# Qualitative Dynamic Behavior of Physical System Models With Algebraic Loops


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## Abstract

TRANSCEND, a system for monitoring and diagnosis of abrupt faults in complex dynamic systems, relies on system models to predict dynamic behavior in response to abrupt faults. The use of qualitative predictions of this transient behavior mitigates complexity issues and convergence problems that exist in numerical diagnosis approaches. In this framework, it is important to model the system under diagnostic scrutiny at a level of detail that relates to the bandwidth of the measurement data. Too much detail results in a model that captures very fast dynamics as continuous transients, whereas they appear as discontinuous changes in the measured signals. In many cases, abstracting away these small parameter values may result in algebraic dependencies between variables, i.e., variables affect one another instantaneously without integrating effects. Because of the compensating effects inherent in physical system behavior, a straightforward deviation propagation results in predictions that are unknown in a qualitative sense. A method is proposed that recognizes such dependencies and propagates compensating effects without introducing unnecessary conflicts by tracking the origins of compensating influences in feedback loops.


## Introduction

The application of functional redundancy techniques is the key to recent advances in model based fault detection and isolation (FDI). This FDI paradigm uses a functional model of the system to relate measurements and detect any discrepancies based on the model information. When such discrepancies occur, model parameters that are part of the functional relations are implicated as possibly deviating from their normal value. Continued monitoring of the system then provides additional measurement information that can be used to find the true deviating parameter, or parameter set, and relate this back to a physical component that embodies these parameters.

Traditional FDI schemes, employ a quantitative approach, which requires a sufficiently detailed model with precise knowledge of system parameters to achieve good results (Isermann 1989; Clark, Frank, & Patton 1989). In many situations this may be hard to achieve because the detailed model may be of high order with complex nonlinearities. Further, inaccuracies in the sensor data may make the analysis difficult. Qualitative approaches seek to overcome these problems by abstracting model relations and expressing them as increasing and decreasing effects. A qualitative FDI scheme can be designed to mitigate the problems of quantitative approaches for complex systems by quantizing measurements as: (i) magnitude values below nominal ($-$), at nominal (0), and above nominal ($+$), and (ii) rate of change as increasing ($\uparrow$) and decreasing($\downarrow$). A qualitative parameter estimation framework can be designed to exploit the functional relations between parameters and system variables, to arrive at qualitative parameter value deviations, i.e., the parameter value is determined to be above ($+$) or below ($-$) normal.

To illustrate, consider the functional relation for flow, $f$, through a pipe, given by

$$f = \frac{p}{R}, \tag{1}$$

where $p$ is the pressure drop over the pipe and $R$ is the pipe resistance. In the case of a blockage in the pipe, the flow is reduced, indicated by $f^-$, and this correctly implicates $R$ as above normal ($R^+$). $R^+$ can be interpreted as a fault hypothesis, and denotes an increase in the resistance of the pipe. For a pipe connected to a tank (see Figure 1), the rate of change in the pressure, $\dot{p}$, at the bottom of the tank is given by

$$\dot{p} = \frac{1}{C}(f_{in} - f), \tag{2}$$

where $f_{in}$ is the flow into the tank and $C$ is the tank capacity. An observed decrease in flow, $f^-$, again implicates $R^+$ as a fault hypothesis from Eq. (1), and also implies $\dot{p}^+$ from Eq. (2), which means $p^+$ over time. However, $p^+$ also implies $f^+$ from Eq. (1), and, therefore, unless properly analyzed, the set of equations generate conflicting predictions for $f$, and the true future behavior cannot be determined.

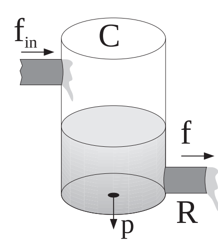



Figure 1: A tank with in and outflow.

This is solved by including the temporal delay as introduced by the time derivative operator acting on $p$. Now, $f^{-}$ predicts $p^{\uparrow}$ from Eq. (2), i.e., $p$ will rise because it has a positive derivative value. Consequently, $f^{-,\uparrow}$, i.e., $f$ is initially below nominal, and its value starts increasing. Therefore, a careful analysis of the transient dynamics even in a qualitative framework reveals that the instantaneous effect of a blockage in the pipe is a decrease in outflow, $f$. This in turn causes the tank to accumulate more fluid and $p$ increases, which causes the flow rate to increase after its initial drop.

The notion of including temporal effects in the qualitative description of system behavior has been formalized in an FDI theory based on a representation of system dynamics as a temporal causal graph (TCG) model. This model representation captures functional relations (edges) between system variables (vertices) in terms of model parameters and their temporal effect. TRANSCEND, a model based diagnosis system for fault detection and isolation of abrupt faults in engineered systems is based on this approach (Mosterman & Biswas 1999a). To avoid the traditional problems of a qualitative framework (Kuipers 1994), the system model must be well constrained to avoid predicting spurious behaviors and prevent a combinatorial explosion of possible behaviors. Bond graph models have proven to be well suited for this task (Mosterman 1997; Mosterman & Biswas 1999a) and can be automatically converted into a TCG representation by the hybrid bond graph modeling and simulation tool HYBRSIM (Mosterman & Biswas 1999b). The approach has been successfully applied to a number of controlled physical systems (e.g., a secondary sodium cooling loop (Mosterman & Biswas 1999a), an automobile engine cooling system (Manders *et al.* 2000), and a three-tank fluid system (Manders & Barford 2000)).

A critical task in constructing the system model for FDI is to determine the fastest dynamic behavior that can be observed in the system. Dynamic behavior that is too fast to be observed in the measurements should not be included in the model as it leads to functional relations that cannot be observed, and may lead to spurious failure hypotheses. For example, small inertial effects of flow in a pipe may not be visible with the available instrumentation system, and, therefore, the model should not include parameters that correspond to those inertial effects.

In lumped parameter models, neglecting small parameters may cause recursive dependencies among system variables without temporal effects in the system model (van Dijk 1994).The model is said to contain algebraic loops. When the model includes these parameters, the integrating effects would temporally decouple signals, and add to the state vector. However, if these parameters are abstracted away, direct relations arise, and, because of the negative feedback between passive model components, this leads to conflicting influences when predicting system behavior. In a qualitative framework, variables with such conflicting predictions are unknown and cannot be used to refine the set of hypothesized faults.

This paper presents a method is to correctly handle algebraic loops in physical system models for diagnosis applications. The TCG representation and the hypothesis generation algorithms of TRANSCEND are introduced, and followed by a detailed analysis of the the algebraic loop problem and its implications for representing a model for FDI as a TCG. A method for handling algebraic loops is then proposed, and presented as an enhancement of the hypothesis generation algorithm. A small example shows that the behavior prediction is richer with the enhanced algorithm.

## Prediction of system behavior in TRANSCEND

TRANSCEND's diagnosis model is a directed graph (TCG) whose edges capture the dynamic relations that govern system behavior. The TCG provides a rich and uniform framework for representing magnitude and temporal constraints among system variables. Component parameter values and their temporal influences on system behavior are defined as attributes of the TCG edges. System behavior variables, defined in terms of the domain independent concepts of *effort* and *flow*, correspond to TCG vertices. In addition, there may be *signal* vertices that denote modulated variables and other constraints that exist between system variables. The hypothesis generation and behavior prediction components of the fault isolation task in TRANSCEND are developed as graph traversal algorithms. These algorithms are discussed in detail in (Mosterman & Biswas 1999a). The problem space is constrained by the assumption that faults do not cause changes in system configuration, and that the system model remains valid when faults occur in the system.

The prediction algorithm computes the qualitative transient behavior of the observed variables under individual fault hypotheses. Transient behavior is expressed as a tuple of qualitative values for magnitude, $1^{st}$ order time derivative and higher order effects. The qualitative values are similar to those of the measured values: '$+$', '$-$', '0' or '$\cdot$'. The '$\cdot$' implies that the value is unknown, a result of opposite qualitative influences on a node. The tuple is called the *signature* for the variable (Mosterman & Biswas 1997; 1999a). The algorithm propagates the effects of a hypothesized fault through the graph to establish a signature for all observations. Energy storage elements cause time integrating effects and introduce temporal edges in which case the cause variable affects the derivative of the effect variable. In the TCG, these edges are marked with a $dt$ attribute. Propagation of a deviation starts with a $0^{th}$ order effect, i.e., a magnitude change. When an integrating edge is traversed, the magnitude change becomes a $1^{st}$ order change, i.e., the first time derivative of the affected quantity changes. Similarly, a first order change propagating across an integrating edge produces a second order change, and so on. The highest predicted derivative order required is a design consideration (Mosterman 1997).

To illustrate, consider again the tank system in Figure 1. A TCG model for this system is shown in Figure 2, where the model includes the combined effects described in Eq. (1) and (2). The variable $v$ is introduced to represent the $f_{in} - f$ summation in Eq. (2). When $R^{+}$ is a possible parameter deviation, hypothesized from a measurement deviation, future behavior is predicted by propagating this value through the graph. The inverse relation implies $R^{+}$ causes $f^{-}$, which

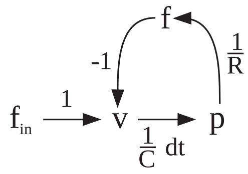



Figure 2: Temporal causal graph of the one-tank system.

causes $v^+$, and the integrating relation '$dt$' on the next edge leads to a first derivative change in $p$, i.e., $p^{\uparrow}$ (the value of $p$ is increasing). Continuing the propagation results in $p^{\uparrow} \rightarrow f^{\uparrow} \rightarrow v^{\downarrow} \rightarrow p^{\downarrow\downarrow}$, i.e., the second time derivative of $p$ is negative. This propagation continues until variable behavior of the highest required order is predicted. Lower order behavior that has no assigned deviation is considered to be normal. In case of the one-tank example, $p$ has a first order behavior that is positive, and a second order behavior that is negative, however, no deviation for the $0^{th}$ order behavior is assigned which is therefore considered to be normal. This is written as $p^{0+-}$, where the subscripts indicate, from left to right, increasing time derivative order of behavior. The implication is that the fault has no effect on $p$ at the time point of failure, but the value of $p$ starts increasing after the time point of failure. The negative value of the second derivative implies that eventually the value of $p$ may reach a steady state or even begin to decrease.

## Analysis of the Algebraic Loop Problem

This section discusses the use of singular perturbation methods to reduce the complexity of a model for FDI by abstracting away detailed behavior. The resulting model is then formulated in a canonical TCG representation and the algebraic loops that have emerged can then be easily recognized.

### Model Abstractions for FDI

Finding the right level of abstraction in the model leads to a crucial issue in the functional redundancy approach to FDI, that involves the resolution of the correspondence between model parameters and physical components. Consider the detailed system behavior to be captured by the system of equations $\langle f, g\rangle$, defined as,

$$\begin{cases} \dot{x} = f(x, z, \epsilon, t) \\ \epsilon\dot{z} = g(x, z, \epsilon, t) \end{cases} \tag{3}$$

where $\epsilon$ is small, and, therefore, $z$ captures fast dynamic behavior that may not be observable given the measurements. To achieve a suitable model for diagnosis, a singular perturbation (Kokotović, Khalil, & O'Reilly 1986) approach can be employed. Letting $\epsilon \rightarrow 0$, Eq. (3) becomes

$$\begin{cases} \dot{x} = f(x, z, 0, t) \\ 0 = g(x, z, 0, t) \end{cases} \tag{4}$$

that may in turn be be simplified to

$$\begin{cases} \dot{x} = f(x, z, t) \\ 0 = g(x, z, t) \end{cases} \tag{5}$$

Here $g$ becomes the *implicit* part, or the *algebraic equations*, and $z$ are the corresponding *algebraic* variables. If it is possible to manipulate the implicit part in an explicit form, i.e.,

$$z = g_x(x, t) \tag{6}$$

then $z$ can be substituted for in the differential equations to get:

$$\dot{x} = f(x, g_x(x, t), t), \tag{7}$$

resulting in the relation,

$$\dot{x} = f_x(x, t). \tag{8}$$

The reduced order model has obvious computational advantages, but the disadvantage of this model representation is that the function $f_x$ is no longer a direct map of the initial model, $\langle f, g\rangle$, anymore, and may contain aggregate parameters, i.e., parameters that correspond to more than one physical component. In an FDI application this is undesirable, because a single deviating parameter would implicate more than one physical component. The functional redundancy information that helped distinguish between components has been removed from the model and replaced by algebraic constraints. Handling the implicit equations directly would result in more discriminating information at the physical component level. To achieve optimal FDI results, it is, therefore, more appropriate to use the functional relations in the implicit equations. However, this results in a more complex model and leads to difficulties in the qualitative analysis techniques for fault hypothesis generation and tracking that was described in the Introduction.

### Singular Perturbation

The time derivative operator acting on $z$ partitions the system into a set of explicit equations. Any feedback effects pass through a temporal delay, and, therefore, only affect higher order temporal behavior and do not cause conflicts. In a causal model, the right hand variables are input to compute their temporal behavior.

If the $\epsilon$ parameter is removed from the model by substituting $\epsilon = 0$, temporal delays are replaced by algebraic couplings between variables, and that may cause conflicting predictions. This is solved by noticing that the entry point of such a set of functional relations may be compensated for by the removed temporal effect, but it is not reversed.

To illustrate, consider the electrical circuit in Figure 3 and

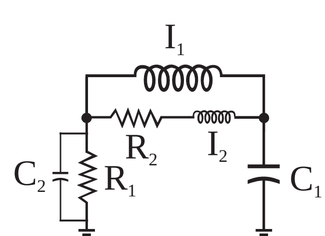



Figure 3: An electrical circuit with parasitic effects ($C_2$,$I_2$).

its model given by:

$$\begin{cases} \dot{p}_1 = \frac{1}{C_1}q_1 - \frac{1}{C_2}q_2 \\ \dot{q}_1 = -\frac{1}{I_1}p_1 + \frac{1}{I_2}p_2 \\ \dot{p}_2 = -\frac{1}{C_1}q_1 + \frac{1}{C_2}q_2 - \frac{R_2}{I_2}p_2 \\ \dot{q}_2 = \frac{1}{I_1}p_1 - \frac{1}{I_2}p_2 - \frac{1}{R_1 C_2}q_2. \end{cases} \tag{9}$$

If $C_2$ and $I_2$ represent parasitic effects, i.e. their values are very small with respect to the measurement bandwidth, they result in modeled time constants that cannot be observed in the measured variables. For FDI purposes, they can be abstracted away. However, for this system, setting $C_2 = 0$ and $I_2 = 0$ results in a system of equations that is singular, with $q_2 = p_2 = 0$. To study the implications of this model abstraction, the model must first be written as a system of equations corresponding to the singular perturbation representation form shown in Eq. (5). The model shows that for $I_2 = C_2 = 0$, $p_2 = q_2 = 0$, and, therefore, $\dot{p}_2 = \dot{q}_2 = 0$. To compute the reduced order system, $\dot{q}_2 = \dot{p}_2 = 0$ can be substituted in the original model in Eq (9) and $C_2$ and $I_2$ eliminated by introducing the algebraic variables $z_1 = \frac{p_2}{I_2}$ and $z_2 = \frac{q_2}{C_2}$. Along with $x_1 = \frac{p_1}{I_1}$ and $x_2 = \frac{q_1}{C_1}$, this gives the following system of equations in the singular perturbation form

$$\begin{cases} \dot{x}_1 = \frac{1}{I_1}x_2 - \frac{1}{I_1}z_2 \\ \dot{x}_2 = -\frac{1}{C_1}x_1 + \frac{1}{C_1}z_1 \\ 0 = -z_1 - \frac{1}{R_2}x_2 + \frac{1}{R_2}z_2 \\ 0 = -z_2 + R_1 x_1 - R_1 z_1 \end{cases} \tag{10}$$

## TCG Representation of a Model

The TCG representation corresponding to Eq. (10) is shown in Figure 4. Because one system parameter occurs on multiple edges, this graph may generate inconsistent predictions. For example, $R_1^+$ predicts $z_2^+$ and $z_2^-$.

To resolve inconsistencies in the predictions, the system of equations must first be transformed into a canonical form, i.e., each system model parameter is present as an edge attribute only once. This results in

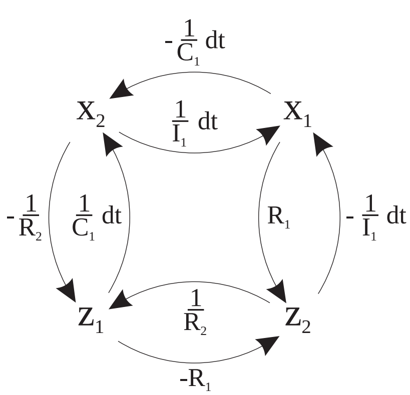



Figure 4: Temporal causal graph representation of Eq.( 10).

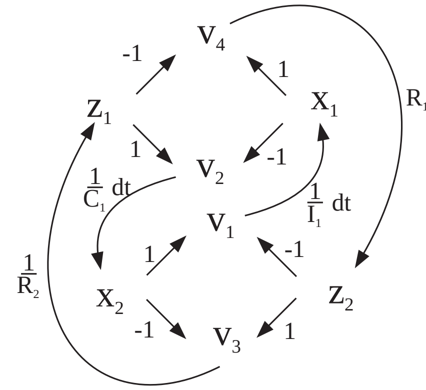



Figure 5: Temporal causal graph representation of Eq.( 11).

$$\begin{cases} \dot{x}_1 = \frac{1}{I_1}(x_2 - z_2) \\ \dot{x}_2 = \frac{1}{C_1}(-x_1 + z_1) \\ z_1 = \frac{1}{R_2}(-x_2 + z_2) \\ z_2 = R_1(x_1 - z_1) \end{cases} \tag{11}$$

where the variable summations become intermediate variables $v_1$, $v_2$, $v_3$, and $v_4$ in the TCG, shown in Figure 5. In this TCG, $R_1^+$ predicts $z_2^+$, which predicts $v_3^+$, $z_1^+$, $v_4^-$, and $z_2^-$, as a result of the algebraic loop. With the parasitic phenomena included in the model, the prediction would be $z_2^{0+-}$, i.e., the negative feedback effect would be temporally delayed, and, therefore, consistent predictions are achieved.

## Algebraic Loops

When the small time constants are abstracted away, $z_1$ and $z_2$ can be solved for algebraically to find their actual sensitivity to changes in $R_1$. This yields

$$\begin{cases} z_1 = \frac{1}{R_1 + R_2}(R_1 x_1 - x_2) \\ z_2 = \frac{1}{R_1 + R_2}(R_2 R_1 x_1 + R_1 x_2) \end{cases} \tag{12}$$

Therefore, $\frac{\partial z_2}{\partial R_1} > 0$ for $R_1 > 0$, and, consequently, $R_1^+$ should predict $z_2^+$.[1]

This sensitivity analysis can be generalized by assuming a linear system in the canonical form with negative loop gain. The general form of a variable $z_i$ then is $z_i = f(p)z_i + g(p)x$, where $x$ are input variables to the causal loop and $p$ represents the unique occurrence of a system parameter. This can be written as $(1 - f(p))z_i = g(p)x$, or $z_i = \frac{g(p)}{1-f(p)}x$. Because the loop gain, $f(p)$, is negative, this function is monotonic with respect to $p$ for $p > 0$.

In terms of the algebraic loop prediction, this implies that an initial deviation that enters a set of algebraic equations may be compensated for by a negative feedback effect, but it cannot reverse the initial deviation. A prediction $z_2^+$ that leads to $z_2^-$ along a negative feedback path results in a partial compensation, and, therefore, does not cause an ambiguity in the analysis of the change in $z_2$. The actual deviation remains $z_2^+$, but in a quantitative sense less $+$ than what it

[1] All variables are assumed to be positive.

would have been without the negative feedback. With this understanding it is now possible to design an algorithm that predicts deviations of variables in algebraic loops that would otherwise be unknown.

## A Method for Handling Algebraic Loops

For physical systems, the system of equations of a model may contain negative feedback effects that would prohibit the use of a qualitative prediction scheme. However, the compensating effect does not reverse the initial deviation that enters the system of algebraic equations. Therefore, the prediction is not unknown, but the compensating effect should be recognized, and not affect the initial deviation in the qualitative framework.

### Tracking the Origin of Compensating Effects

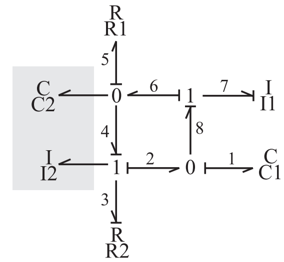
The solution algorithm is illustrated on the electrical circuit example from Figure 3. Figure 6(b), derived from the bond graph model of the circuit in Figure 6(a). The TCG is generated from the bond graph with the bond graph modeling tool HyBrSim. Note that a TCG that is derived from a bond graph model by use of the Sequential Causality Assignment Procedure (van Dijk 1994) is automatically in its canonical form.

In Figure 6 vertices $e_3$, $e_4$, $e_5$, $f_3$, $f_4$, and $f_5$ are part of an algebraic loop. Note that this loop has a negative gain, but no temporal, '$dt$', edges. Therefore, any straightforward propagation of a deviating value eventually causes all values along the loop to become unknown, which in turn affects predicted deviations in the rest of the model.

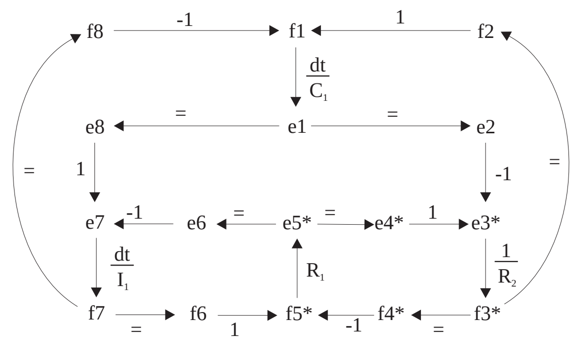
The solution consists of a special treatment of the vertices in an algebraic loop in the TCG. It is assumed here that those vertices have already been identified. HyBrSim automatically identifies and tags vertices that are part of an algebraic loop in the generated TCG, but this information can also be obtained by analyzing the equations. This algorithm is based on the observation that compensating effects in an algebraic loop in the physical model affect the rate of change of a root deviation, but not its sign. Therefore, in a qualitative sense, an initial '$+$' deviation is still '$+$' if the corresponding vertex is part of an algebraic loop. The compensating '$-$' effect generated by the algebraic loop only decreases the '$+$' magnitude. Conversely, an initial '$-$' deviation is still '$-$', though not as much when a compensating '$+$' value is propagated along the algebraic loop. This does not mean that an algebraic loop vertex can never become unknown. In the situation where a vertex in an algebraic loop is assigned a '$+$' value and a '$-$' deviation is propagated along a path other than the algebraic loop, the predicted deviation for the vertex still becomes unknown.

The algorithm must keep track of which vertex is the entry point to the algebraic loop of the initial deviation. If a deviation with opposing sign is assigned to a vertex, it is checked to determine whether the present deviation has the same algebraic loop entry point. If so, the present deviation is maintained and propagation along this branch is terminated. However, if the entry point differs, the vertex is assigned an unknown value and propagation of the unknown deviation continues. This is illustrated by analysis of a hypothesized candidate $C_1^-$ and observed variable $e_3$ in Figure 6. In case $C_1^-$, the deviation $e_1^+$ is propagated. Along $e_2$ it predicts a deviation $e_3^-(e_3)$, where $e_3$ in parentheses indicates this was the algebraic loop entry point for the vertex deviation. This value is propagated along the algebraic loop $f_3$, $f_4$, $f_5$, $e_5$, $e_4$, $e_3$, and because of the negative gain, predicts $e_3^+(e_3)$. This prediction conflicts with the already assigned deviation, and because both predictions have the same entry point, $e_3$, the initially assigned deviation is kept, i.e., $e_3^-(e_3)$, and propagation terminates.

It can also be seen that the algorithm must keep track of the entry point for *each order* of the propagated deviation from Figure 6 by analysis of hypothesized candidate $C_1^-$ and observed variable $f_5$. Along $e_2$, $e_3$, $f_3$, and $f_4$ the deviation $C_1^-$ predicts a deviation $f_5^+(e_3)$. Along $e_8$, $e_7$, $f_7$, $f_6$ the algorithm predicts $f_5^{\uparrow}(f_5)$. Now, when the $f_5^+$ deviation is propagated along $e_5$, $e_4$, $e_3$, $f_3$, $f_2$, $f_1$, $e_1$, $e_2$, $e_3$, $f_3$, $f_4$ a deviation $f_5^{\downarrow}(e_3)$ is predicted, and so an opposing effect is found. In this case, because the entry points for $f_5^{\downarrow}(e_3)$ is not the same as the entry point for $f_5^{\uparrow}(f_5)$ a conflict arises and the first order derivative of $f_5$ is unknown, i.e., $f_5^{+\cdot}$.




(a) Bond graph model with parasitic elements shown in shaded area (without causality).




(b) Temporal causal graph representation of the bond graph model with parasitic elements removed. Vertices that are in an algebraic loop are marked with an '$*$'.

Figure 6: Model representations for the electrical circuit in Figure 3.

step 0

| | | |
|---|---|---|
| **actual** | $e_1$: | $+ \cdot$ |
| $C_1-$ | $e_1$: | $+ \cdot \cdot$ |
| $R_1+$ | $e_1$: | $0 \cdot \cdot$ |
| $R_2+$ | $e_1$: | $0 \cdot \cdot$ |
| $I_1-$ | $e_1$: | $0 \cdot \cdot$ |

(a) Normal algorithm.

step 0

| | | |
|---|---|---|
| **actual** | $e_1$: | $+ \cdot$ |
| $C_1-$ | $e_1$: | $+ - \cdot$ |
| $R_1+$ | $e_1$: | $0 + \cdot$ |
| $R_2+$ | $e_1$: | $0 + \cdot$ |
| $I_1-$ | $e_1$: | $0 \cdot \cdot$ |

(b) Enhanced algorithm.

Figure 7: Comparison of signature generation with and without algebraic loop handling, for a positive deviation observed on vertex $e_1$.

## Algorithm

The algorithm is described in terms of graph propagation in the TCG, which must be given in its canonical form. The pseudo code for the algorithm is given in Algorithm 1. This algorithm is the behavior prediction phase of the fault isolation stage in TRANSCEND, and as such it supersedes Algorithm 2 in (Mosterman & Biswas 1999a). The algorithm handles the algebraic loop entry points by first determining of a successor vertex to the current vertex is in an algebraic loop and then determining if this is a new entry point or a continuation of a propagation through a loop. The enhanced prediction result is obtained in the conditional statements on lines 28 and 35 where it is determined whether an encountered opposing relation should be interpreted as a compensating effect within the same algebraic loop, or as an actual conflict.

As an example of the improved result a deviation on vertex $e_1$ is input to the prediction algorithm in TRANSCEND. A comparison of the prediction with and without special treatment of the algebraic loop shows that the prediction is improved, and that the signatures contain fewer unknown values. For an initially positive deviation on vertex $e_1$ in Figure 6, four fault candidates are hypothesized. Results are shown in Figure Note that '$\cdot$' indicates an unknown value. Also note that in general '$0$' predictions are not used for fault refutation, and, therefore, have less discriminative power. For three out of four hypothesized candidates, the first order prediction is known when algebraic loops are handled with the enhanced algorithm. First order predictions are unknown for all four candidates when the enhanced algorithm is not applied. Thus this provides richer information, and, therefore, more functional redundancy and improves the accuracy of fault isolation.

## Conclusions

Our system for monitoring, prediction, and fault isolation of abrupt faults in dynamic physical systems, TRANSCEND (Mosterman & Biswas 1999a), relies on a qualitative diagnosis model of the system under scrutiny. The level of detail of this model needs to be well balanced and adapted to the bandwidth of the measurement system. Behaviors that cannot be measured should not be included in the model as they complicate the diagnosis process.

Therefore, small time constants that are present in the physical system are abstracted away and this may result in models that contain *algebraic loops*, i.e., instantaneous causal feedback paths. If such a loop has a negative gain, straightforward propagation of deviations in qualitative, '$+$' and '$-$', terms results in conflicting predictions, and, therefore, observed future behavior cannot be compared with predicted future behavior and much discriminating power is lost.

This paper evaluates the effect of such instantaneous negative feedback effects and shows that because of the monotonous behavior of the loop transfer, qualitative predictions are not changed by a negative feedback effect. This feedback may affect the behavior quantitatively but not qualitatively, e.g., a '$+$' deviation is still '$+$', though less '$+$' than without the negative feedback.

This notion forms the basis of an algorithm to generate qualitative predictions based on a qualitative model with instantaneous negative feedback. It requires the edges of the feedback paths to be marked as being part of an algebraic loop. Keeping track of the entry point into the algebraic loop is crucial to correctly generate unknown future behaviors. In addition, this entry point needs to be kept separately for the order of each propagated deviation. The algorithm generates richer sets of predictions, and, therefore, results in better diagnosable systems.

## Acknowledgments

This work has been supported in part by a grant from Agilent Laboratories, Palo Alto, California. Gautam Biswas has been partially supported by DARPA SEC grant number F33615-99-C-3611.

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```
initialize v_predict[i, j] ← no_mark
add initial vertex, i.e., immediate consequence of the fault to list v_list
mark vertex 0^th order derivative with qualitative value
mark v_predict accordingly
while v_list is not empty do
  v_current ← the first vertex in v_list
  while v_current has successors not determined to sufficient order do
    if successor relation includes a time integral effect then
      increase current derivative order
    if derivative order ≤ maximum order and successor derivative is not conflict then
      if successor derivative is no_mark then
        successor derivative value ← new_value(current value, relation)
        if successor in algebraic loop then
          if v_current in algebraic loop then
            add v_current loop entry point to set of successor loop entry points
          else
            add v_current to set of successor loop entry points
      else if successor derivative value is opposite of current value then
        if relation is inverse then
          if successor in algebraic loop then
            if current loop entry point not in set of successor loop entry points then
              if v_current in algebraic loop then
                add current loop entry point to set of successor loop entry points
              else
                add v_current to set of successor loop entry points
        else
          if successor in algebraic loop then
            if current loop entry point not in set of successor loop entry points then
              successor derivative value ← conflict
          else
            successor derivative value ← conflict
      else
        if relation is inverse then
          if successor in algebraic loop then
            if current loop entry not in set of successor loop entry points then
              successor derivative value ← conflict
          else
            successor derivative value ← conflict
        else
          if successor in algebraic loop then
            if current loop entry point not in set of successor loop entry points then
              if v_current in algebraic loop then
                add current loop entry point to set of successor loop entry points
              else
                add v_current to set of successor loop entry points
      if attributes of successor changed then
        add the successor to end of v_list
for all vertex derivatives do
  if value = no_mark and any higher order derivative ≠ no_mark then
    replace no_mark with normal
  if value = conflict then
    replace conflict with no_mark
```

**Algorithm 1:** Predict future behavior for a fault

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