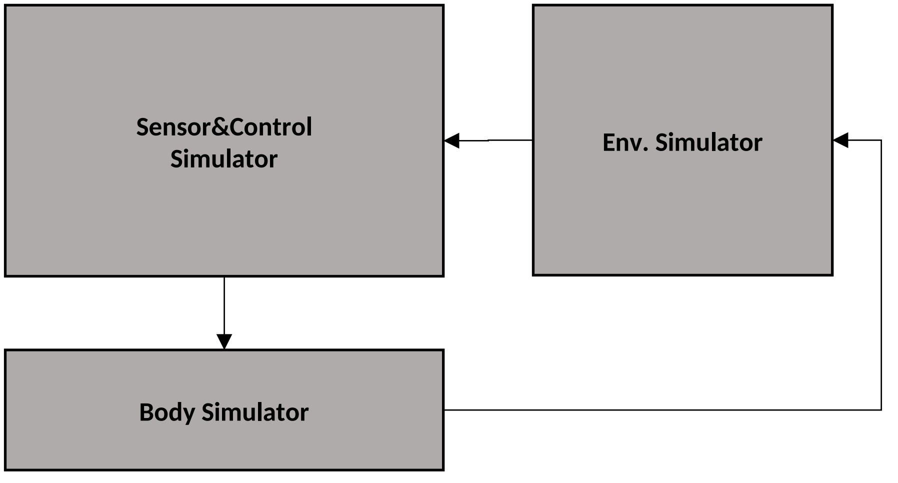
  $logicalTime \leftarrow logicalTime + \Delta t$

end while

CBD simulation algorithm can also be used for scheduling at level of Master

need “zero-delay feedthrough” information (algebraic loops)

Caveat: naive



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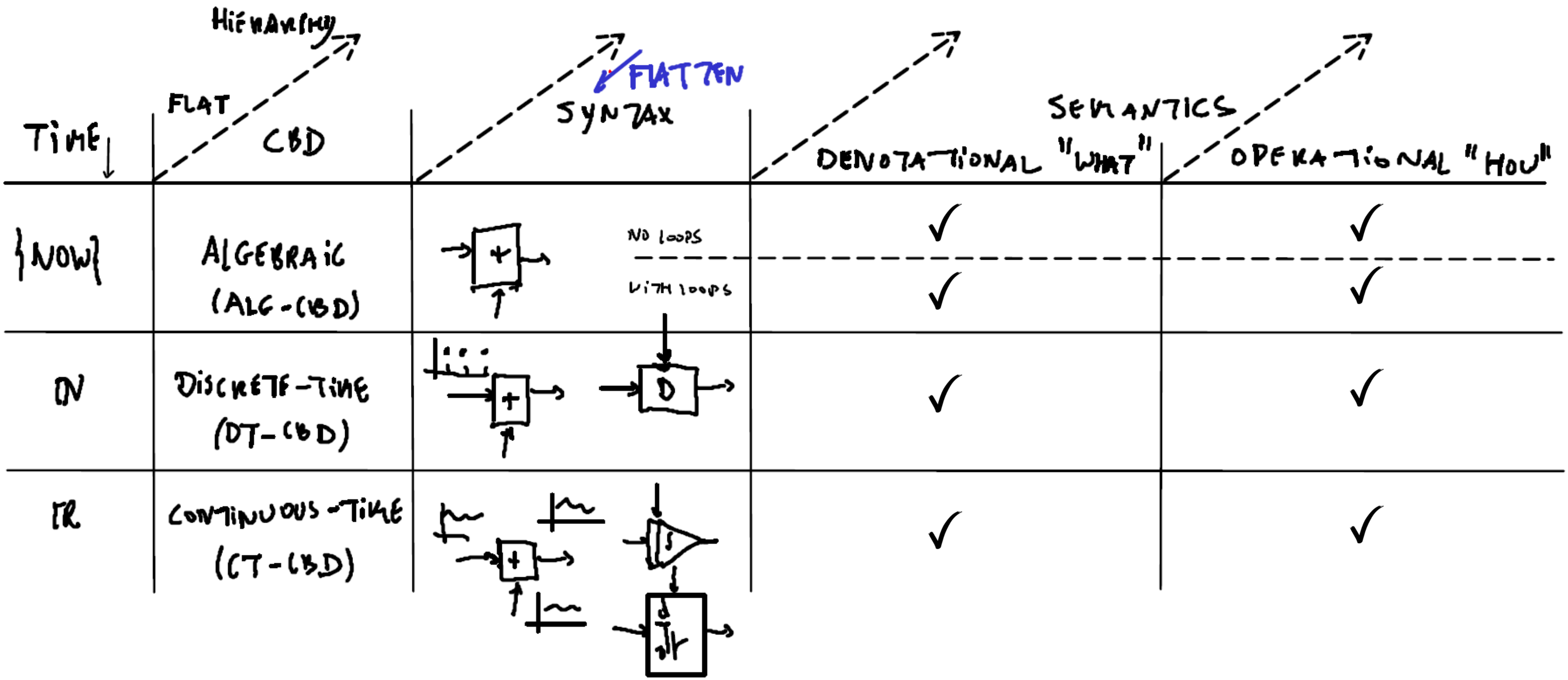
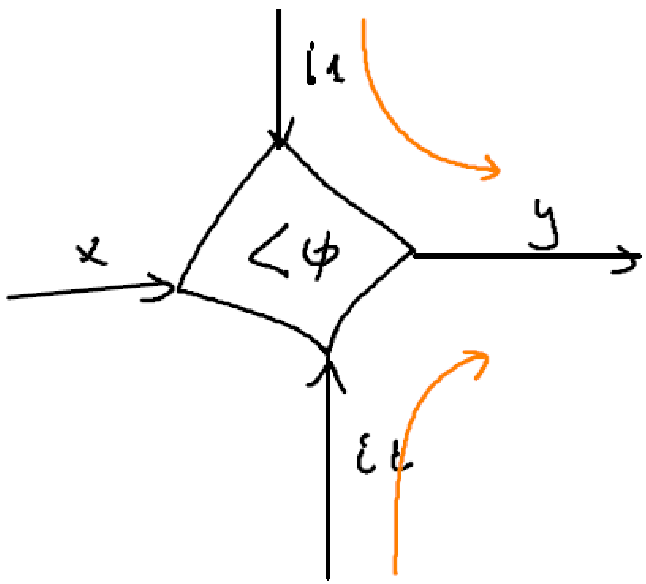
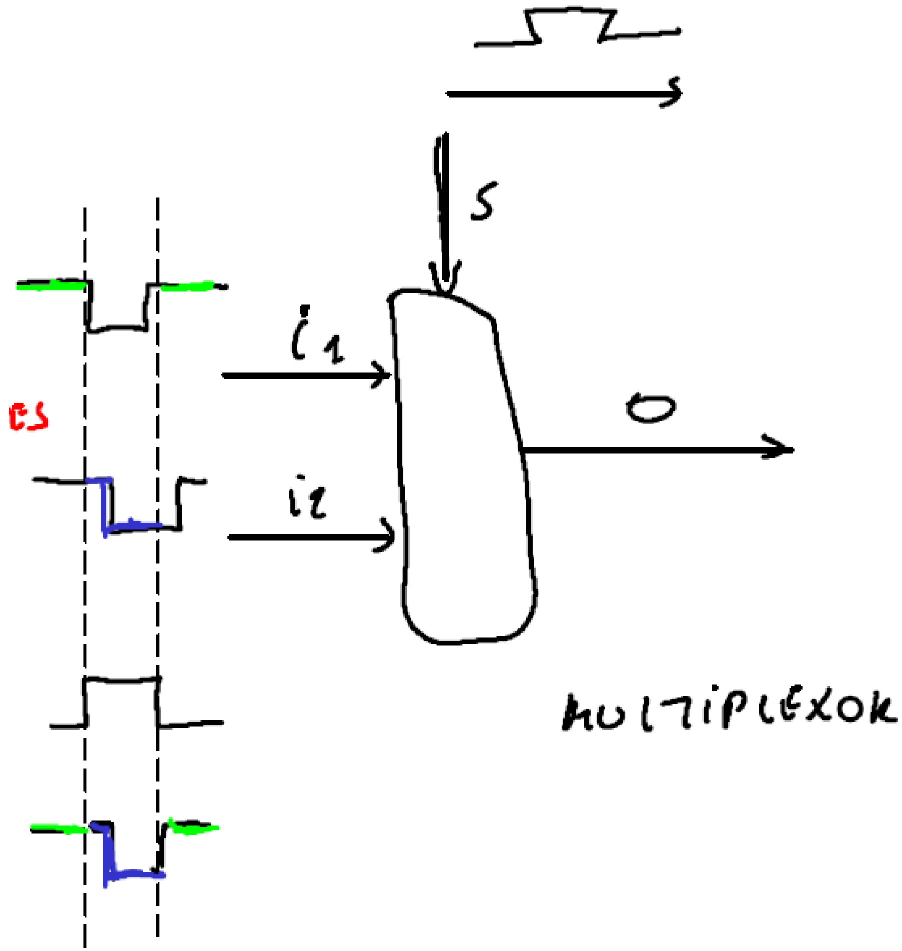
- (general) Laws of Physics
- Power Flow/Bond Graphs (physical: energy/power)
- Computationally a-causal
  (Mathematical and Object-Oriented) ← **Modelica**
- Causal Block Diagrams (data flow)
- Numerical (Discrete) Approximations
- Computer Algorithmic + Numerical
  (Floating Point vs. Fixed Point)
- As-Fast-As-Possible vs. Real-time (XiL)
- Hybrid (discrete-continuous) modelling/simulation
- Hiding IP: Composition of Functional Mockup Units (FMI)
- Dynamic Structure

++ Dynamic Structure

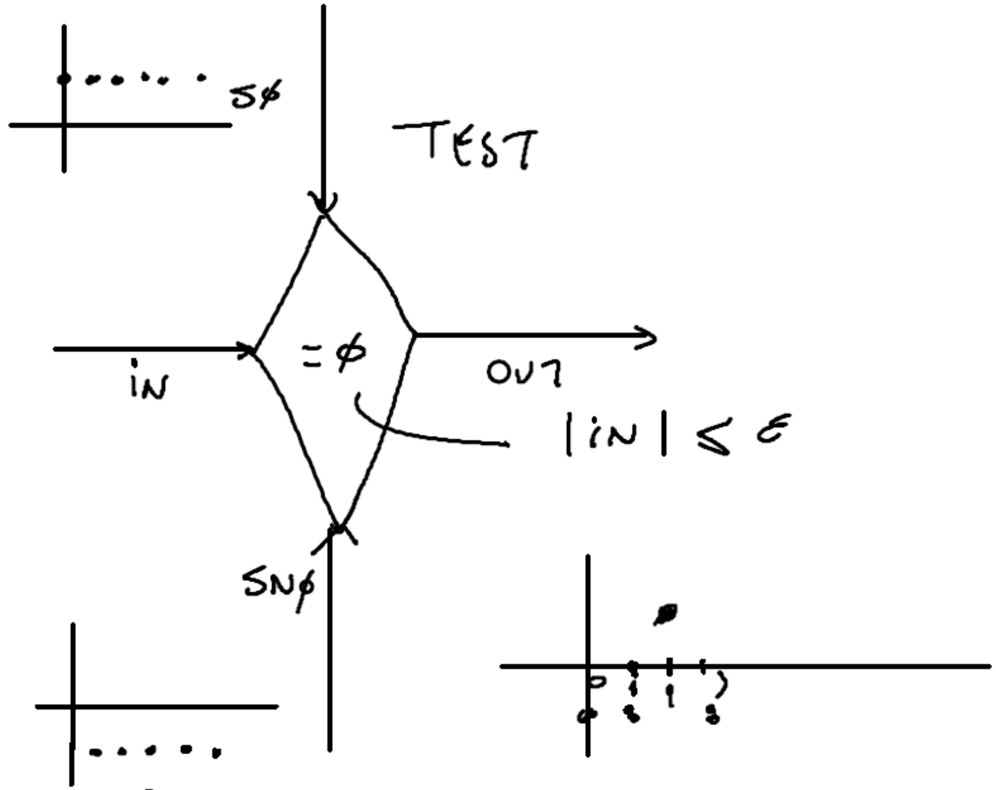


DYNAMIC STRUCTURE





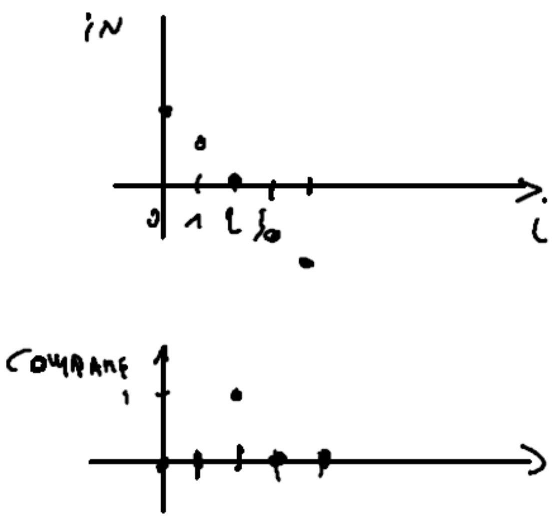
s
i1
ES
i2
o
MULTIPLEXOR

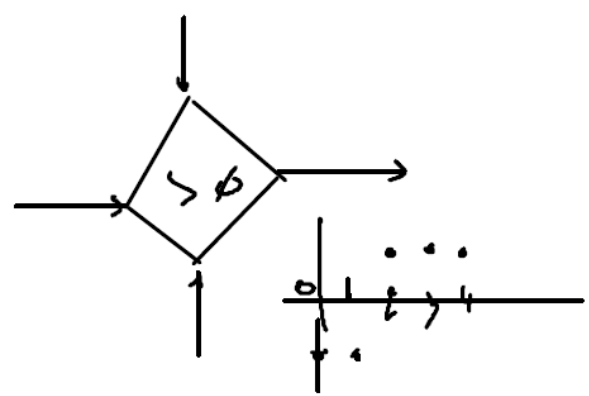


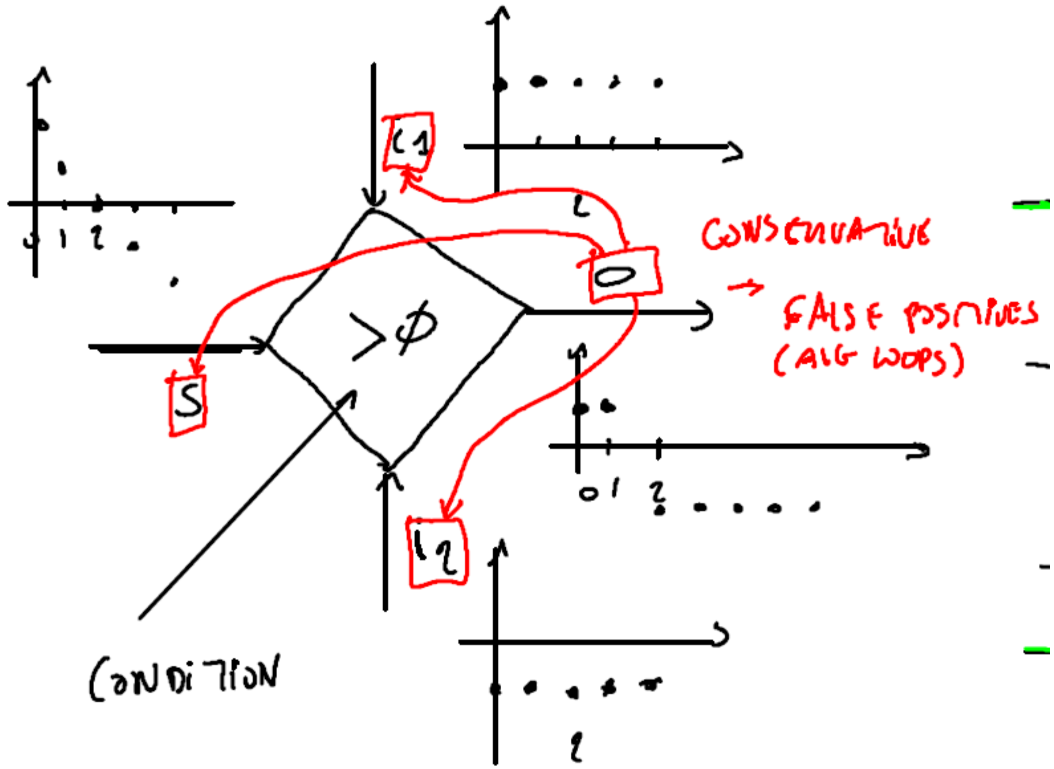
ℕ, ℤ $i_1 == i_2$
init

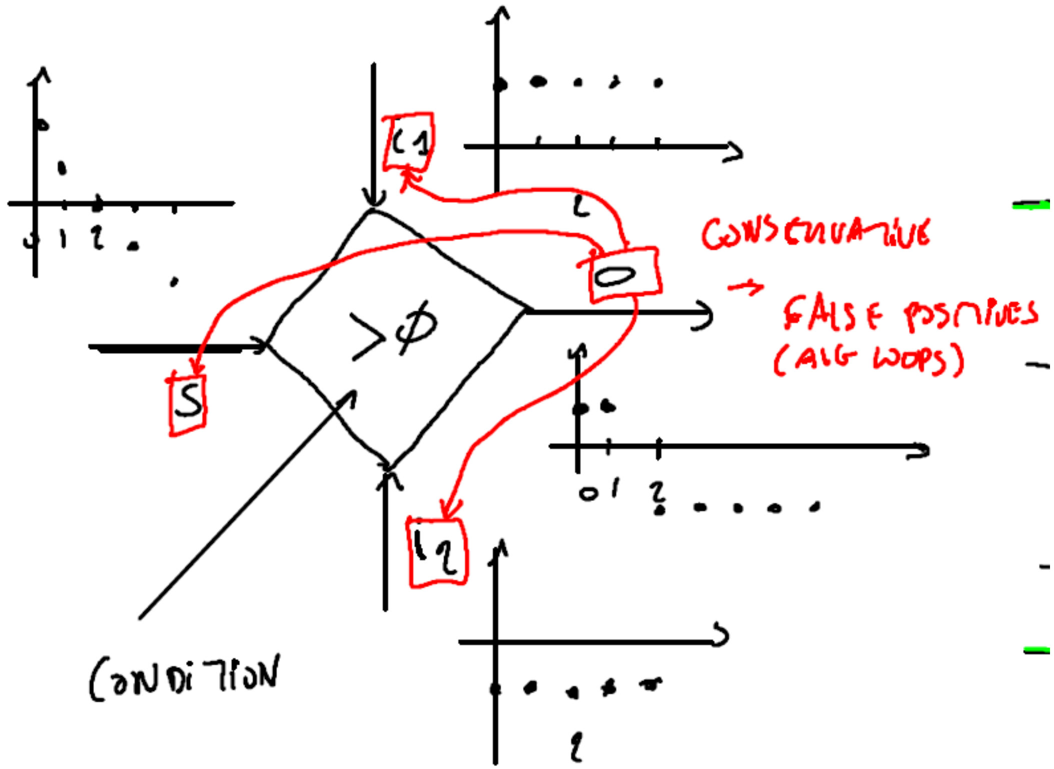
ℝ $r_1 == r_2$
float, double

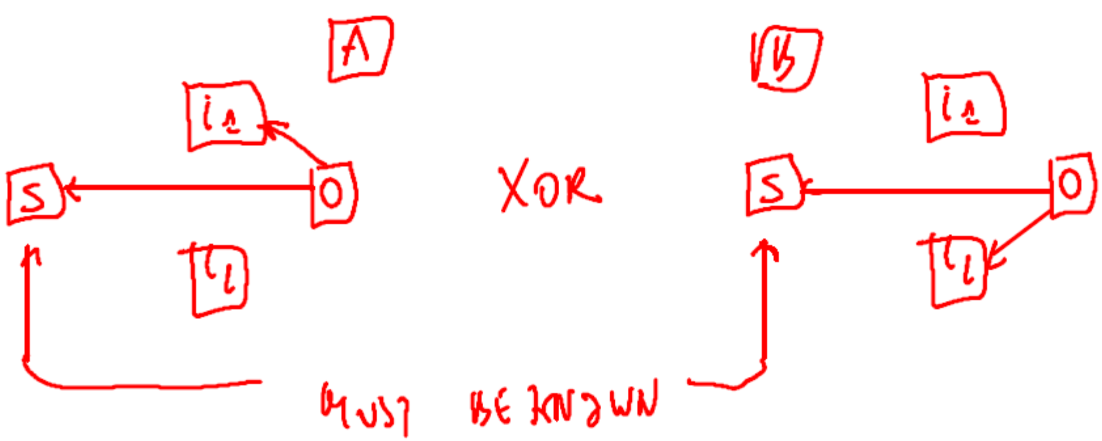
$|r_1 - r_2| \leq \varepsilon$






>Φ
CONDITION
S
C1
C2
O
CONSERVATIVE
→ FALSE POSITIVES
(ALG LOOPS)

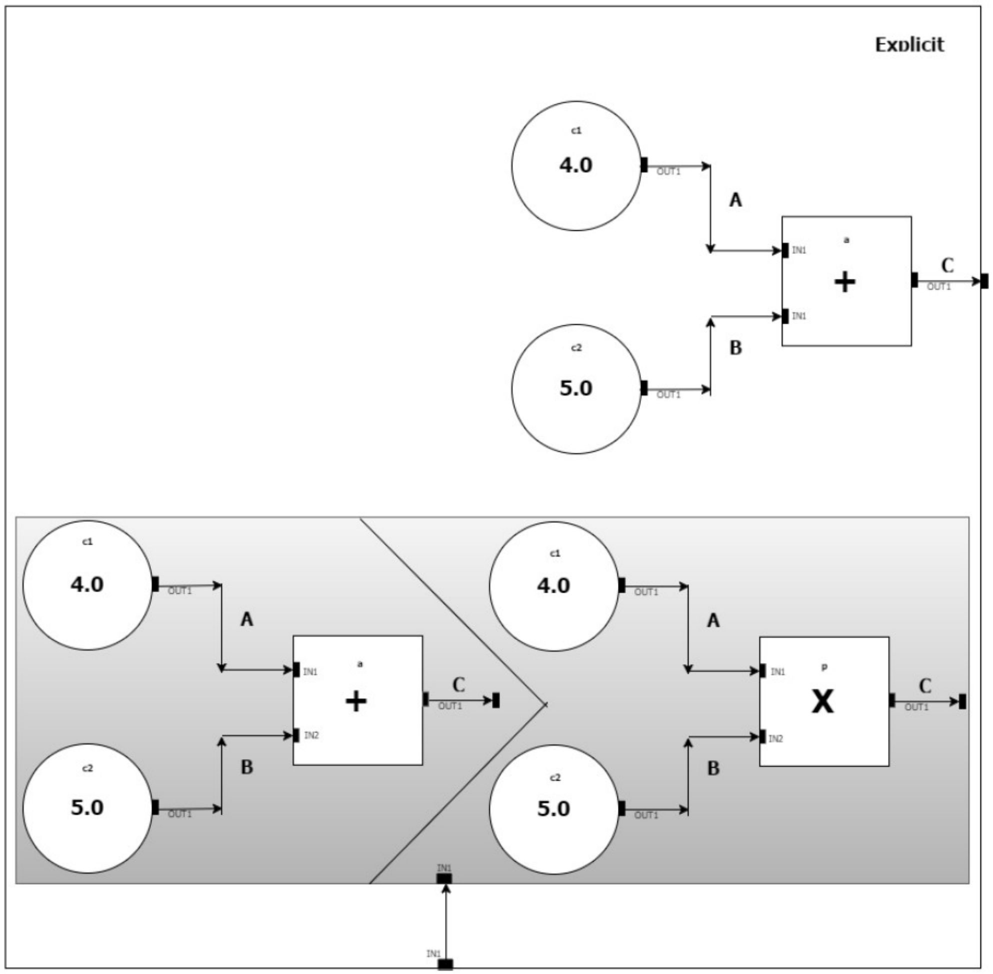


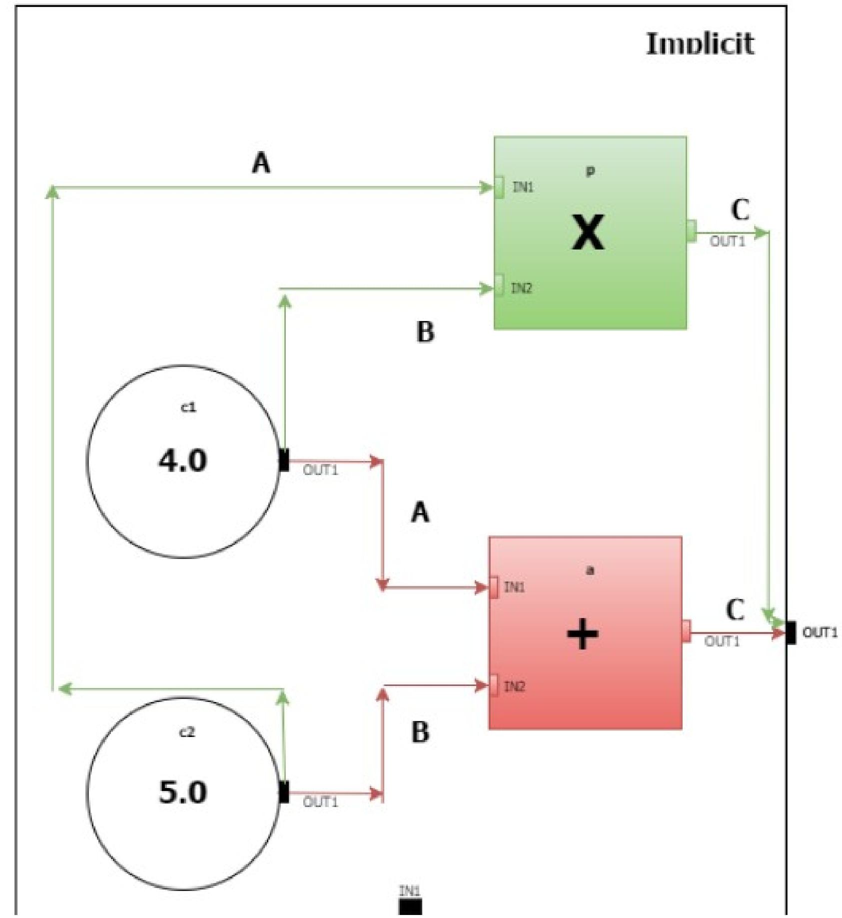


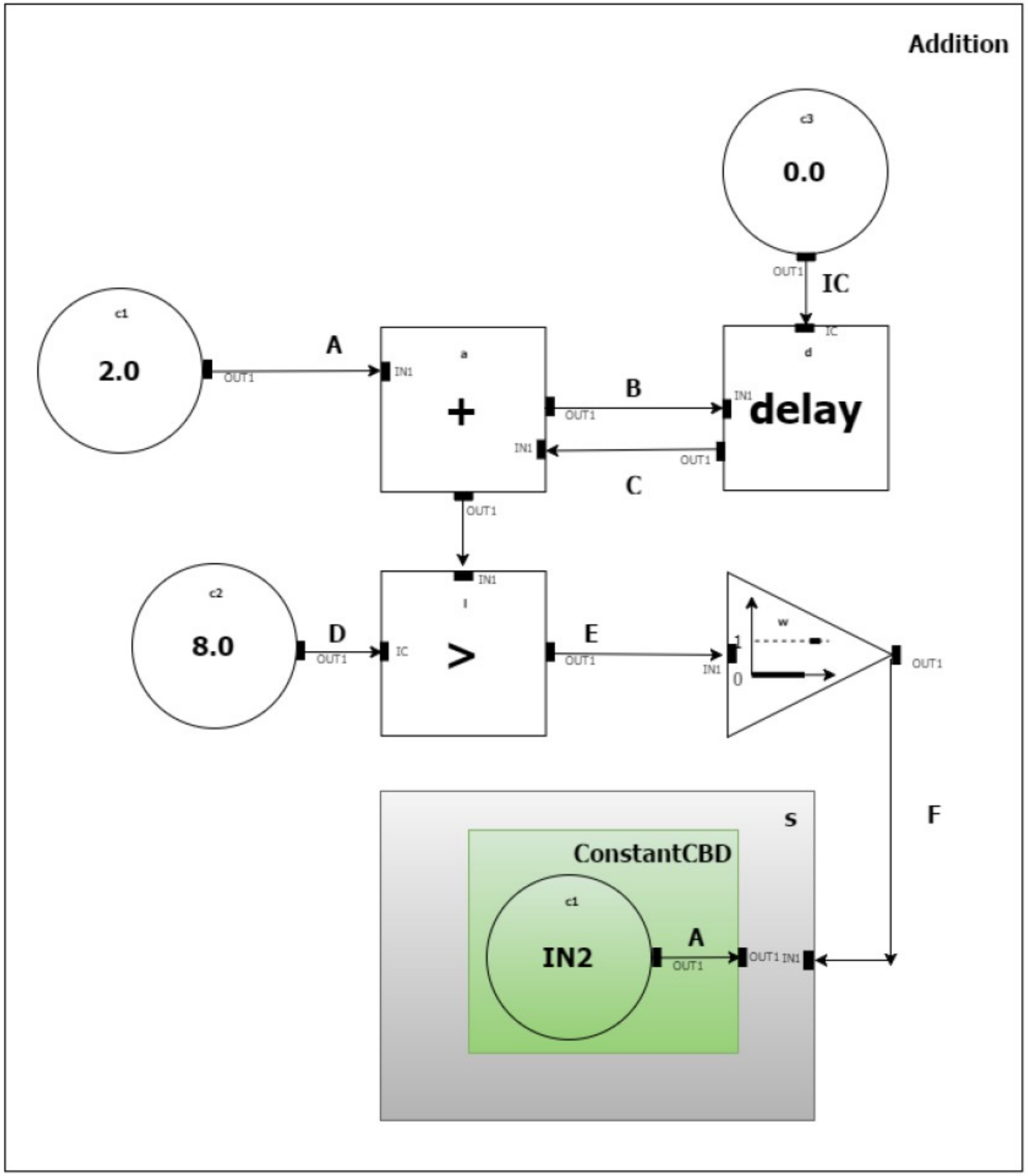
1. BUILD DEP. GRAPH UPTO [S]
2. PARTIAL SORT/LOOP DET UPTO [S]
3. COMPLETE DEP GRAPH WITH [A] OR [B]
4. FULL SORT/LOOP DETECT

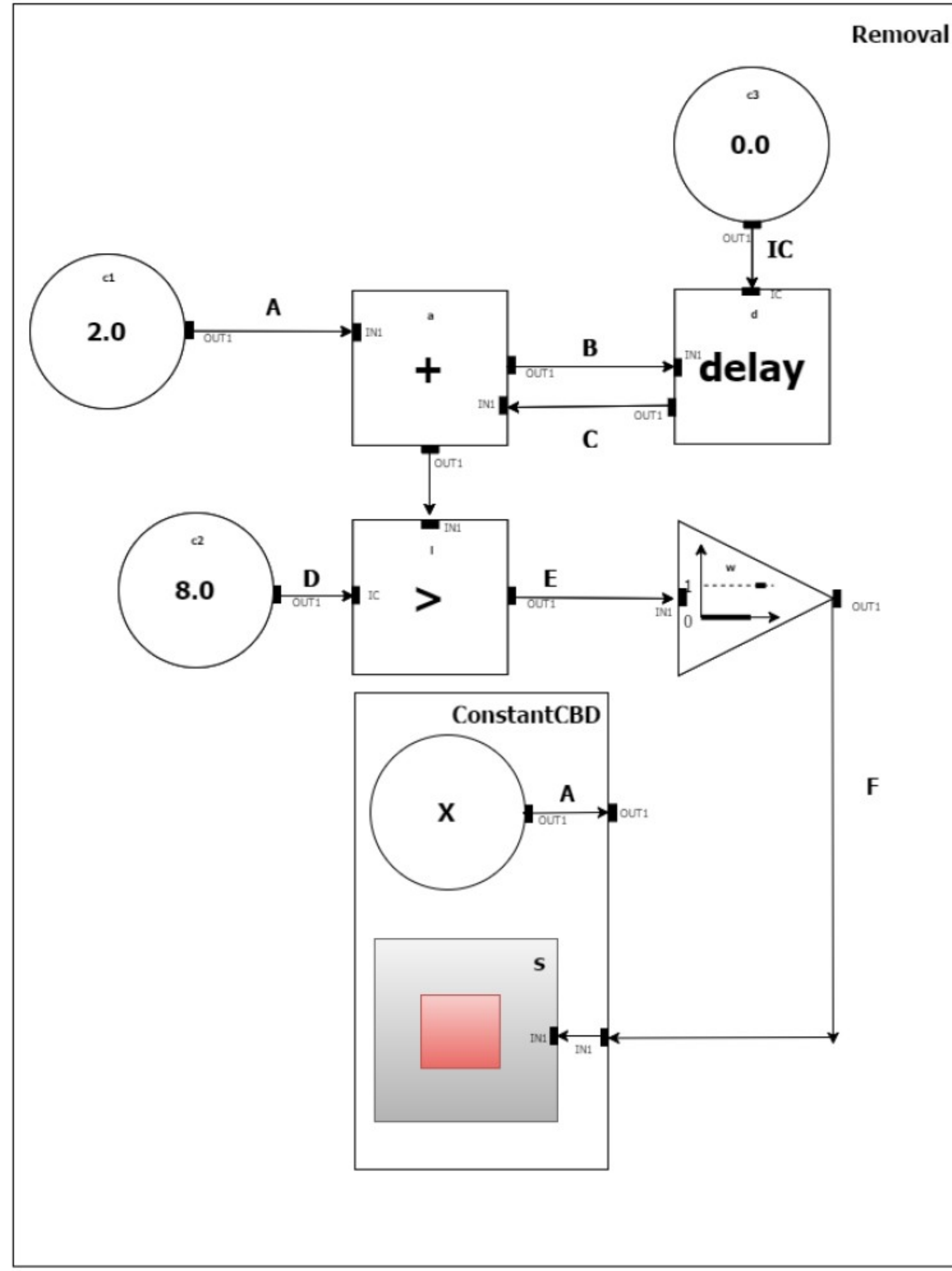
MULTIPLE TEST BLOCK

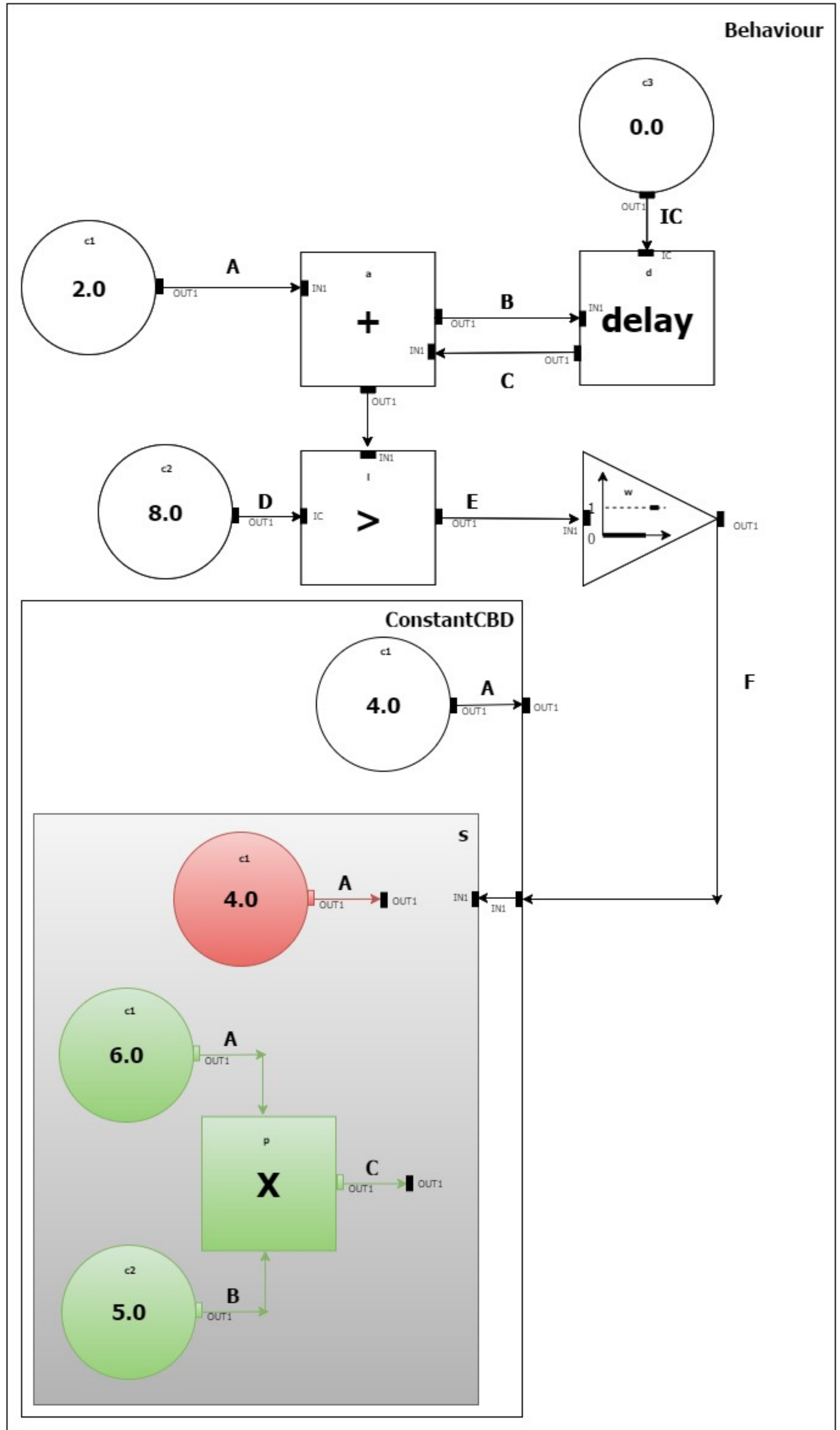
FIXED POINT

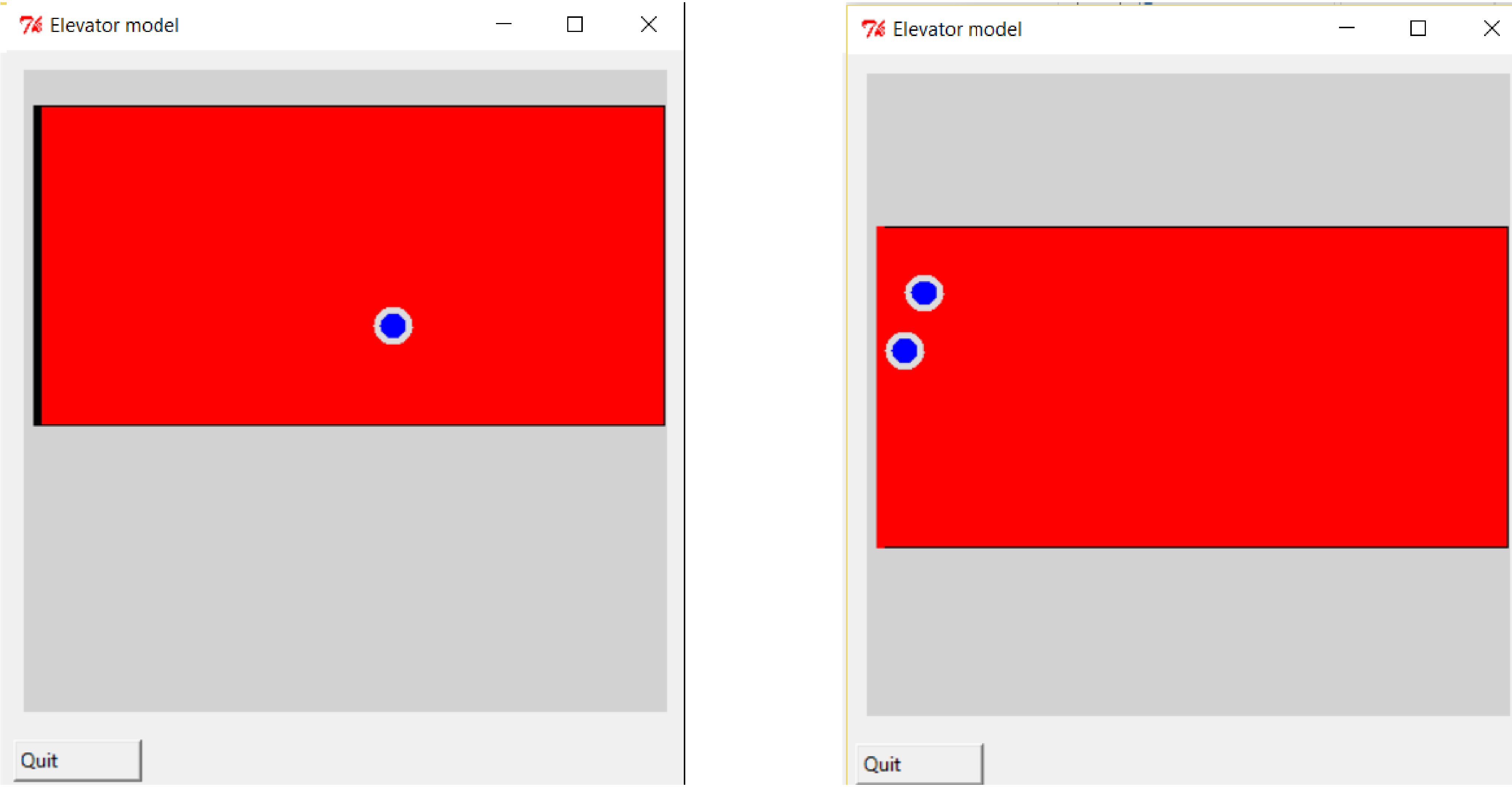

Explicit
c1
4.0
OUT1
A
a
+
IN1
IN1
C
OUT1
c2
5.0
OUT1
B
c1
4.0
OUT1
A
IN1
a
+
C
OUT1
IN2
B
c2
5.0
OUT1
c1
4.0
OUT1
A
IN1
p
X
C
OUT1
IN2
B
c2
5.0
OUT1
IN1
IN1

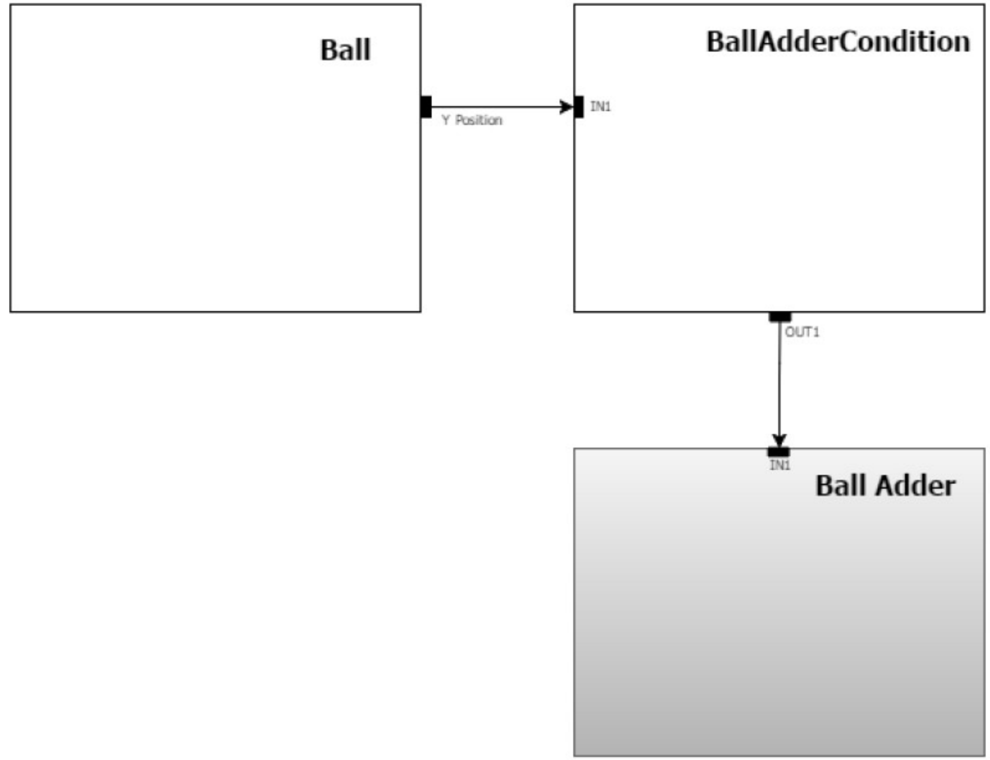

Implicit
A
IN1
p
X
C
OUT1
IN2
B
c1
4.0
OUT1
A
IN1
a
+
C
OUT1
OUT1
IN2
B
c2
5.0
OUT1
IN1

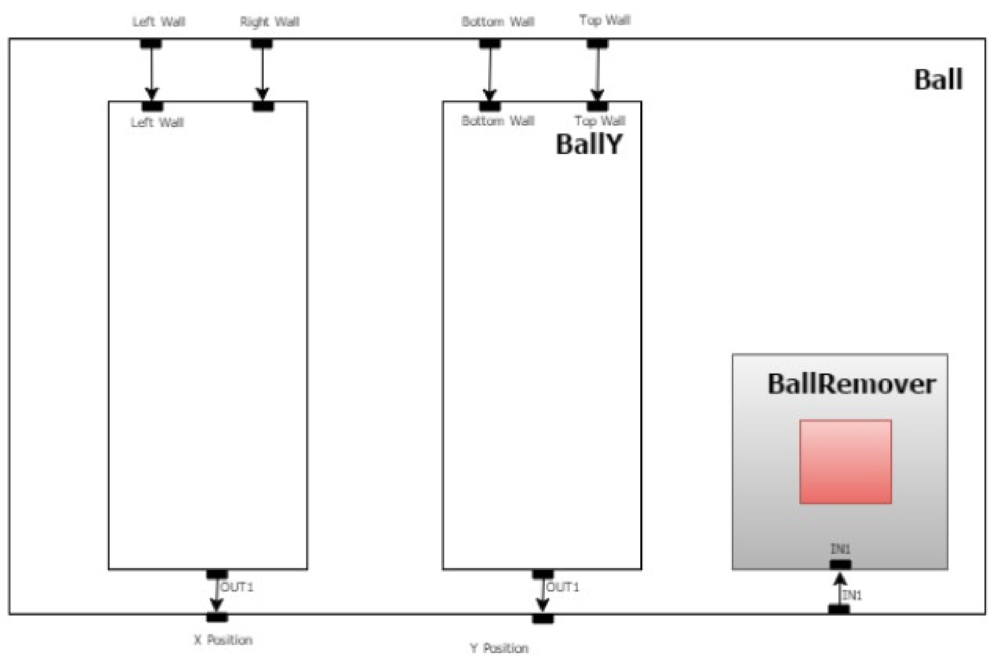

Addition
c3
0.0
OUT1
IC
IC
c1
2.0
OUT1
A
IN1
a
+
B
OUT1
IN1
d
delay
IN1
OUT1
C
OUT1
IN1
c2
8.0
D
OUT1
IC
>
E
OUT1
IN1
w
1
0
OUT1
F
s
ConstantCBD
c1
IN2
A
OUT1
OUT1
IN1


Removal
c3
0.0
OUT1
IC
IC
c1
2.0
A
OUT1
IN1
a
+
B
OUT1
IN1
d
delay
IN1
C
OUT1
OUT1
IN1
c2
8.0
D
OUT1
IC
>
E
OUT1
IN1
w
1
0
OUT1
ConstantCBD
X
A
OUT1
OUT1
F
s
IN1
IN1


Behaviour
c3
0.0
OUT1
IC
c1
2.0
OUT1
A
IN1
a
+
B
OUT1
IN1
d
delay
IN1
OUT1
C
OUT1
IN1
c2
8.0
D
OUT1
IC
>
E
OUT1
IN1
w
1
0
OUT1
ConstantCBD
c1
4.0
A
OUT1
OUT1
F
s
c1
4.0
A
OUT1
OUT1
IN1
IN1
c1
6.0
A
OUT1
p
X
C
OUT1
OUT1
c2
5.0
B
OUT1


Elevator model
Quit
Elevator model
Quit


Ball
BallAdderCondition
Y Position
IN1
OUT1
IN1
Ball Adder


Left Wall
Right Wall
Bottom Wall
Top Wall
Ball
Left Wall
Bottom Wall
Top Wall
BallY
BallRemover
IN1
OUT1
OUT1
IN1
X Position
Y Position

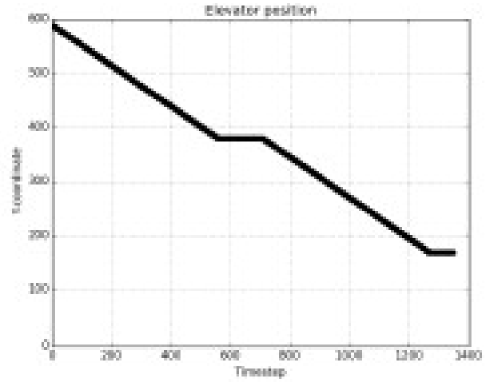


Figure 4.10: Y-position of the elevator

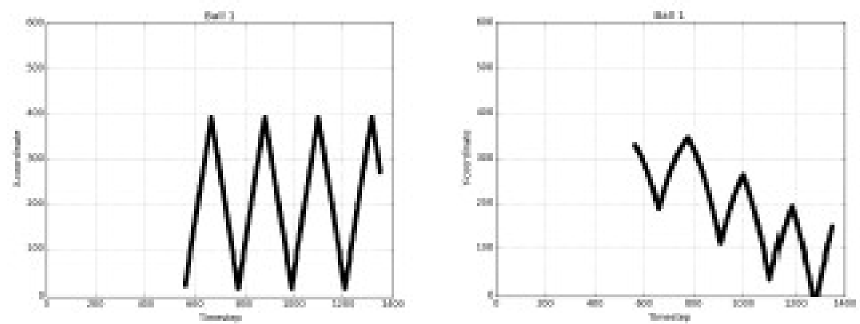


Figure 4.11: Position of ball 1

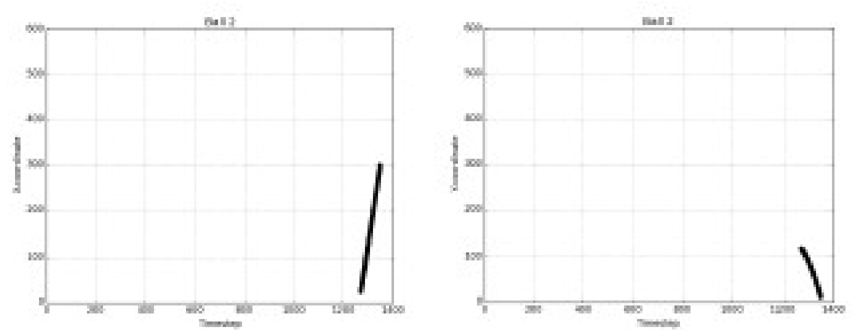


Figure 4.12: Position of ball 2