#### Statecharts aka Harel Charts

- visual formalism
- application of higraphs
- diverse applications;

in particular: concurrent, reactive systems behaviour modelling

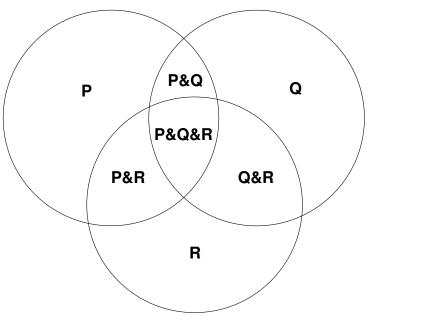
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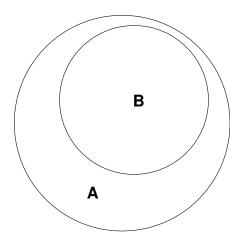
#### Visualising Information

- complex
- non-quantitative, structural
- topological, not geometrical
- Euler
  - Venn diagrams (Jordan curve: inside/outside): enclosure, intersection
  - graphs (nodes, edges: binary *relation*); hypergraphs

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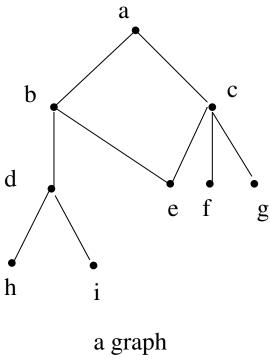
#### Venn diagrams, Euler circles

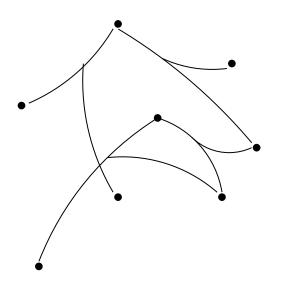




- topological notions:
   enclosure, intersection, exclusion
- Used to represent *mathematical set operations*: union, intersection, difference

#### Hypergraph





a hypergraph

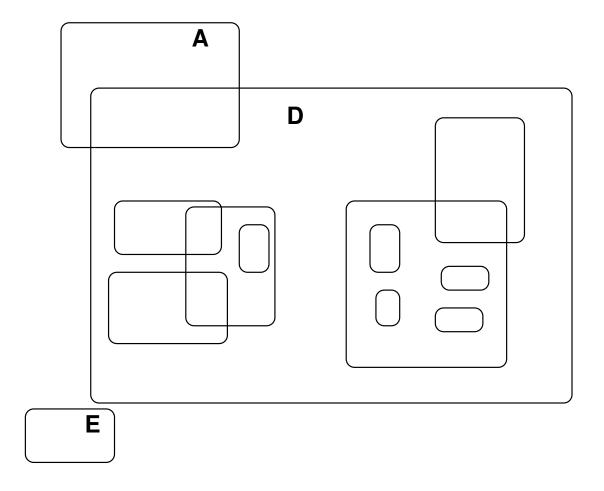
- topological notion: connectedness
- Used to represent *relations* between sets.
- Hyperedges: non longer binary relation ( $\subseteq X \times X$ ):  $\subseteq 2^X$  (undirected),  $\subseteq 2^X \times 2^X$  (directed).

# Higraphs: combining hypergraphs and Venn diagrams

- sets + cartesian product
- hypergraphs

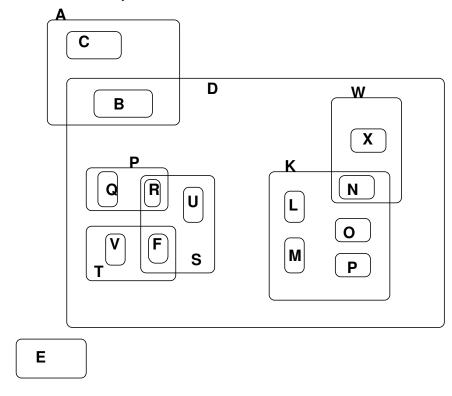
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#### Blobs: set inclusion, not membership



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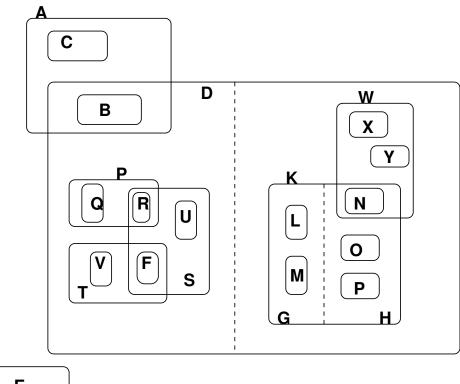
#### Unique Blobs (atomic sets, no intersection)



- empty space has no meaning, intersection must be identified
- atomic blobs are identifiable sets
- other blobs are union of enclosed sets

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# Unordered Cartesian Product: Orthogonal Components

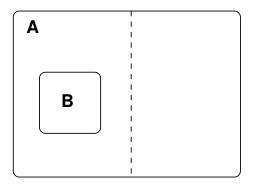


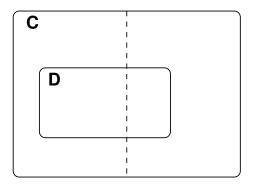
E

$$K = G \times H = (L \cup M) \times (N \cup O \cup P)$$

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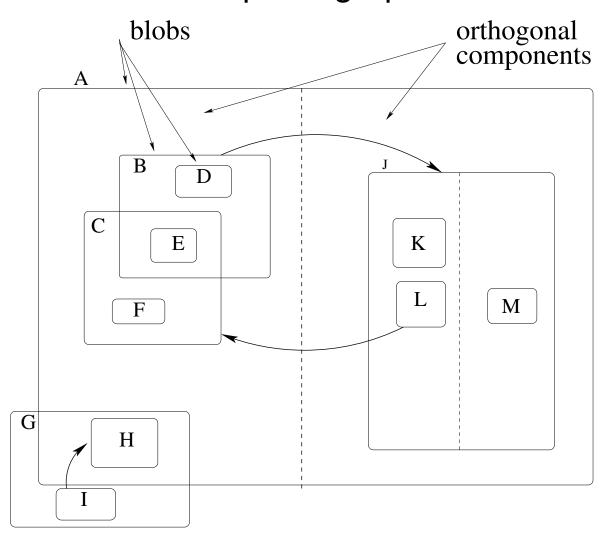
# Meaningless constructs





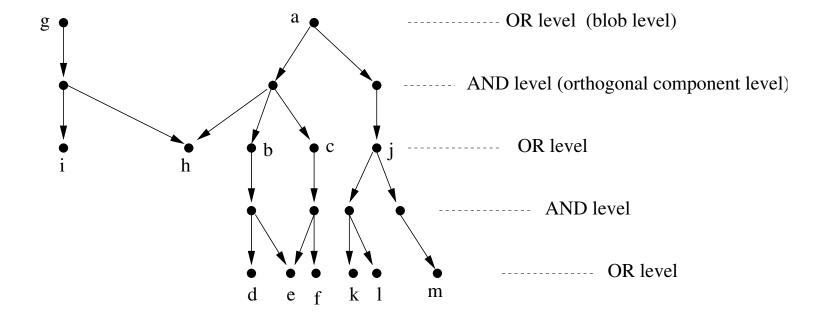
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# Simple Higraph



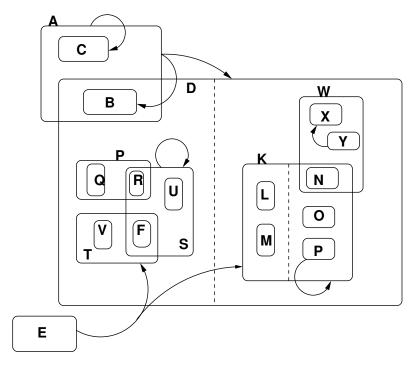
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#### Induced Acyclic Graph (blob/orth comp alternation)



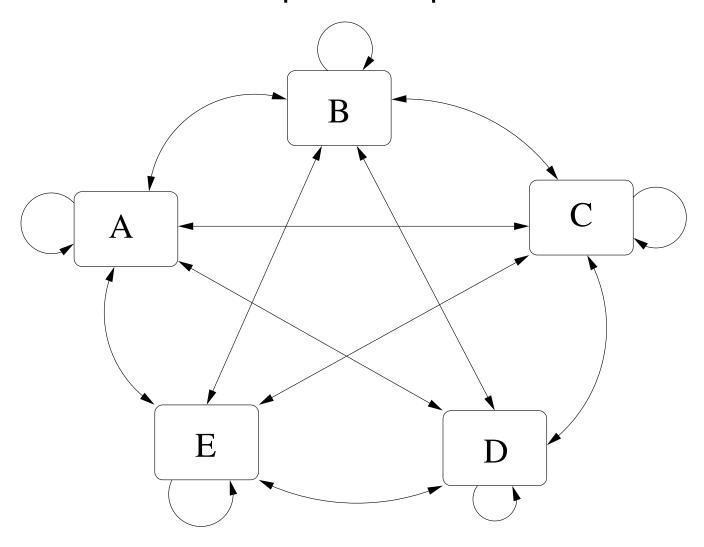
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### Adding (hyper) edges

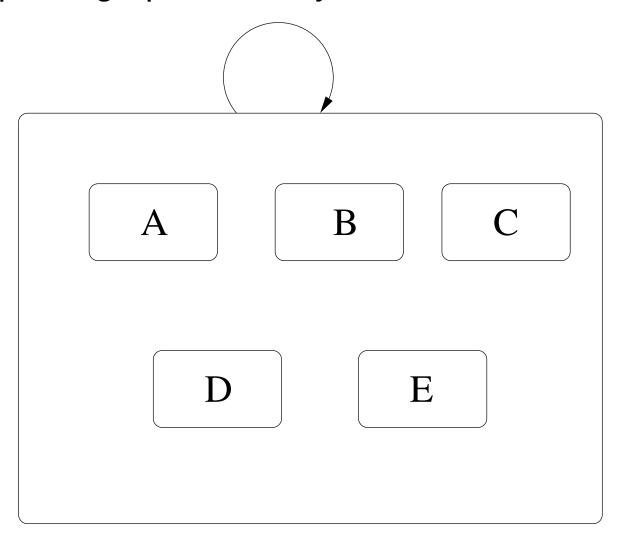


- hyperedges
- attach to contour of any blob
- inter-level possible

# Clique Example

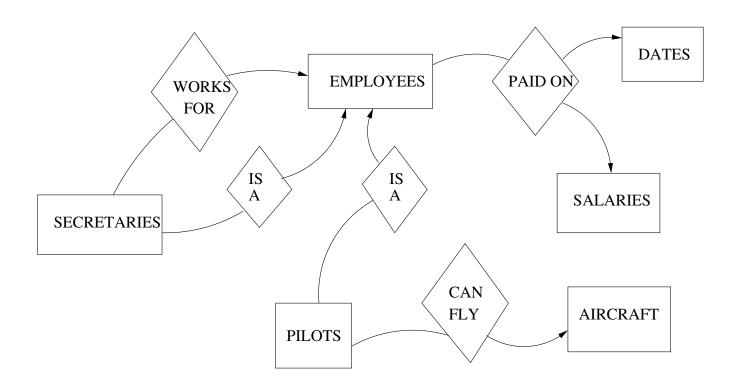


### Clique: higraph with fully connected semantics



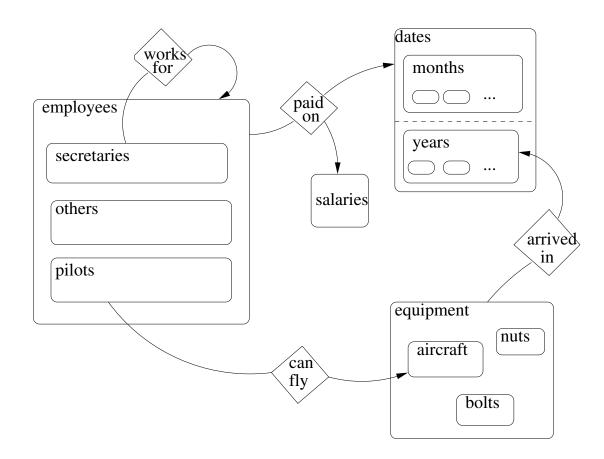
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# Entity Relationship Diagram (is-a)



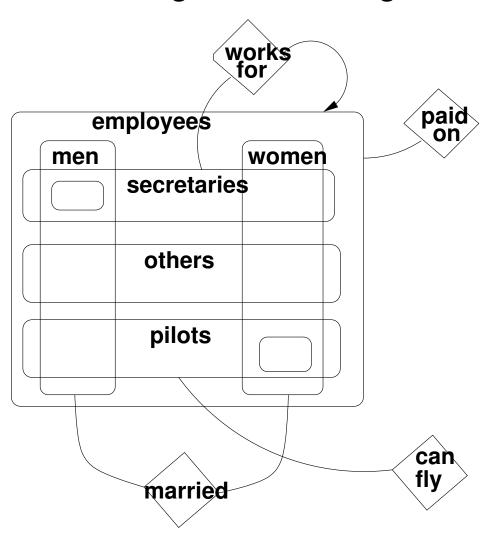
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### Higraph version of E-R diagram



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### Extending the E-R diagram



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#### Formally (syntax)

A higraph H is a quadruple

$$H = (B, E, \sigma, \pi)$$

B: finite set of all unique blobs

*E*: set of hyperedges

$$\subseteq X \times X, \subseteq 2^X, \subseteq 2^X \times 2^X$$

The subblob (direct descendants) function  $\sigma$ 

$$\sigma: B \to 2^B$$

$$\sigma^{0}(x) = \{x\}, \ \sigma^{i+1} = \bigcup_{y \in \sigma^{i}(x)} \sigma(y), \ \sigma^{+}(x) = \bigcup_{i=1}^{+\infty} \sigma^{i}(x)$$

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Subblobs<sup>+</sup> cycle free

$$x \not\in \sigma^+(x)$$

The partitioning function  $\pi$  associates equivalence relationship with x

$$\pi: B \to 2^{B \times B}$$

Equivalence classes  $\pi_i$  are orthogonal components of x

$$\pi_1(x), \pi_2(x), \ldots, \pi_{k_x}(x)$$

 $k_x = 1$  means a single orthogonal component (no partitioning)

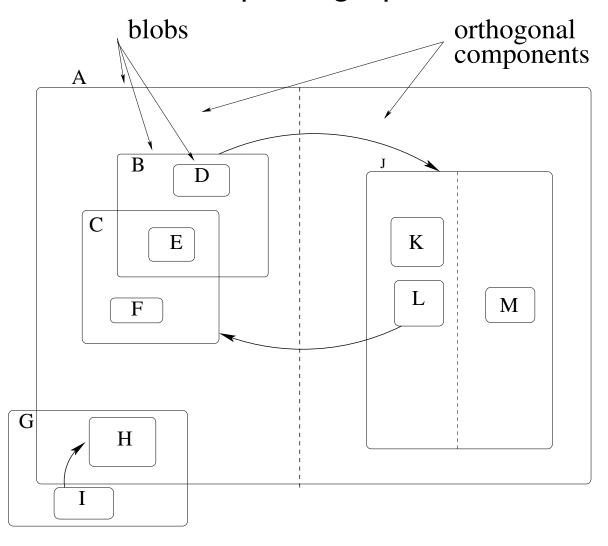
Blobs in different orthogonal components of *x* are *disjoint* 

$$\forall y, z \in \sigma(x) : \sigma^+(y) \cap \sigma^+(z) = \emptyset$$

unless in the same equivalence class

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# Simple Higraph



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#### Induced Orthogonal Components

$$B = \{A, B, C, D, E, F, C, G, H, I, J, K, L, M\}$$

$$E = \{(I, H), (B, J), (L, C)\}$$

$$\rho(A) = \{B, C, H, J\}, \rho(G) = \{H, I\}, \rho(B) = \{D, E\}, \rho(C) = \{E, F\},$$

$$\rho(J) = \{K, L, M\}$$

$$\rho(D) = \rho(E) = \rho(F) = \rho(H) = \rho(I) = \rho(K) = \rho(L) = \rho(M) = \emptyset$$

$$\pi(J) = \{(K,K), (K,L), (L,L), (L,K), (M,M)\}$$

Induces equivalence classes  $\pi_1(J) = \{K, L\}$  and  $\pi_2(J) = \{M\}, \ldots$ These are the *orthogonal components* 

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#### Higraph applications

- E-R diagrams
- data-flow diagrams (activity diagrams)
   edges represent (flow of) data
- inheritance
- Statecharts

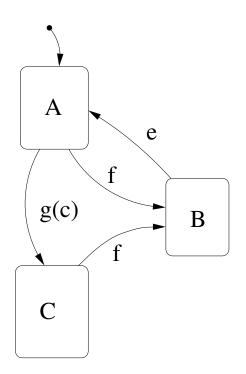
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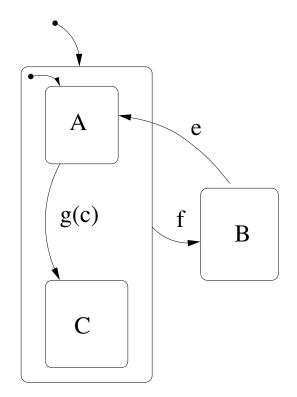
# StateCharts = state diagrams + depth + orthogonality + broadcast

- Reactive Systems (event driven, react to internal and external stimuli)
- like Petri Nets, CSP, CCS, sequence diagrams, ...
- graphical but formal and rigourous for
  - analysis
  - code generation
- solve FSA problems:
  - flat  $\Rightarrow$  hierarchy  $\Rightarrow$  re-use
  - represent large number of transitions concisely
  - represent large number of states concisely
  - sequential  $\Rightarrow$  concurrent

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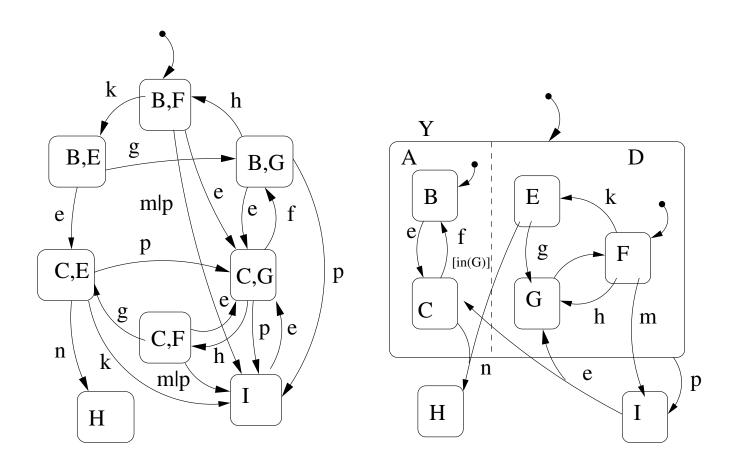
# Depth (XOR)





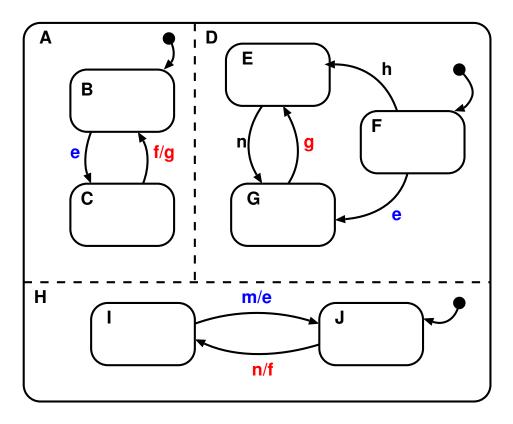
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## Orthogonality (AND), flattening ⇒ semantics



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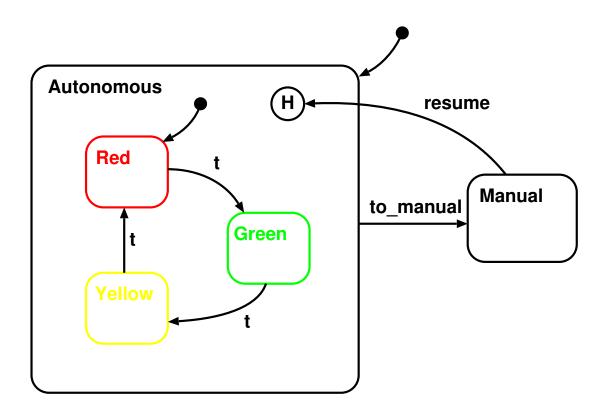
## Broadcasting (output events)



Input Segment: nmnn

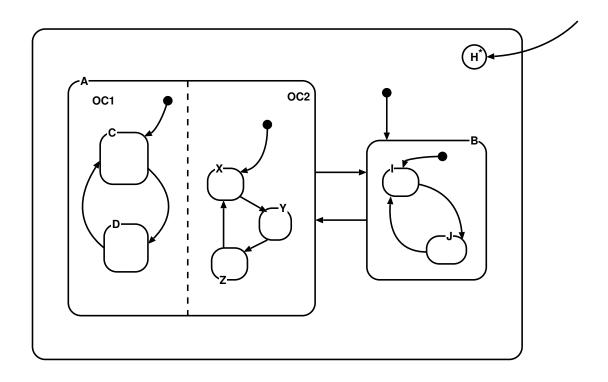
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## **History States**



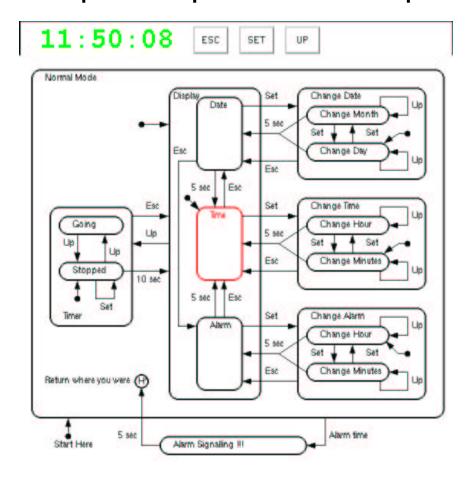
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# **Deep History**



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#### Simple Stopwatch Example

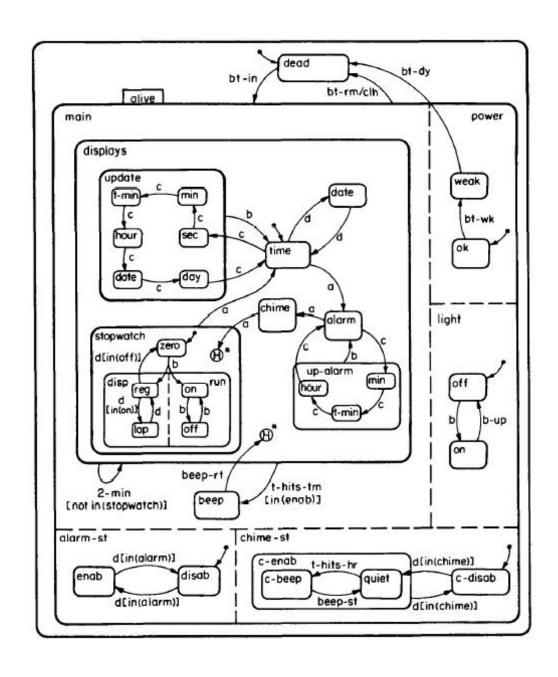


http://www.xjtek.com/products/xjcharts/demo/

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# Stopwatch Example

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#### Messages

- object.method(...) synchronous method invocation
- object->method(...) queued, a-synchronous

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