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SELF-ORGANIZATION OF COMPLEX SYSTEMS

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The basic laws of physics are simple, so why is the world complex? The theory of self-organized criticality posits that complex behavior in nature emerges from the dynamics of extended, dissipative systems that evolve through a sequence of meta-stable states into a critical state, with long range spatial and temporal correlations. Minor disturbances lead to intermittent events of all sizes. These events organize the system into a complex state that cannot be reduced to a few degrees of freedom. This type of "punctuated equilibrium" dynamics has been observed in astrophysical, geophysical, and biological processes, as well as in human social activity.

1 Introduction

Scientific inquiry in the second millennium has focused almost exclusively on discovering the fundamental constituents, or building blocks, of nature. The most innermost secrets have been revealed down to ever smaller scales. Matter is formed of atoms; atoms are composed of electrons, protons, and neutrons, and so on down to the smallest scale of quarks and gluons. These basic elements interact through simple physical laws.

In the realm of biology, it is known that life on earth is based on the DNA double helix. But even though we understand perfectly the laws governing the interaction of atoms, we cannot directly extrapolate these laws to explain the beginning of life, or the auto-catalysis of complex molecular networks, or why we have brains that can contemplate the world around us. Due to the overwhelming unlikeliness of random events leading to complex systems like ourselves, it seems as if an organizing agent or "God" must be invoked who puts the building blocks together.

It isn't necessary to delve into the biological realm to see the ultimate inadequacy of a purely reductionist approach. For instance, the surface of the earth is an intricate conglomerate of mountains, oceans, islands, rivers, volcanoes, glaciers, and earthquake faults, each with its own dynamics. The behavior of systems like these cannot be deduced by examining ever smaller scales to derive microscopic laws; the dynamics and form is "emergent." Unless one is willing to invoke an organizing agent of some sort, all these phenomena must be self-organized. Complexity must emerge from a self-organizing dynamics. But how?

A few ideas have been proposed that begin to address this problem, which can be characterised as "How do we take God out of the equations." The most pessimistic

view is that one has to describe each and every feature in nature on a case by case basis. Indeed, such a "stamp collection" approach has prevailed in sciences such as biology and geophysics, and attempts to look for a unifying description have in the past been met by very strong scepticism among the practitioners of those sciences, although there have been exceptions such as plate tectonics theory, Kauffman's work on autocatalytic networks [1], and Gould and Eldridge's theory of punctuated equilibrium in biological evolution [2].

Perhaps nature does not need to invent a multitude of mechanisms, one for each system. The view that only a limited number of mechanisms, or principles, lead to complexity in all its manifestations (from the galactic or universal to the molecular) is supported by the observation of regularities that appear in the statistical description of complex systems. These statistical regularities provide hope and encouragement that a science of complexity may eventually emerge.

For example, river networks, mountain ranges, etc. exhibit scaling behavior, both in the spatial and in the temporal domain, where landslides or sediment deposits interrupt the quiet steady state. These landslides have been observed to be scale free [3]; similarly the Gutenburg Richter law for earthquakes states that they are also a scale free phenomena, with avalanches (quakes) of all sizes [4]. The distribution of energy released during earthquakes is a simple power law, despite the enormous complexity of the underlying system, involving a multitude of geological structures. Forest fires have a similar behavior [5], as does volcanic activity [6]. In astrophysical phenomena, there are star quakes, which we observe as pulsar glitches [7], interrupting quiet periods. Black holes are surrounded by accretion disks, from which the material collapses into the black hole in intermittent, earthquake-like events, which interrupt the otherwise steady evolution and occur over a wide range of scales [8].

Biological evolution also exhibits long periods of stasis punctuated by extinction events of all sizes. The paleontologists Stephen Jay Gould and Niles Eldredge [2] coined the term "punctuated equilibrium" to describe the pace of evolution. Gould also argues that the record of extinction of species is contingent on seemingly minor accidents, and if the tape of the history of life were to be rerun an entirely different set of species would emerge [9].

We assert that punctuated equilibrium dynamics is the essential dynamical process for everything that evolves and becomes complex, with a specific behavior that is strongly contingent on its history [10]. The periods of stasis allow the system to remember its past, the punctuations allow change in response to accumulated forcing over long time scales, and the criticality assures that even minor perturbations can have dramatic effects on the specific outcome of a particular system, making it possible to have distinct individual histories and forms.

Perhaps the greatest challenge is to find the mechanism by which the big bang has led to ever increasing complexity in our universe, rather than exploding into a simple gas-like fragmented substance, as explosions usually do, or imploding into a simple solid or black hole. Some intricately balanced feature of the initial state must have existed that allowed this to happen. How that "fine tuning" could have appeared remains a mystery, with Lee Smolin's speculation of universes created by Darwinian selection being the only attempt so far [11].

Complexity is a hierarchical phenomenon, where each level of complexity leads to the next: astrophysics, with its own hierarchy of scales, leads to geophysics, which is the prerequisite for chemistry, biology, and ultimately the social sciences. Although the origin of the hierarchy is not understood, we do have the rudiments of a theory for the emergence of one level out of the previous one. Due to this hierarchy of emergence, it isn't necessary to understand the mechanism of the big bang in order to understand the dynamics of earthquakes.

A common feature of the systems mentioned thus far, and perhaps of all complex systems, is that they are driven by slowly pumping in energy from a lower level of the hierarchy. For instance, biological life is driven by the input of energy from the sun. The energy is stored and later dissipated, in an avalanche process like an earthquake. Even a small increment in energy can trigger a large catastrophe, making these systems strongly contingent on previous history. They operate far from equilibrium, which is necessary since systems in equilibrium tend to become more and more disordered (rather than complex) over time, according to the second law of thermodynamics.

2 Complexity and Criticality

One view of systems driven out of equilibrium is that they should tend to a uniform "minimally" stable state generated by some type of optimization process. In traffic flow such a state would correspond to a uniform flow of cars with all cars moving at maximum velocity possible. But these optimized states often are catastrophically unstable, exhibiting breakdown events or avalanches, such as traffic jams [12]. In tokamaks [13], this means that the ideal state of the plasma with the highest possible energy density is locally stable, but globally unstable with respect to explosive breakdown events. The surface of the sun is unstable with respect to formation of solar flares emitting energy in terms of light or gamma rays. In fact, the actual sets of states that emerge are those which are organized by the breakdown events.

A possible self-organized state is one that is critical in the sense that it has power law spatial and temporal correlations, like equilibrium systems undergoing a second order phase transition. The breakdown events in that state then must also be critical in the sense of a nuclear chain reaction process. In a supercritical system, a single local event, like the injection of a neutron, leads to an exponentially exploding process. A sub-critical process has exponentially decaying activity, always dying out. In the critical state, the activity is barely able to continue indefinitely, with a power law distribution of stopping times, reflecting the power law correlations in the system and *vice versa*.

It is intuitively clear that complex systems must be situated at this delicately balanced edge between order and disorder in a self-organized critical (SOC) state. In the ordered state, every place looks like every other place. Think of a crystal where the atoms are lined up over millions of inter-atomic distances. In the disordered state, there are no correlations between events that are separated in time or space: we have white noise. Again, it makes no sense to talk about complex behavior. Chaotic systems belong to this latter category. Sub-critical or supercritical states can usually be understood quite easily by analysing the local properties. Only

at the critical state, does the compromise between order and surprise exist that can qualify as truly complex behavior. There are very large correlations, so the individual degrees of freedom cannot be isolated. The infinity of degrees of freedom interacting with one another cannot be reduced to a few. This irreducibility is what makes critical systems complex.

Thus, self-organized criticality provides a general mechanism for the emergence of complex behavior in nature. It has been proposed that granular piles [14], traffic [12], magnetic fusion plasmas [13], the crust of the earth [15,16], river networks [17] and braided rivers [18,19], superconductors in a magnetic field [20], etc., all operate in a self-organized critical state.

The sandpile was the first model introduced by Bak, Tang, and Wiesenfeld to demonstrate the principle of self-organized criticality [14,21]. This model has subsequently received a great deal of attention due in part to its potential for having a theoretical solution. Dhar showed that certain aspects of its behavior could be calculated exactly based on the Abelian symmetry of topplings [22]. The sandpile was thought of as a paradigmatic gedanken experiment, but there has also been experimental confirmation of self organized criticality in granular piles. Fig. 1 shows an experiment on a pile of rice by Frette et al. [23]. Grains of rice were dropped between two glass plates by a seeding machine, and the avalanches were monitored by a video camera connected with a computer for data analysis. A power law distribution of avalanches was found, indicating SOC.

Over the past decade there has been a great deal of theoretical work on other models of SOC. Much of this work has focussed on other idealized models of sandpiles. These models typically involve a sequence of nodes to which sand is added until a critical gradient or height is reached locally, triggering redistribution of sand to nearest neighbors. Then a chain reaction of instabilities may occur encompassing all scales up to the system size. Self-organized critical systems evolve toward a scale-free, or critical state naturally, without fine tuning any parameters. This gives rise to power law distributions for the breakdown events. Minimal SOC models have been developed to describe a diverse set of phenomena including earthquakes [15,16,24,25,26], solar flares [27], forest fires [28], magnetically confined plasma [13,29], fluctuations in stock-markets [30] and economics [31], black hole accretion disks [8,32], traffic [12], biological evolution [33,34], braided rivers formed by vortex avalanches in superconductors [35], and disease epidemics [36], among others [21].

Given the preliminary nature of current understanding of complex systems, we are forced to consider one type of system at a time, looking for general principles. Some advancement has come from developing and studying simple computer models which help to conceptualize the essential attributes of the specific phenomena, and eventually to relate those to other phenomena. In the following we shall review a couple of these applications from widely different scientific domains: one from biology (co-evolution of species), one from solid state and geophysics (vortex avalanches and braided rivers), one from the social sciences (traffic), and one from cognitive science (brain function).

Figure 1: Avalanche in Ricepile Experiment

3 Braided Rivers and Superconducting Vortex Avalanches

Magnetic flux penetrates type II superconductors in quantized vortices which can move when an electrical current is applied, overcoming pinning barriers. When magnetic flux is forced in or out of the superconductor, vortices have been observed to intermittently flow [20] through preferred channels [37]. Using a simple cellular model [35] to mimick this experimental situation, it has been found that the vortex flow makes rivers strikingly similar to aerial photographs of braided fluvial rivers, such as the Brahmaputra [38]. This suggests that a common dynamical mechanism exists for braiding, namely, avalanches of stick-slip events, either sliding sediment or vortices, which organize the system into a critical braided state [19].

The cellular model [35] includes basic features of vortex dynamics: over-damped motion of vortices, repulsive interactions between vortices, and attractive pinning interactions at defects in the material. It is a coarse grained description at the scale of the range of intervortex interactions, the so-called London length, and throws out most microscopic degrees of freedom (specific information about the vortex cores). As in experiments, vortices are slowly pushed into the system at one boundary (the left) and allowed to leave at the other boundary (the right). The vortex-vortex repulsions cause a gradient to build up in the vortex density across the system. Even-

tually, as vortices are constantly added, a critical slope is achieved where the force from the gradient of vortex density is opposed by pinning forces, making a delicately balanced vortex pile reminiscent of a pile of sand. Then adding new vortices slowly at the boundary triggers avalanches of vortex motion, where one moving vortex can cause others to become unstuck, leading to a chain reaction. Avalanches of all sizes occur, limited only by the physical size of the system. Since the avalanches have no other characteristic spatial or temporal scale, the model exhibits self-organized criticality. Similar behavior has been observed experimentally[20], and in molecular dynamics simulations of the microscopic equations of motion[39].

The spatial variation of the overall vortex flow is measured in terms of the number of vortices moving in each cell, averaged over a long time interval representing many vortices flowing through the system. Fig. 2. represents a "time-lapsed" photograph of vortex motion. Rather than exhibiting uniform flow, the vortices clearly have preferred channels to move in. The braided vortex river resembles networks of interconnected channels formed by water flowing over non-cohesive sediment. Such braided fluvial systems have been observed from aerial photographs to exist for many different length scales and types of sediment [38,40,41]. In fact, braiding has been proposed to be the fundamental instability of laterally unconstrained free surface flow over cohesionless beds, and has been found to be a robust feature in simulations of river flow with sediment transport that includes both erosion and redeposition [42].

A quantitative scaling analysis reveals that the vortex river pattern is a self-affine multifractal with scaling dimensions close to those measured for a variety of braided rivers [19]. Given the vastly different length scales and materials involved, this apparent universality may seem surprising. Nevertheless, it is known that this type of universality can exist in systems which evolve by avalanches into a self-organized critical state [43]. In the case of braided vortex rivers, the patterns are due to a slip-stick process consisting of vortex avalanches, that self-organizes to a critical state resulting in the observed long-range correlations of the braided pattern. It has been postulated that braiding of fluvial rivers is due to a self-organized critical process[18].

Are there avalanches in fluvial rivers that could self-organize and produce the observed braiding? In fact there are. "Pulses" in bedload transport have been observed to occur on all spatial and temporal scales up to those limited by the size of the river studied [44]. Analogous pulses in the vortex model are seen by measuring the vortex flow through individual lattice cells as a function of time. The flow in a small region of the system is temporally intermittent; there is a broad distribution of intervals between pulses, and the pulses themselves can have a broad range of sizes. These pulses are a consequence of avalanche dynamics in a self-organized critical state in the model. Thus, vortices of magnetic flux are analogous to sediment in fluvial rivers. The elementary stick-slip process is that of sediment slipping and then resticking at some other point, like intermittently moving vortices. The elementary slip event can dislodge nearby sediment leading to a chain reaction of slip events, or avalanches. Sediment transport can be triggered when the local sediment slope is too high; the same is true for vortices in a superconductor. Thus, in both magnetic flux and fluvial rivers it appears that the braiding emerges from a stick-slip process

Figure 2: A "time-lapsed" photograph of vortex motion with average flow from left to right. The lattice size is 600×500. Sites containing an average amount of flow are shown in red. Yellow sites have a flow level greater than 20 times the average. Dark blue sites have almost no vortex flow, although virtually every site has some minimal amount. The intricate braiding pattern is remarkably similar to the pattern formed by braided fluvial rivers.

consisting of avalanches of all sizes [19].

4 Is Life a Self-organized Critical Phenomenon?

Evolution has taken place in a highly intermittent way. Periods with little activity have been pierced by major extinction events where many species disappeared, and other species emerged. About 50 million years ago the dinosaurs vanished during such an event, but this is far from the biggest. 200 million years ago we had the Permian mass extinction, and 500 million years ago the Cambrian explosion took place.

Traditional scientific thinking is linear. Nothing happens without a reason. The bigger the impact, the stronger the response. Thus, without further ado paleontologists and other scientists working on early life took it for granted that those extinctions were caused by some external cataclysmic events. Several have been suggested, including climatic changes and volcanic eruptions. The prevailing view

on the Cretaceous event is that it was caused by a meteorite hitting earth.

The linear point of view is correct for a simple system near equilibrium, such as a pendulum nearly at rest. But we do know that large events can happen without external impact in geophysical and astrophysical processes. No meteorite is needed in order to have large earthquakes, for instance. Actually, there is some striking statistical regularities indicating that the mass extinctions are part of a self-organized critical process.

Species do not evolve in isolation, so biology is a cooperative phenomenon! The environment of each individual is made up of other individuals. The atmosphere that we breathe is of biological origin, with an oxygen content very different from that at the time of the primordial soup. Species interact in food webs. The interaction can be through competition for resources, as parasites, or by symbiosis. This allows for the possibility that the extinction events can be viewed as co-evolutionary avalanches, where the death of one species causes the death (and birth) of other species, just as the toppling of one grain of rice in the rice pile leads to toppling of other grains.

Let us take a look at the fossil record. Fortunately, Jack Sepkoski has devoted a monumental effort to mapping out the rate of extinction during the last 500 million years.[47] It is extremely important to have as much data as possible, since we cannot make accurate theories for specific events, and therefore must confront theories with observations at the statistical level. The insert in Fig. 3 shows the temporal variations of the number of Ammonoida families. If part of the curve shown is enlarged, the pattern seen on the finer scale looks the same as that seen on the coarser scale.[48] Thus, there is no typical scale for the variations. This *scale-independent* or *self-similar* behavior is a strong indication of criticality—it cannot occur in simple systems with few components, including those exhibiting low-dimensional chaotic behavior.

Self-similarity, or scaling, can be expressed more quantitatively in terms of the power spectrum $p(f)$ of the time series. The power spectrum is the Fourier Transform of the autocorrelation function. When plotted with log-log axis, Fig. 3, it shows an approximately straight line over a couple of decades. This indicates that the spectrum is a power law, $p(f) = f^{-\alpha}$. The slope α is approximately unity. This type of dynamics is called one-over-f ($1/f$) noise. It is completely impossible to explain the smooth $1/f$ behavior with a set of arguments tailored each to events on a separate scale. Even in the absence of any theory, the smooth $1/f$ behavior is an empirical indication that the underlying mechanisms are the same on all scales. How else to explain that the curve has the same slope on all scales, and that segments corresponding to different scales join smoothly to form a straight line spanning all scales? Figure 4 shows the distribution of life times T of genera, also from Sepkoski's data. This is another power-law, $N(T) \sim T^{-2}$, giving further evidence that life is a critical process.

Because of the complexity of the phenomenon that we are dealing with—the global biological evomlution on all time scales—mathematical modelling is an extremely delicate affair. It is difficult to go from micro-evolution where the mechanisms (genetics) are relatively well understood, to macro-evolution at the largest scale. Geneticists may understand what goes on within a few generations of a few

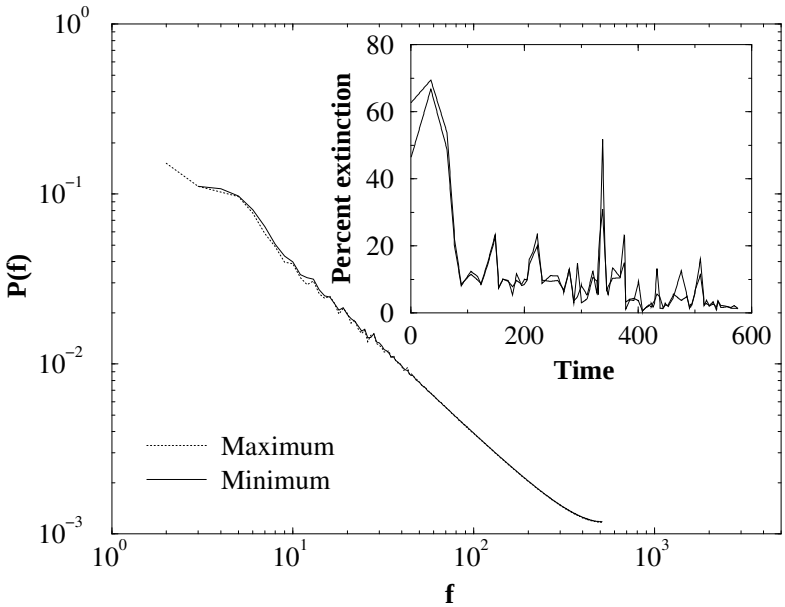

Figure 3: Spectrum of the extinction rate for Ammonoida families. The temporal variations of the number of families is shown in the insert (Sole, Monrubia, Benton, Bak [48]).

hundreds or a few thousands of rats, but they have little to say about the behavior of an evolving global ecology of millions of species, each with hundreds of millions of individuals.

Kauffman and Johnsen [49] were the first to suggest that the Darwinian dynamics of an ecological network with all species connected through their interactions, positive or negative, could lead to a critical state. The first model for evolution to show SOC was the Bak-Sneppen (BS) model [33,51,10].

The Bak-Sneppen model represents an entire species by a single fitness number. Selection acts on the level of the individual, of course, but to achieve simplification we consider the evolution at the "coarse-grained" species level. Consider a number, N, of species placed on a circle. Each species interacts with its two neighbors. Each species is assigned a random fitness $0 < f < 1$ which represents its ability to survive in a given environment. Time is discrete, and at each time step the species with the lowest fitness goes extinct, and is replaced by another species with a random fitness f, $0 < f < 1$. Alternatively, one could view the process as a pseudo extinction where a species is replaced by a mutated variant. Whatever the view, this change in one species affects the fitnesses of its two neighbors: their fitnesses, which might originally have been high, are also replaced by new random fitnesses, reflecting the fact that their existence has become a new ball game. This process of changing the fitnesses of the least fit species and the two it interacts with is continued ad infinitum.

Most of the species have fitnesses above a threshold that has established itself with value approximately 0.67, forming a rather stable network (Figure 5). However,

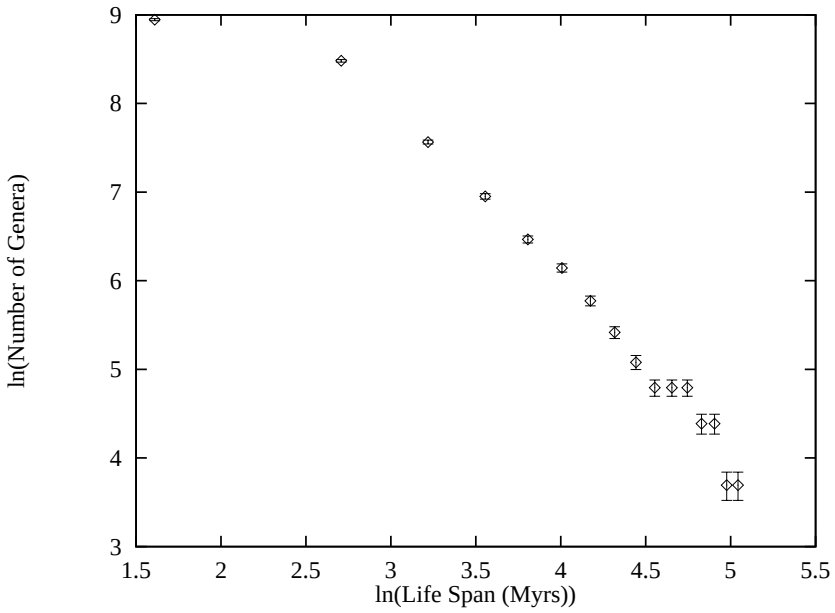

Figure 4: Lifetime distribution for genera as recorded by Sepkoski and Raup. The distribution can be well fitted by a power law $N(T) \propto 1/T^{-2}$ except at its lowest T-values (Sneppen, Bak, Flyvbjerg, Jensen [51]).

there is a localized region with species of lower fitnesses. These are the species, or niches, that are currently undergoing changes or extinctions as part of a *co-evolutionary avalanche*.

During an avalanche, nature "experiments" with the species involved, changing many of them several times, until they all have achieved fitnesses above the threshold. If the changes experienced by any given species is measured vs. time, one finds punctuated equilibrium behavior, with periods of stasis interrupted by intermittent bursts. This can be characterized by the power-spectrum of the local activity, which is a $1/f$ spectrum with exponent $\alpha \sim 0.59$.[34]

Note that in the BS model evolution progresses by elimination of the least fit species, and not by propagation of strong species. This distinction is not merely semantics. One can not have a process of evolution, where the individual species out-competes their environment, the popular view of Darwinian evolution. The complexity of Life is intimately related to the existence of large interactive networks. Actually, extremal dynamics associated with removing the weakest link is essential for the emergence of complex or critical phenomena. The criticality of the SOC earthquake models can also be traced to the breakdown of the weakest site, and not an arbitrary site.

Thus, the mechanism of evolution is "extinction of the least fit" rather than "survival of the fittest"! The best a species can hope for is to be a participant of the global ecological network. In the final analysis, being fit simply means being a self-consistent part of a complex structure.

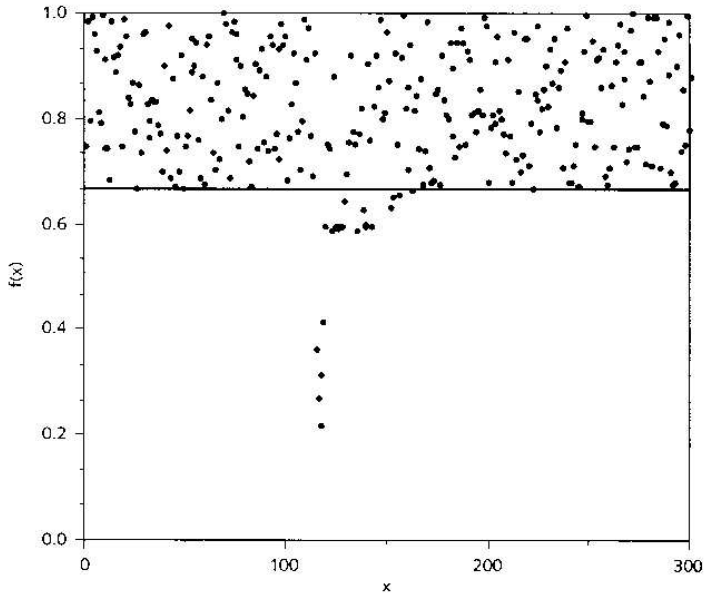

Figure 5: Illustration of the BS model for 300 species. A snapshot of the 300 fitness values is shown. Most values are above the threshold of 0.67. The species with fitnesses below 0.67 participate in an avalanche. In the next step, the species with the lowest barrier, here number 113, will evolve, together with its two nearest neighbors, nos. 112 and 114 (Sneppen, Bak, Flyvbjerg, Jensen [51]).

4.1 Ecology dynamics

Perhaps the dynamics of evolution can found in a smaller scale by studying local ecologies or food webs. Keitt and Marquet [52] have studied the dynamics of birds introduced into Hawaiian islands. They measured the extinction rate between successive periods of 10 years, (to be compared with 4 million year intervals used for the analysis of the fossil record) and found a power law distribution and also extracted the lifetime distribution of species, yielding another power law with exponent near unity. A total of 59 extinctions on six islands were included in their statistics. Because of the scant amount of data available, no firm conclusions could be reached, but everything was consistent with an ecology operating at criticality. In a very comprehensive study, Lockwood and Lockwood [53] have analyzed grasshopper infestations in several regions of Idaho and Wyoming. Histograms of annual infestations, measured as the area involved, shows a power law distribution. Although numerous external factors affect the infestation rate, the results suggest criticality.

5 Traffic Jams and the Most Efficient State

Our everyday experience with traffic jams is that they are annoying and worth avoiding. Intuitively, many people believe that if we could somehow get rid of jams

then traffic would be more efficient with higher throughput. However, this is not necessarily true. By studying a simple model of highway traffic, it is found that the state with the highest throughput is a critical state with traffic jams of all sizes. If the density of cars were lower, the highway would be underutilized; on the other hand, if it were higher there would inevitably be a huge jam lowering throughput. This leaves us with the critical state as the most efficient state that can be achieved. Finding a real traffic network operating at or near peak efficiency may seem highly unlikely. To the contrary, as found in the model, an open network self-organizes to the critical state [12].

The Nagel-Schreckenberg [45] model is defined on a one dimensional lattice with cars moving to the right. Cars can move with integer velocities in the interval $[0, v_{max}]$. The maximum velocity v_{max} is typically set equal to 5. This velocity defines how many "car lengths" each car will move at the next time step. If a car is moving too fast, it must slow down to avoid a crash. A slow moving car will accelerate, in a sluggish way, when given an opportunity. The ability to accelerate is slower than the ability to break. Also, cars moving at maximum velocity may slow down for no reason, with probability p_{free}. A "cruise-control" limit of the model exists where $p_{free} \to 0$. This means that all cars which have reached maximum velocity, and have enough headway in front of them to avoid crashes, will continue to move at maximum velocity. Thus it is possible for the motion in the system to be completely deterministic.

If the cars are moving on a ring starting from random initial conditions, at low densities the initial jams will "heal" and the system will reach a deterministic state where the current is equal to the density of cars multiplied by the maximum velocity. This will hold up to some maximum density above which jams never disappear and the current is a decreasing function of density.

Remarkably, maximum throughput, j_{max}, is selected automatically when the left boundary condition is an infinitely large jam and the right boundary is open.[12] Traffic which emerges from the megajam operates precisely at highest efficiency. This situation is shown in Fig. 6.

The horizontal axis is space and the vertical axis (down) is increasing time. The cars are shown as black dots which move to the right. The diagram allows us to follow the pattern in space and time of the traffic. Traffic jams show up as dense regions which drift to the left, against the flow of traffic. The structure on the left hand side is the front of the megajam (cars inside the megajam are not plotted). Cars emerge from the big jam in a jerky way, before they reach a smooth outgoing pattern operating at j_{max}. Far away from the front of the megajam all cars eventually reach maximum velocity.

If the outflow is perturbed slightly, traffic jams of all sizes occur. No cataclysmic triggering event, like a traffic accident, is needed to initiate large jams. They arise from the same dynamical mechanism as small jams and are a manifestation of the criticality of the outflow regime. Our natural intuition that large events come from large disturbances is violated. It does not make any sense to look for reasons for the large jams. The large jams are fractal, with small sub-jams inside big jams ad infinitum. Between the subjams are "holes" of all sizes where cars move at maximum velocity. This represents the irritating slow and go driving pattern that

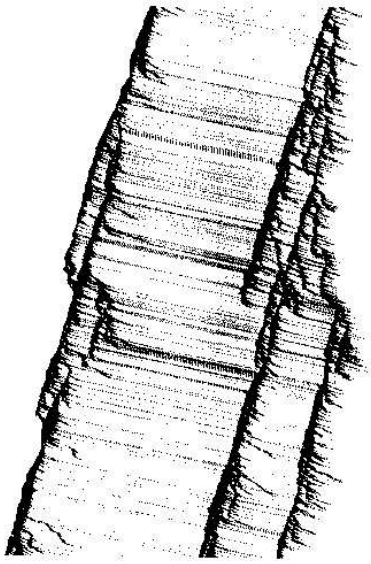

Figure 6: Traffic jams. The horizontal direction indicates a highway. Cars are shown as black dots. Time progresses in the downward direction. The dots form trajectories of individual card. The dark areas with a high density of cars indicate traffic jams. Note that the jams are moving backwards. (Nagel and Paczuski, 1995 [12]).

we are all familiar with in congested traffic. On the diagram, it is possible to trace the individual cars and observe this intermittent pattern. This behavior gives rise to $1/f$ noise, as seen in real traffic flow [46]. This $1/f$ behavior can be calculated exactly for this model by formulating the jams as a cascade process [12]. The picture of avalanche dynamics as a fractal in space and time has application to many complex dynamical systems in addition to traffic.[34]

The conventional view is that one should try to get rid of traffic jams in order to increase efficiency and productivity. However, the critical state, with traffic jams of all sizes, is the most efficient state that can actually be achieved. A carefully prepared state where all cars move at maximum velocity would have higher throughput, but it would be dreadfully unstable. The very efficient state would catastrophically collapse from any small fluctuation. A similar situation occurs in the familiar sand pile models of SOC.[14] One can prepare a sand pile with a supercritical slope, but that state is unstable to small perturbations. Disturbing a supercritical pile will cause a collapse of the entire system in one gigantic avalanche.

But there is perhaps even a deeper relationship between traffic and economics [12,21]. In an economy, humans interact by exchanging goods and services. In the real world, each agent has limited choices, and a limited capability to monitor his changing environment. This is referred to as bounded rationality. The situation of a car driver in traffic can be viewed as a simple example of an agent trying to better his condition in an economy. Each driver's maximum speed is limited by the other cars on the road and posted speed limits. His distance to the car in front of him is

limited by his ability to stop and his need for safety in view of the unpredictability of other drivers. He is also exposed to random shocks from the road or from his car. He may be absent minded. If traffic is a paradigm for economics in general, then perhaps we have found a new economic principle: the most efficient state that can be achieved for an economy is a critical state with fluctuations of all sizes.

6 The Critical Brain

Why do we need a brain at all? In a sub-critical world everything would be simple and uniform - there would be nothing to learn. In a supercritical world, everything would be changing all the time in a chaotic way - it would be impossible to learn. The brain is necessary for us in order to navigate in a complex, critical world.

A brain is able not only to remember, but also to forget and adapt to a new situation. In a sub-critical brain memories would be frozen. In a supercritical brain, the patterns would change all the time so no long term memory would be possible. This leaves us with one choice - the brain itself has to be in the in-between critical state. Using physics terminology, it is the high susceptibility of the critical state which makes it adaptable.

Actually, Alan Turing [54], some time ago, speculated that perhaps the working brain needs to operate at a barely critical level, in order to stay away from the two extremes - namely the too correlated sub-critical level, and the too explosive supercritical dynamics.

In traditional neural network models, the goal has typically been to have the desired patterns represented by very stable states. In the Hopfield model [55], for instance, the patterns correspond to deep energy minima in a spin glass model. This represents the traditional Hebbian [56] picture where synapses connecting firing neurons are strengthened. Once the desired memory has been encoded, it is hard to adapt to a new situation when the environment changes, because the deep minima have to be removed by a dynamical process. Traditional models are sub-critical. Moreover, the learning process takes place by having an external teacher, a computer algorithm that sets the strengths of the neural network connections. It is hard to see how this can be accomplished without the intervention of an external agent. The learning process of the neural network is not self-organized. Chialvo and Bak [57] have suggested an alternative scheme, which at least in principle could act as a paradigm for real brain processes.

6.1 Learning from Mistakes

Recall that in the evolution model, criticality, and hence complexity, was achieved by extremal dynamics where the least fit species were weeded out. Chialvo and Bak used a similar mechanism for brain functioning, with the synapses playing the role of the individual species. Whenever a poor result is achieved, all the synapses which fired in the process are democratically punished. However, good behavior is not rewarded at all; the reward system is all stick and no carrot. There is no Hebbian strengthening of successful synapses. While the model is grossly simplified, the features of the model are all biologically plausible.

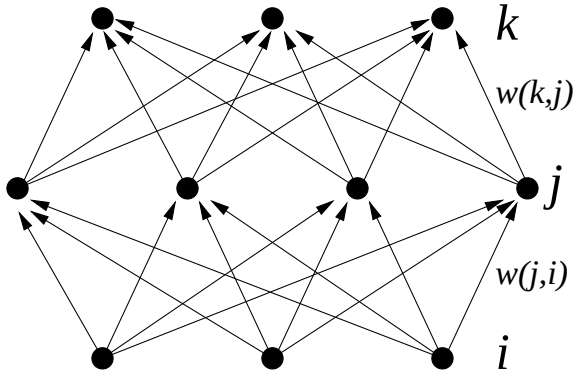

Figure 7: Neurons in layer J have synaptic connections with all inputs in I and connect with all neurons in layer K.

The topology of the network is not very important. For simplicity, let us consider neurons arranged in the layered network in Fig. 7, where K represents the outputs, I the inputs and J the middle layer. Each input is connected with each neuron in the middle layer which, in turn, is connected with each output neuron, with weights W representing the synaptic strengths. The network must learn to connect each input with the proper output (which is pre-determined) for any arbitrary associative mapping. The weights are initially randomised, $0 < W < 1$.

The dynamical process in its entirety is as follows:
An input neuron is chosen. The neuron j_m in the middle layer with the largest $w(j,i)$ is firing. Next, the output neuron k_m with the maximum $w(k,j_m)$ is firing. If the output k happens to be the desired one, **nothing** is done, otherwise $w(k_m,j_m)$ and $w(j_m,i)$ are both depressed by a fixed amount. The iterative application of this rule leads to a convergence to any arbitrary input-output mapping. Since there are no further changes once the correct result has been achieved, the proper synapses are only barely stronger than some of the incorrect synapses.

Supposed now that the environment changes, so that a different connection between input and output is correct. The neurons which fire and led to the previously correct output are now punished, allowing new connections. Eventually that pattern will also quickly be learned.

The reason for quick re-learning (adaptation) is simple. The rule of adaptation assures that synaptic changes only occur at neurons involved in wrong outputs. The landscape of weights is only re-shaped to the point where the new winners barely support the new correct output, with the old pattern only slightly suppressed. Thus, only a slight suppression of a currently active pattern is needed in order to generate new patterns when need be. In particular, re-learning of "old" patterns which have been correct once in the past is fast. This feature can be strengthened if the synapsis which have never been firing when a good result was achieved are punished more than synapses whose firing has previously led to a good result.

The landscape of synaptic strength in our model after many learning cycles

consist of very many values which are very close to those of the active ones, a manifestation of the critical nature of the state. Figure 8 shows a snapshot of the synaptic strengths. The synapse indicated by an arrow is a currently active one, associated with a correct response. Other neurons near the active surface have strengths located slightly below the critical surface. One can imagine that "thinking" is the process of sifting through, and suppressing, patterns which once have been correct, until a combination leading to a good result is achieved. Bits and pieces of patterns that have previously been successful are utilized. Old memories are located at the same spot where they have always been - they have simply been slightly suppressed by more recent patterns.

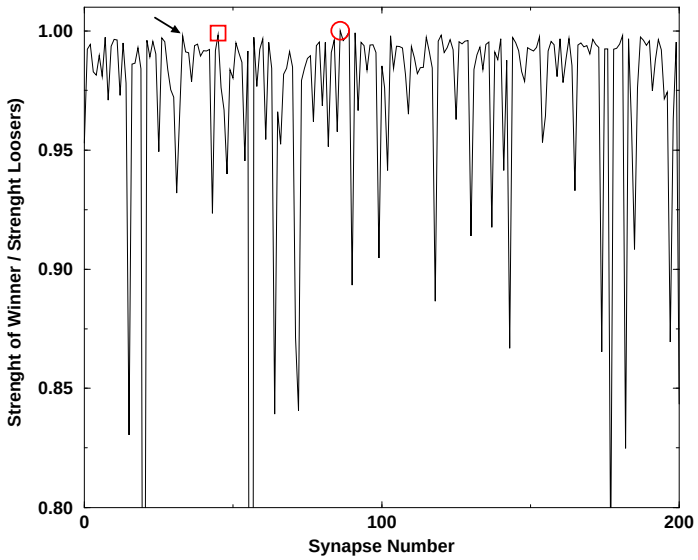

Figure 8: Landscape of synaptic strenghts between one input and 200 neurons in the middle layer. The encircled value corresponds to the strongest connection between the input neuron and the output layer. The arrow and the box indicate other connections likely to be used in the future (see text).

The biological plausibility of the schema depends on the realization at the neuronal level of two crucial features:

a) Activity propagates through the strongest connections, i.e. extremal, or winner-take all, dynamics. This can be fulfilled by a local circuit organisation, known to exist in all cortices, where the firing of other neurons is shut off by lateral inhibitory connections.

b) Depression of synaptic efficacy involves the entire path of firing neurons. A process must exist such that punishment can be relayed long after the neuron has fired, when the response from the outer world to the action is known. Chialvo and Bak conjectured a mechanism of "tagging" synapses for subsequent punishment, or long term depression (LTD), analogous to (but mirroring) recently reported tagging of synapses for long term potentiation (LTP) [58]. The feed-back probably takes

place through the limbic system of neurons, situated in the neck, which spray the large areas of the brain. One could imagine that this global feed-back signal affects all neurons which have recently fired, causing plastic changes of the synaptic connections. The limbic system is disconnected when dreaming, which could explain why we generally do not remember our dreams. Actually, long-distance, long-term synaptic depression has been directly demonstrated by Fitzsimons et al [59] in cultured hippocampal neurons from rat embryos.

In addition to giving insight into mechanisms for learning in the brain, the ideas presented here could be useful for artificial learning processes, for instance in adaptable robots. These possibilities are currently being investigated and appear promising.

Historically, many processes that were considered to be examples of directed learning have been shown to be caused by selection. The Larmarquean theory of evolution as a learning process, where useful acquired features are strengthened, was replaced by the Darwinian theory of evolution as a selection process, where the unfit species are weeded out. A similar paradigm shift occurred in immunology through the theory of clonal selection. Ironically, if the philosophy represented by the Chialvo-Bak model is correct, learning in the brain is not a (directed) learning process either. It is also an example of a co-evolutionary selection process where incorrect connections are weakened.

The paradigm of science in the second millenium, reductionism, is insufficient to explain complexity in nature. There appears to be a need for an outside organizing agent who fine tunes the natural world and puts the building blocks together. We speculate that, instead of this agent, co-evolutionary selection leading to a critical state by removing untenable parts may be the fundamental organizing principle leading to all the possible complexity in the universe.

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